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Three Essays in Agent-Based Market Analysis

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The undersigned Lucia Milone, in his quality of doctoral candidate for a Ph.D. degree in Economics granted by the Università Ca' Foscari Venezia attests that the research exposed in this dissertation is original and that it has not been and it will not be used to pursue or attain any other academic degree of any level at any other academic institution, be it foreign or Italian.

a Francesco

alla famiglia:
l'unico vero rifugio,
il porto sicuro,
l'amore incondizionato

a Elisa, Fabrizio e tutti miei compagni di dottorato,
per aver diviso con me tutto questo

E un grazie speciale al Prof. Marco LiCalzi e al prof. Paolo Pellizzari... per i loro preziosi suggerimenti, per il loro evidente amore per la ricerca, per la fiducia e la loro sincera umanità

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Preface

In the recent decades, increasing attention has been devoted to the analysis of market protocols, with the aim to both analyze the performance of the market and to search for an improving design. This Thesis proposes a contribution in this direction. We concentrate on a specific protocol, the continuous double auction (CDA), given its diffusion in real market contexts.

We focus on two important concepts that may be considered as the key determinants of the results from exchange: the agents' behavior and the market protocol itself. In fact, we are convinced that the dynamic of the market is primarily affected and determined by the interaction between the strategy chosen and the rules that govern the exchange. Appear as immediately clear the high level of complexity of the system under consideration.

As main topics, we investigate the determinants of order flow in limit order markets, the trade-off between increase probability to trade and exploit the most from transaction, the effects of the risk of being picked off and cost of non execution on bid-ask spreads, the value of information.

In the literature we are able to find many attempts to analize the dynamic of a limit order market. Similar works differ in the type of approach that they choose to follow: empirical, experimental, theoretical and/or computational. The empirical analysis is really effective in taking a "picture" of the real situations in order to identify which are the different causality relationships between the forces that play a role in determining the dynamics of the market. However, it is not able to close the circle and give useful insights on what is not directly extractable from the data. The experimental analysis permits, thanks to the use of laboratory experiments, to deeply investigate the agents' behavior. However, since the experiment is conducted in a lab and not in real life, there is an intrinsic necessity to reduce complexity and simplify the framework. In addition, many possible external noises might arise by chance, such as an improper use of information available or a wrong choice of the sample. Similar considerations apply to theoretical papers. They are mainly devoted to search for equilibrium strategies; in order to do that and find close form solutions, they need to impose many assumptions that simplified the model. As a consequence, the results are valid to determine qualitatively the role played by the forces in the game but sometimes they cannot be applied to more complex frameworks that try to replicate reality. Finally, the computational approach is useful to overcome this limitations but it is insufficient to answer the other issues commented above. We can say that a complete analysis is possible if we allow interactions between these different approaches. We choose to follow adopt computational approach and refer to the agent-based methodology; agent-based models simulate the simultaneous interactions of multiple agents. Inspiration for modeling traders' behavior is taken by the findings in the empirical and experimental literature. Theoretical models are inspired by previous related works. This choice is motivated by the desire to study, re-create and predict complex phenomena.

In the first chapter (Market behavior under Zero Intelligence Traders and Price Awareness) we studies the consequences on market performance of different behavioral assumptions about agents' trading strategies. The investigation is conducted according to different criteria, such as efficiency, volume and price dispersion. We obtain the following results. Information disclosure is not always beneficial, depending on the trading behavior. The agents' behavior also play an important role in reducing the source of inefficiency in the market and determine the average volume of transactions. We use a zero-intelligence behavior as a benchmark; as a general result, a trading behavior that reacts to information available in the market performs better than a completely random choice.

In the second chapter (Allocative Efficiency and Traders' Protection under Zero Intelligence Behavior) we studies the continuous double auction from the point of view of market engineering: we tweak a resampling rule often used for this exchange protocol and search for an improved design. We assume zero intelligence trading as a lower bound for more robust behavioral rules and look at allocative efficiency, as well as three subordinate performance criteria: mean spread, cancellation rate, and traders protection. This latter notion measures the ability of a protocol to help traders capture their share of the competitive equilibrium profits. We consider two families of resampling rules and obtain the following results. Full resampling is not necessary to attain high allocative efficiency, but fine-tuning the resampling rate is important. The best allocative performances are similar across the two families. However, if the market designer adds any of the other three criteria as a subordinate goal, then a resampling rule based on a price band around the best quotes is superior.

One of the most interesting issue in the recent literature on limit order market deals with the attempt to describe the traders' strategies in equilibrium. An analytically tractable model that leads to a close form solution requires important simplifications that restrict the explanation power of the model itself. In order to overcome this limitation a large use of computational analysis emerges in the recent literature; this will be the approach used in this paper. We rely our analysis on the paper by Foucault et al. (2005) that develops (under strict assumptions) a dynamic model of a limit order market in which agents that differ in their level of impatience interact. The authors derive analytically equilibrium strategy results and identify the two variables that are the key determinants of the market dynamics: the proportion of patient traders and the order arrival rate. The aim of the last chapter (Learning Cancellation Strategies in a Continuous Double Auction Market) is twofold. Firstly, a genetic algorithm is proposed and computationally implemented in order to test if learning agents' behavior converges to the same equilibrium strategy that is derived for traders that play strategically. Secondly, we propose a model that relaxes one of the assumptions in Foucault et al. (2005): traders are allowed to cancel a placed order. We study how strategies are affected by the new rule. Results with and without cancellation are also compared (and conclusions are drawn) on the basis of different performance criteria both at individual and aggregate level, such as individual profits and efficiency of the market as a whole. Direction of results are mixed.

Chapter 1

Market Behaviour under Zero Intelligence Traders and Price Awareness

1.1 Introduction

Market protocol and trading behavior may influence economic performance. There is a growing literature that has focused on mechanism evaluation and design with the aim to try to isolate the effect of market rules, market environment and agent behavior on market performance.

Information disclosure is commonly recognized as a fundamental issue in the design of markets. Despite increasing research attention, there is still little consensus on how quantity and type of information available influence market behavior; results are mixed and no definitive answer has emerged.

In this paper, we study the consequences of (partial) pre-trade quote disclosure on market performance according to different criteria, such as efficiency, transaction volume and average spread. I follow the spirit of the seminal paper by Gode and Sunder (1993), commonly recognized as a benchmark in the study of the algorithms for continuous double auction markets. In order to keep the model as simple as possible, I replace human traders by Zero-Intelligence programs that submit random bids and offers following different specified rules of behavior.

In particular, my attempt is to try to find an answer to the following questions:

1. How does price awareness (and, in particular, pre-trade information disclosure) affect CDA market outcomes?

2. Do agents' differences in trading behavior matter? And, if yes, how do they influence performance of the markets in two different scenarios with and without information disclosure?

The *design* issue in (1) is addressed by comparing CDA outcomes in two different frameworks; in the closed-book scenario there is no public or private information accessible, in the open-book scenario private information on the outstanding ask and bid is made available. The issue in (2) deals with the *evaluation* of the role played by three behavioral rules. See Section 1.2.3.

I present results from a simulation study that leads to the following main findings. Price awareness may affect CDA outcomes but its positive effect is strictly related to the trading strategies that characterize the agents' behavior; alone, it is not always enough. Traders that behave more aggressively and/or more often as a truth-teller, submit offers in favor of the market and help to increase the total level of allocative efficiency achieved by the institution. *Pure zero-intelligent* traders are usually overperformed by agents that follow different strategies, independently from information available.

The paper is organized as follow. Section 1.2 presents the model. It focuses on description of the behavioral assumptions and illustrates the simulation setup. Section 1.3 lists the outcome variables and summarizes results. Section 1.4 concludes, offers interpretations and suggests future possible directions of research.

1.2 The Model

Following Smith (1982), we identify three distinct components for our (simulated) exchange markets. Section 1.2.1 describes the set of agents' preferences and endowments (environment), Section 1.2.2 summarizes the institutional structure that governes exchange (protocol) and Section 1.2.3 provides detailed description about the assumptions made on traders' behavior (trading strategies).

1.2.1 The environment

We consider an economy with N traders (we set $n = 1000$); every agent is initially endowed with a zero (buyer) or a single (seller) unit of one good. Each buyer i receives a private valuation v_i; similarly, each seller j receives a private cost c_j. At the beginning of each trading session[1], Both valuations and costs are drawn from the same uniform distribution on [0,1]. There is no cost of delaying transaction; hence, utility of players are simply defined by the difference between their private valuations and the transaction price (if buyer, $v_i - p$) or between the transaction price and their private cost (if seller, $p - c_j$). Payoffs from the trading session equal zero if no trade occurs.

1.2.2 The protocol

I analyze the performance of a continuous double auction sequential protocol, given its relevance in real contexts. Each agent can exchange at most one unit of the good at a time. Price offers to buy and to sell are submitted in a randomly selected order. Two order books list asks and bids sorted from the lowest to the highest and from the highest to the lowest, respectively. The lowest (highest) offer in the ask (bid) queue is called *best ask* (*best bid*). As soon as one buyer (seller) offers a price that is greater (lower) or equal to the lowest sell (highest buy) offer, the CDA matches these offers; the transaction price is equal to one of the outstanding offers, depending on which of them come first. Then, these orders are removed from the books and the agents become inactive for the rest of the trading session. The books are completely cleared at the end of each session[2], when all agents have placed their offer and summary data from the period are stored.

1.2.3 Trading Strategies

Following in spirit the seminal paper by Gode and Sunder (1993), commonly recognized as a benchmark in the study of the algorithms for continuous double auction markets, I replace human traders by Zero-Intelligence programs that submit random offers. The impossibility to control the trading behavior of individuals is at the root of the choice of this approach; from an *evaluation* perspective, we try to investigate if a certain market protocol is able to achieve a decent level of performance without considering more sophisticated behavioral rules, such as a profit-oriented trader behavior.

[1] See appendix A

[2] In this respect, this setup differs from the one in Gode and Sunder (1993). In fact, as pointed out in Licalzi et al. (2008), they release their conjectures on the assumption for which books are cleared after each transaction and this significantly increases the level of efficiency achieved by the market.

We kept fixed throughout this paper the individual rationality assumption for ZI-C traders (Gode and Sunder 1993, p.123). Agents are not allowed to trade at a loss; hence, they submit only offers below their valuations (if buyer) or above their costs (if seller).

The Make-Take Tradeoff and The Order Aggressiveness. In a continuous limit order market, a trader has to decide between placing a market or a limit order. In the first case he behaves as a price taker since execution is immediate at a price equal to the outstanding offer. In the second case he becomes a price maker deciding to sustain the risk of no execution waiting on the book for a more favourable transaction in the future. This tradeoff is deeply investigate in the recent literature[3].

At the same time, placing an order he has to face a second tradeoff between trying to exploit the most from transaction and increase the probability to trade. Such a tradeoff might be viewed as a measure of order aggressiveness.

We push aside the first one (that is behind the scope of this paper) treating the choice between limit and market orders as endogenous and decide to model the second one introducing an individual aggressiveness parameter γ_i randomly drawn from a uniform distribution on $[0, 1]$.

A trader who wants to place an offer is supposed to formulate a conjecture on the current spread that will drive his decision. Such a conjecture might be based on some knowledge about the state of the book (see below) or, when no information is available, it might reflect a worst case scenario in which the spread is the largest possible[4].

Empirical evidence shows how a large fraction of the order placements improves upon the best bid or ask quote; eg., Biais et al.(1995, p.1657) "find that the flow of order placements is concentrated at and inside the bid-ask quote". On the base of this consideration, kept fixed the assumption of random behavior, we move from a pure ZI behavior switching from the use of the uniform distribution in drawing offers to the use of a triangular distribution that assign a higher probability to prices inside the spread, according to a reference point defined as follow

$$rp = b + [(a - b)\gamma_i]$$

where b and a are the best bid and the best ask (respectively) currently listed on the books[5].

Modelling Information. In addition to the *evaluation* scope of the paper described above, from a *design* perspective we try to evaluate if -ceteris paribus- perfomance outcomes might be improved by the introduction of a specific market rule concerning information available. In order to do that, I study two different scenarios (in isolation). In one (*closed* book), no information, public or private, is available. In order to define their trading strategies, agents behave on the basis of the worst possible scenario as mentioned above and in footnote 4. In the other (*open* book), I introduce in the model a market maker who communicates a pre-trade private advice concerning the outstanding bid and ask to the trader that is in charge to submit his offer. Information disclosure has to be defined as "partial", since the source of information made available concerns only the best (lowest) ask and the best (highest) bid currently listed on the book; nothing is known about other asks and bids in the queue. Note that no formal differences in beliefs and behaviour exist between the case in which there is no information at all and the case in which information is allowed but books are empty.

Behavioural Rules. We propose two different types of traders. All these trading strategies do not respond to an intention to mimic human behavior. The aim is to propose a variation of the

[3]See Preface for a brief literary review on this topic.

[4]ie, best ask equals 1 and best bid equals 0. Remember that, without loss of generality, prices are assumed to lie in $[0, 1]$

[5]As anticipated above, the parameter γ_i deals with the mentioned tradeoff since it determines the distance from the best outstanding offer on the opposite side of the market.

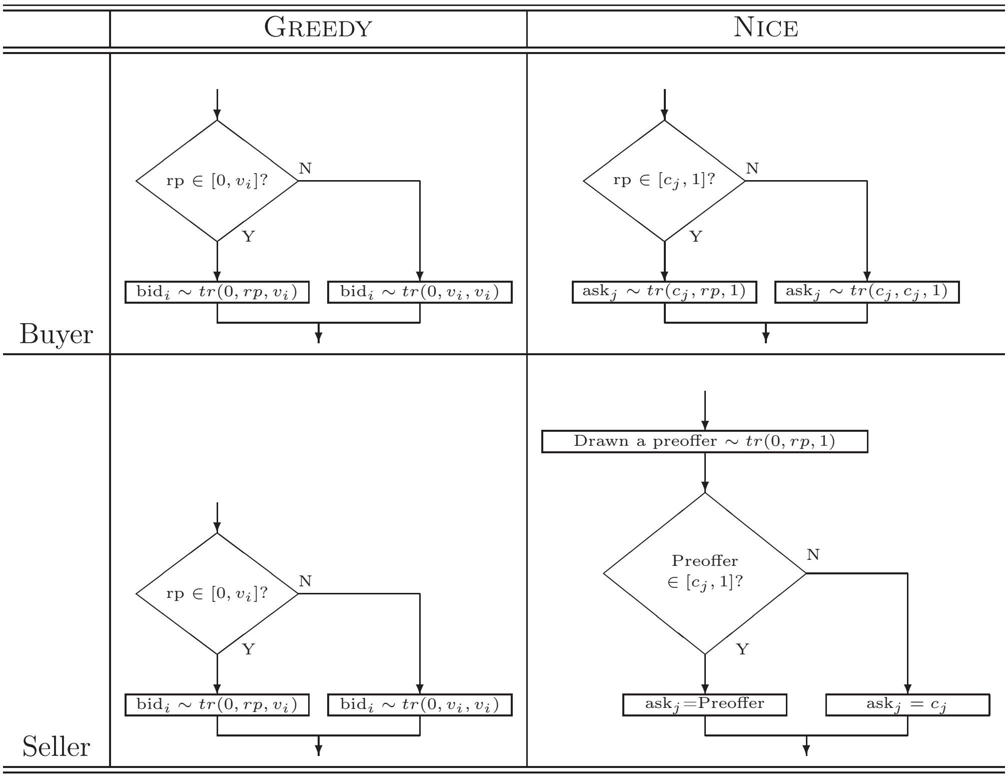


Table 1.1: Reference point (rp): $b + [(a - b)\gamma_i]$ [OPEN-BOOK], γ_i [CLOSED-BOOK]

ZI trading in which agents continue to behave randomly but react in some way to beliefs on price or to information available on it (price awareness). Both these rules model traders that submit improving offers with higher probability, for the reasons mentioned above; the main difference consists in the way in which they deal with the no loss constraint. In one case (henceforth, *greedy*) individual rationality is fulfilled ex-ante; this is the simplest natural extension of pure ZI behavior. A buyer trades over the support $[0, v_i]$ using a triangular distribution to draw his offer in order to increase his probability to choose a price in the spread (and trade). If the reference point doesn't lie in the feasible set and it cannot be used as the mode of the triangular distribution, their own valuation is used instead. In the second case (henceforth, *nice*) we allow to relax the assumption for which the support has an upper (lower) side corresponding to the private value (cost); individual rationality is fulfilled ex post. The random drawn is done on the entire interval $[0, 1]$ and, when the random offer is not feasible, the trader decides to play "in favor of the market" and places his own valuation. Hence, in the latter case (nice), the probability attached to non-feasible random drawn puts mass on the true value; in the former (greedy) such probability is split over the entire (feasible) support. This difference, jointly with the relative value of their own valuation, will be crucial in interpreting the results. Seller follow by symmetry.

1.3 Experimental Design and Results

We are interested in study the CDA market protocol with respect to different performance criteria and different behavioral assumptions in isolation (i.e., we run 500 rounds of simulation for each pair of information setup and trading behavior separately).

Allocative Efficiency. We start measuring allocative efficiency by the ratio of the gains from trade by all traders to the maximum total profit that could have been earned by all traders in equilibrium (i.e., Competitive outcome or Marshallian Profits[6]), formally defined as in Zhan et al. (2002, p.678).

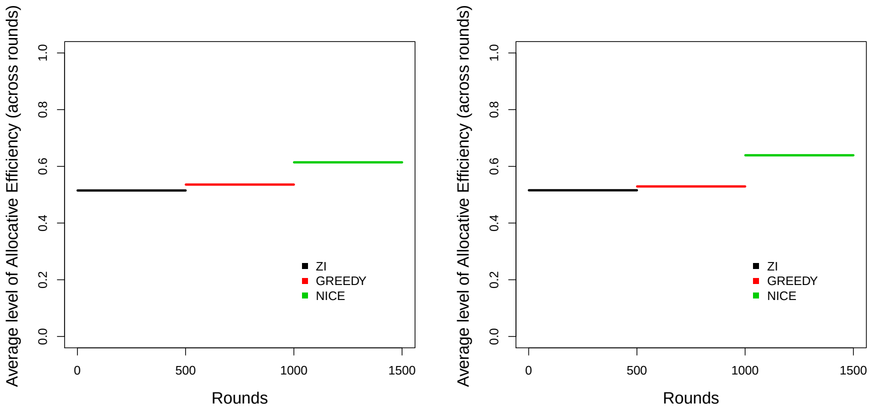


Figure 1.1: Allocative Efficiency without and with information release

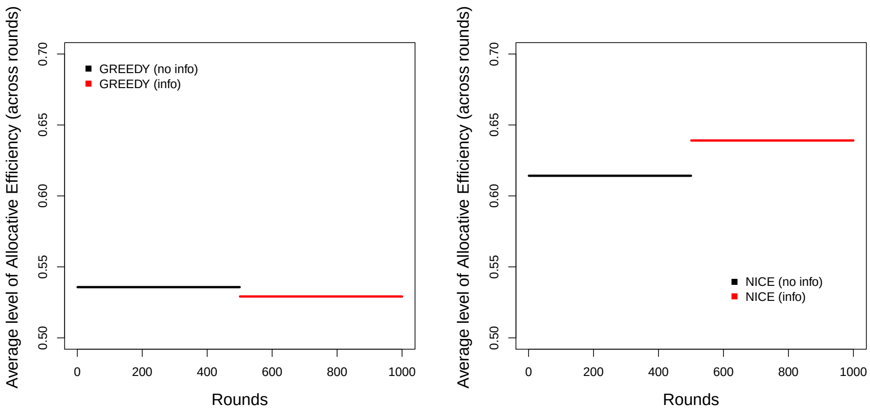


Figure 1.2: Allocative Efficiency - Comparison

The left-hand side of Figure 1.1 reports data on average level of allocative efficiency of the CDA market for 500 independent rounds when no information is available. The average allocative efficiency levels are 0.515 (ZI), 0.536 (greedy), 0.614 (nice). The right-hand side reports the data on average level of allocative efficiency of the CDA market for 500 independent rounds when information is available. The average allocative efficiency levels are 0.515 (ZI), 0.529 (greedy), 0.639 (nice).

A first significative observation is that ZI pure behavior is systematically overperformed. This means that a minimum reaction to a beliefs (or knowledge) about the state of the book increases (significantly) the aggregate performance of the market. Figure 1.2 allow us to investigate the role of the information release. Results are mixed; the market performs better under nice behavior but looses in efficiency under greedy behavior.

[6]"*through the Marshallian path, the buyer whose redemption value is the highest trades with the seller whose unit cost is the lowest; next, the two second ranked traders trade; and so forth*", Zhan et al. (2002)

The Wilcoxon signed-rank test rejects the hypothesis that the means are equal at a level of significance of 10^{-3} for all the possible pair comparisons.

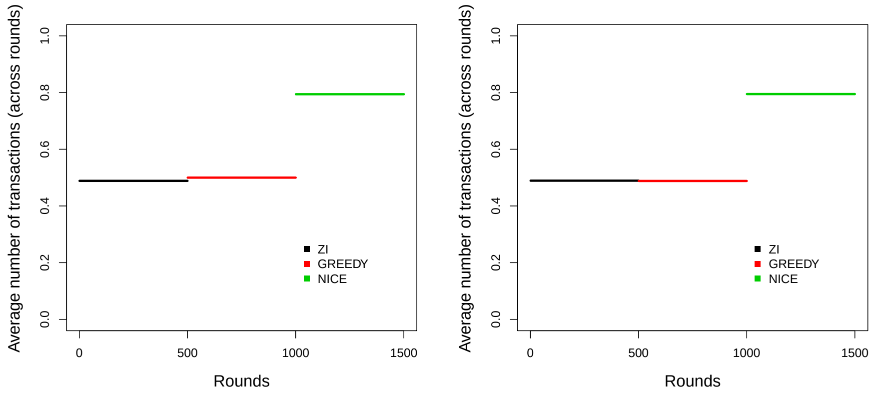


Figure 1.3: Volume

Volume. It is defined as the number of transaction per trading session; it is normalized using CE Volume equal to one[7]. The left-hand side of Figure 1.3 reports data on average level of volume of the CDA market for 500 independent rounds when no information is available. The average volume levels are 0.489 (ZI), 0.450 (greedy), 0.794 (nice). The right-hand side reports the data on average level of volume of the CDA market for 500 independent rounds when information is available. The average volume levels are 0.489 (ZI), 0.488 (greedy), 0.795 (nice).

	ZI	*nice-closed*	*nice-open*	*greedy-closed*
nice-closed	0.0000*			
nice-open	0.0000*	**0.808**		
greedy-closed	0.0000*	0.0000*	0.0000*	
greedy-open	**0.791**	0.0000*	0.0000*	0.0000*

Table 1.2: **Volume** - * *p-values* of the Wilcoxon test, *significant at the 0.05 level

The volume increases under information, independently from the behavioral assumption considered. Moreover, the difference between transaction volume in the open and the closed book scenario under nice behavior is not significant. Furthermore, this is the only case throughout the paper in which pure ZI behavior is not overperformed by greedy behavior with information; their performances are undistinguishable.

1.3.1 Allocative Efficiency: who win and who looses? The sources of inefficiency

Results about efficiency and volume lead to different questions. The first, why different behavioral trading strategies conduct to opposite results in terms of overall efficiency? Secondly, the increase in allocative efficiency due to information is not a good result *per se*; we deal with the interesting issue to analyze how allocative efficiency might be decomposed and how information affects the effect of each of these components on the allocative efficiency itself. Finally, if there is no significant difference in transaction volume between an open and a closed book scenario under

[7] To facilitate computations, in this setup CE volume is equal to a half of the number of intramarginal traders.

nice behavior how can we justify significance of the difference in levels of allocative efficiency achieved? Before to speculate over an answer, we need some preliminary definitions.

Let p^* be the market-clearing price. This equilibrium price is defined by the intersection between demand and supply functions obtained sorting buyers' valuations from the highest to the lowest and sellers' costs from the lowest to the highest. Given the fact that stepwise supply and demand functions are used, we decide to define the equilibrium price as the midpoint in the (possible) range of CE prices. Buyers (sellers) with a private valuation (cost) greater (lower) than p^* are called *intramarginal* traders (IB, IS). All other traders are *extramarginal* (EB, ES).

Distributive Criteria. The Allocative Efficiency can be decomposed into three different components as described in Table 1.3. The realized fair shares (henceforth, *rfs*) are the shares of the competitive outcome[8] that are realized by intramarginal traders during a transaction. When two traders transact at a price different from p^* -with respect to the individual competitive outcome- one of them sustains a loss, the other exploits a gain. We focus on gains since they are components of the level of allocative efficiency achieved. If it is exploited by an intramarginals we will cal it *gainA*, otherwise we label it *gainB*.

	IB $\bigoplus$ IS	IB$\bigoplus$ES	EB$\bigoplus$IS	EB$\bigoplus$ES
$p > p^*$	rfs=$(v_i - p) + (p^* - c_j)$ gainA=$p - p^*$	rfs=$v_i - p$ gainB=$p - c_j$	–	–
$p < p^*$	rfs=$(v_i - p^*) + (p - c_j)$ gainA=$p^* - p$	–	rfs=$p - c_j$ gainB=$v_i - p$	–

Table 1.3: IB: intermarginal buyer, IS: intermarginal seller, EB: extramarginal buyer, ES: extramarginal seller, p: transaction price, p^*: equilibrium price, v_i: buyer's value, c_j: seller's cost, rfs: realized fair-shares, gainA: gains for intermarginals, gainB: gains for extramarginals

We normalize average allocative efficiency to one in order to study how its composition varies as a consequences of information released. Figure 1.6 shows that the impact of the realized fair shares over the allocative efficiency achieved overall under both greedy and nice behavior increase. The main difference between the two behavioral assumptions is that the effects of both gains of extramarginal and intramarginal traders derease more under greedy behavior. All results are statistically significant.

	ZI	GREEDY (no info)	GREEDY (info)	NICE (no info)	NICE (info)
mean rfs	0.459	0.4872	0.4918	0.5559	0.5838
mean gainA	0.0505	0.0447	0.0349	0.0538	0.0513
mean gainB	0.0054	0.0038	0.0025	0.0045	0.004

Table 1.4: Shares of the Allocative Efficiency Outcome

Sources of inefficiency. We also compute[9] an alternative measure of a source of inefficiency due to intramarginals traders that fail to trade. It is called $VI - inefficiency$ and it is given by the ratio between the loss of total surplus over the marshallian profits[10]. It can be formulated

[8]It is the shares of CE outcome attributed to an intramarginal trader that transact at p^*; competitive share outcome of extramarginal is zero by definition. See Licalzi et al. (2009) for a formal definition.

[9]Following the approach proposed by Zhan and Friedman (2007)

[10]Zhan et al. (2007) demonstrate that the realized CDA surplus (i.e., allocative efficiencylevel achieved in the market) plus the loss due to VI-inefficiency and plus the loss related to trades that involve an extramarginal trader sum up to the marshallian profits, normalized to 1.

as follow:

$$VI - inefficiency = (\sum_{i \in IMBN} (v_i - p^*) + \sum_{j \in IMSN} (p^* - c_j))/\sum_{i,j=1}^{q^*} (v_i - c_j) \quad (1.1)$$

where $IMBN$ and $IMSN$ are (respectively) the intramarginal buyers and sellers that fail to trade, q^* and p^* are the equilibrium quantity and price.

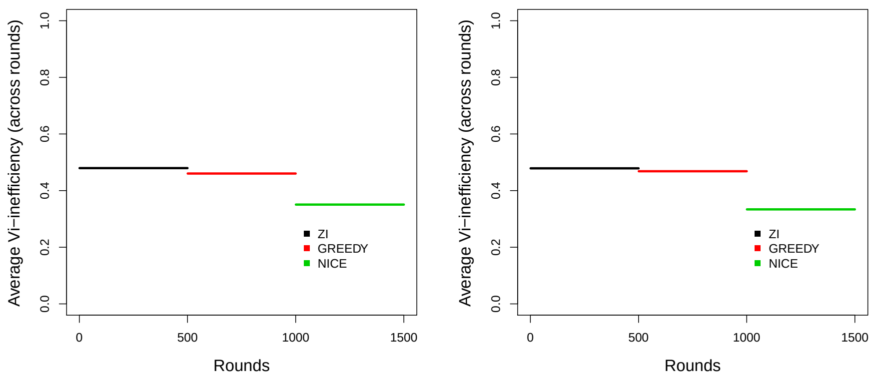


Figure 1.4: VI-inefficiency: intramarginals that fail to trade

The left-hand side of Figure 1.4 reports data on average level of VI-inefficiency of the CDA market for 500 independent rounds when no information is available. The average VI-inefficiency levels are 0.479 (ZI), 0.460 (greedy), 0.351 (nice). The right-hand side reports the data on average level of VI-inefficiency of the CDA market for 500 independent rounds when information is available. The average VI-inefficiency levels are 0.479 (ZI), 0.468(greedy), 0.334 (nice). The Wilcoxon signed-rank test rejects the hypothesis that the means are equal at a level of significance of 10^{-3} for all the possible pair comparisons.

VI-inefficiency for *nice* traders is lower in the open book scenario and this fact can be used as a partial explanation for an higher level of allocative efficiency. Note also that VI-inefficiency is lower under nice behavior than under greedy behavior independently from information availability. This result leads to the conclusion that trades between intramarginal traders is made easier by a more aggressive behavior in favor of the market and this supports the intuition that better performance of *nice* traders versus *greedy* traders is driven not only by an increase in transaction volume but also by a rise in the number of "fair" transaction[11].

There are a couple of observations to be made.

In absolute terms the realized fraction of the competitive outcome (as well as the overall allocative efficiency) is greater under nice behavior. Moreover, in all the cases (ie, independently

[11] A robustness test has been conducted. *EMI-inefficiency* is defined by Zhan et al. (2007) as the total loss due to transactions that involve extramarginal traders; namely, it is equal to $p^* - v_i$ when an extramarginal buyer trades with an intramarginal seller and to $c_j - p^*$ when an extramarginal seller trades with an intramarginal buyer. Has been proved that (normalized) allocative efficiency in the CDA market = 1 - VI*inefficiency* - EMI*inefficiency* (see footnote 6). As a consequence, when VI*inefficiency* lowers, the allocative efficiency of the market increases if EMI*inefficiency* decreases or remains constant; if it increases, the total allocative efficiency increases if $\triangle EMI < \triangle VI$, remains constant if $\triangle EMI = \triangle VI$ and decreases if $\triangle EMI > \triangle VI$. Situation when VI*inefficiency* increases follows by symmetry. Results show that EMI*inefficiency* is lower in an open book scenario both for *nice* and for *greedy* traders; significance in differences is supported by a Wilcoxon test for every level of confidence. Allocative efficiency for *nice* traders is higher in an open book scenario; allocative efficiency for *greedy* traders is higher in a closed book scenario since $\triangle EMI$ is significantly lower than $\triangle VI$. This confirms that "identity" of the traders matter.

from the agents' behavior) the release of information lower (in absolute value) this measure and increases significantly some source of VI-inefficiency. This suggests that extramarginal traders take advantage from information available.

Relative to the impact of the different components of the allocative efficiency on the efficiency itself (hence, in relative terms), the results above suggest another possible reason for the fact that information relesed increases allocative efficiency under nice behavior but not under greedy behavior. It might be given by an higher number of transaction that involved intramarginal with high valuations (low costs) under nice behavior. Note that this does not mean (necessarily) that more intramarginal trades. Let us observe that when VI-inefficiency increases it means that (a) more intramarginals fails to trade or (b) less or an equal number of intramarginal fails to trade but agents that fail have high value or low cost. In other words, according to what is shown in Figure 1.6, it might be the case in which under greedy behavior we observe less trades (because the bid is on average less aggressive) but intramarginals agents involved in transactions under greedy behavior show low values or high costs (since under greedy behavior they have less chance to trade).

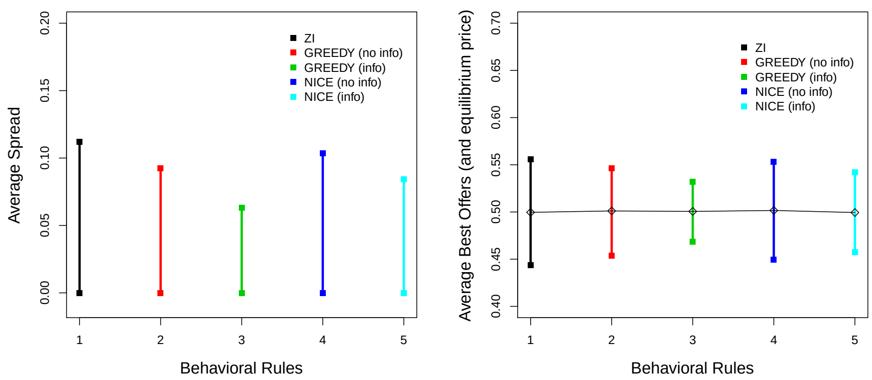


Figure 1.5: Average Spread and Average Best Offers under different Behavioral Assumption

Average Spread. The last performance criteria analyzed in this paper is the average spreads across 500 rounds of simulations. Table 1.5 shows that the spread is smaller under greedy behavior and information contributes a lot to decrease it.

By visual direct inspection we can assert that information narrows the spread independently from the behavioral rules taken into consideration. This results is intuitive since offers, with these modifications of the pure ZI behavior, are concentrated in and around the spread.

	ZI	GREEDY (no info)	GREEDY (info)	NICE (no info)	NICE (info)
Average Spread	0.112	0.0925	0.0633	0.1037	0.0843

Table 1.5: Average Spreads

1.4 Conclusions (and Open Scenarios)

This investigation is dealing with two different issues. On one side it refers to a *design* problem: is or is not convenient to make information on outstanding bid and ask privately available? On the other side it makes an attempt to evaluate the role played by different trading behaviors in different scenarios.

Simulation evidence leads to the following main conclusions.

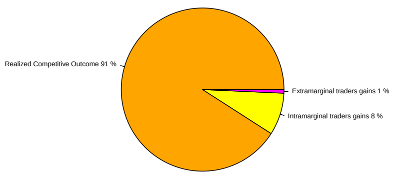
- ZI are almost always overfperformed
- Information release:
  - might both increase or decrease the level of allocative efficiency achieved, dependently from the behavioral rule under consideration. Behavior matters.
  - increase the rfs and decrease the extragains (with respect to the competitive outcome) of both intramarginal (gainA) and extramarginal traders (gainB). This happens independently from the trading strategies considered; moreover, the level of magnitude is different under different behavioral assumptions. Information matters.
  - Transaction volume is higher with nice behavior but this is not enough to justify entirely the higher level of allocative efficiency achieved. Identity of the traders matters.

Open Scenarios. There are many possible future direction for research; in particular, I would like to signal three relevant extensions of this work.

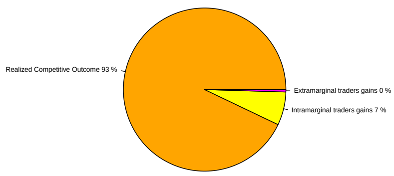
The first is to investigate the role of γ_i, in particular the way in which it is distributed across simulations. Our intuition is that the joint effect of the value of γ_i and the type of traders to which it is associated (intramarginal-high value, intramarginal-low value, extramarginal, etc.) plays a crucial role

Secondly, this setup is organized over a single trading session, where all agents participate. At this stage there is no role for dynamics and each round works independently from each other. It could be interesting to investigate whether more realistic assumptions about agents' behavior affect results. My claim is that learning (on the basis of past history and past trades) in a dynamic context might significantly help in increasing efficiency moving trades closer to the equilibrium price (probably after an initial adjustment).

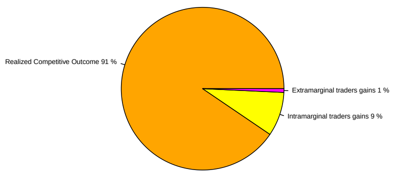
Finally, another possible interesting direction for future reasearch could be to try to investigate deeply the balance between nice and greedy behavior of the agents in the market and try to figure out which is the right proportion between them that permits to increase efficiency.



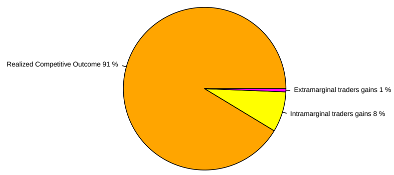
(a) Greedy (no info)



(b) Greedy (info)



(c) Nice (no info)



(d) Nice (info)

Figure 1.6: Average Fair Shares

Chapter 2

Allocative Efficiency and Traders' Protection Under Zero Intelligence Behaviour

2.1 Introduction

In a seminal paper, Gode and Sunder (1993) define a *zero intelligence* (ZI) trader as an agent that "has no intelligence, does not seek or maximize profits, and does not observe, remember or learn." (p. 121) Such zero intelligence assumption is not meant to provide a descriptive model of individual behavior: on the contrary, it is used to instantiate severe cognitive limitations that should impede the overall performance of the market.

A ZI agent is usually modeled as a robot player that submits random offers in an exchange market, under a minimal assumption of *individual rationality*: he never takes actions that can lead him to trade at prices below his cost or above his valuation. To the best of our knowledge, the first (unnamed) use of individually rational zero intelligence behavior in economic theory goes back to the B-process studied in Hurwicz, Radner and Reiter (1975); they prove that, if the market protocol allows unlimited retrading, an economy without externalities must converge to a Pareto optimal allocation. Throughout this paper, we take the postulate of individual rationality for granted and speak simply of zero intelligence behavior.

By simulating the actions of (individually rational) ZI traders in a continuous double auction, Gode and Sunder (1993) achieved levels of allocative efficiency similar to the outcomes generated by human subjects in laboratory experiments. This was used to argue that the main feature leading to a high allocative efficiency is the market protocol rather than the trading strategies used by the agents. More boldly put, the market can substitute for the cognitive limitations of the individuals. This conclusion has spawned a large literature venturing in different directions, including experimental economics and computer science; see Duffy (2006) for a thorough survey.

In general, it is widely acknowledged that the interpretation of Gode and Sunder's results is controversial. Gjerstad and Shachat (2007) emphasize the role of individual rationality as the key crucial assumption for allocative efficiency. A recurrent theme is the robustness of Gode and Sunder's conclusion: it is not difficult to produce environments where the allocative efficiency reached by ZI agents badly underperforms humans' results; see e.g. Brewer et al. (2002). On the other hand, the literature has shown that even minor improvements to the basic ZI trading rules suffice to achieve convergence to the competitive equilibrium; see Cliff and Bruten (1997) or Crockett et al. (2008).

Clearly, humans' cognitive abilities provide more leverage than zero intelligence. Therefore, we do not expect that the performance of a market protocol in an environment populated with

ZI agents would be the same as with human traders. On the other hand, it is not unreasonable to postulate that the performance of a market protocol under a ZI behavioral assumption provides a plausible benchmark for its evaluation in view of use by human subjects. In his recent discussion of the "market-centered theory of computational economics", Mirowski (2007) attributes to the zero intelligence literature the computational insight that human cognitive abilities can be ignored under controlled circumstances to focus on the causal capacities of the market protocols. In a similar vein, Sunder (2004, p. 521) states that "[w]hen seen as human artifacts, a science of markets need not be built from the science of individual behavior." The implicit claim is that we may learn about the properties of markets regardless of the agents operating in them.

Our viewpoint is the following. Market protocols are complex artifacts; see Subrahmaniam and Talukdar (2004). Their design requires a special attention to details and minutiæ that partakes of the engineering attitude advocated in Roth (2002): we need to complement theory with experiments and computational simulations. In order to make fine-grained comparisons among different protocols, it is necessary to pin down agents' behavior to a simple standard. The ZI assumption provides a rough simplification under which it is possible to evaluate markets protocols *in silico* in order to select more promising designs.

The purpose of this paper is to exemplify this approach with regard to the continuous double auction. We replicate the results produced in Gode and Sunder (1993) and show that they depend crucially on a subtle assumption about the market protocol that has gone unnoticed in the literature. They write: "There are several variations of the double auction. We made three choices to simplify our implementation of the double auction. Each bid, ask, and transaction was valid for a single unit. *A transaction canceled any unaccepted bids and offers.* Finally, when a bid and a ask crossed, the transaction price was equal to the earlier of the two." (p. 122, emphasis added.) As discussed below, the second emphasized assumption forces a frequent resampling of agents' quotes that is crucial (under zero intelligence) for allocative efficiency. We call this assumption *full resampling*: speaking figuratively, it mandates to toss away the book after each transaction. This seems both unrealistic and unpalatable for practical market design.

We are thus left to ask whether Gode and Sunder's implementation of the continuous double auction is a promising design. Taking the viewpoint of a market designer who is interested in allocative efficiency, we evaluate alternative market protocols that enforce different degrees of resampling. As it turns out, the assumption of full resampling is not necessary to achieve very high allocative efficiency under zero intelligence. There is a continuum of protocols, ordered by the strength of their resampling properties, that attain comparable levels of efficiency. This makes it possible to search for more effective protocols than Gode and Sunder's (1993) without renouncing the objective of allocative efficiency.

To refine our selection, we introduce a subordinate criterion. While allocative efficiency is desirable from an aggregate point of view, a single trader in an exchange market is likely to be more interested in getting a fair deal. Let the *competitive share* of a trader be the profit he would make by transacting at the (competitive) equilibrium price. A market protocol that is more effective in helping traders realize their competitive share offers a superior *traders' protection.* Therefore, we study the traders' protection offered by comparably efficient market protocols to devise a practical and simple implementation of the continuous double auction.

We study two families of resampling rules and identify a design that delivers a significant improvement over Gode and Sunder's (1993). However, barring an experimental validation with human subjects, we can only claim that the lower bounds on its performance with regard to both allocative efficiency and traders' protection are higher under zero intelligence.

The organization of the paper is the following. Section 2.2 describes the model used in our computational experiments and clarifies some technical details in the implementation of Gode and Sunder's (1993) continuous double auction. The zero intelligence assumption is maintained throughout the paper. Section 2.3 proves that some (possibly not full) resampling is a necessary condition for allocative efficiency in the continuous double auction; see also LiCalzi and Pellizzari (2008). Section 2.4 shows that partial resampling may be sufficient for allocative efficiency. Based on this result, we study a family of resampling rules for the implementation of the continuous double auction protocol that modulates the probability of clearing the book after a transaction. Several rules within this family attain comparable levels of allocative efficiencies. Section 2.5 introduces an alternative way to effect resampling that is based on the use of a price band. Section 2.6 compares the alternatives and argues that the second method delivers a better protocol. Section 2.7 recapitulates our conclusions.

2.2 The Model

We use a setup very similar[1] to Gode and Sunder (1993), who consider a simple exchange economy. Following Smith (1982), we identify three distinct components for our (simulated) exchange markets. The environment in Sect. 2.2.1 describes the general characteristics of our simulated economy, including agents' preferences and endowments. Section 2.2.2 specifies how agents make decisions and take actions under the zero intelligence assumption. This behavioral rule is kept fixed throughout this paper to let us concentrate on the effects of tweaking the market design. Finally, Sect. 2.2.3 gives a detailed description of the institutional details that form the protocol of a continuous double auction (and its variants) which regulate the exchange.

2.2.1 The Environment

There is an economy with a number n of traders, who can exchange single units of a generic good. (We set $n = 40, 200, 1000$ to look at size effects.) Each agent is initialized to be a seller or a buyer with equal probability. Each seller i is endowed with one unit of the good for which he has a private cost c_i that is independently drawn from the uniform distribution on $[0, 1]$. Each buyer j holds no units and has a private valuation v_j for one unit of the good that is independently drawn from the uniform distribution on $[0, 1]$. Without loss of generality, prices are assumed to lie in $[0, 1]$.

2.2.2 Zero Intelligence Behavior

Zero intelligence reduces behavior to a very simple rule: when requested a quote for an order, a trader draws a price from a random distribution (usually taken to be uniform). We assume that traders' behavior abides by *individual rationality*: each seller i is willing to sell his unit at a price $p \geq c_i$ and each buyer j is willing to buy one unit at a price $p \leq v_j$. Therefore, throughout this paper, the zero intelligence assumption pins down behavior as follows: when requested a quote for an order, a seller i provides an ask price that is an independent draw from the uniform distribution on $[c_i, 1]$; similarly, a buyer j makes a bid that is an independent draw from the uniform distribution on $[0, v_j]$. This behavioral rule is called ZI-C in Gode and Sunder (1993).

Note that the only action requested by an agent is to issue a quote: it is left to the market to process traders' quotes and execute transactions on their behalf. This is consistent with an approach of market engineering: we are not interested in the performance of more sophisticated behavioral rules, but rather in the design of protocols that take decent care even of simple-minded agents.

In particular, this paper studies protocols that implement variants of the continuous double auction, where agents sequentially place quotes on the selling and buying books. Orders are im-

[1] There are negligible differences. We consider n agents who can trade at most one unit, while they have 12 traders who can exchange several units but must trade them one by one. Our setup is simpler to describe because it associates with each trader a single unit and a one-dimensional type (his cost/valuation).

mediately executed at the outstanding price if they are marketable; otherwise, they are recorded on the books with the usual price-time priority and remain valid unless a cancellation occurs. When a transaction takes place, the orders are removed from the market and the traders leave the market and become inactive.

The zero intelligence assumption places a second restriction on agents' behavior. In a sequential protocol like the continuous double auction, an agent can choose both his action and the time at which to take it; see Gul and Lundholm (1995). Zero intelligence robs agents of the opportunity to make decisions about the timing at which to issue a quote. Agents are exogenously arranged in a queue and reach the market one at a time, until the queue is exhausted or some exogenous event triggers the formation of a new queue.

The standard implementation is the following. At the beginning of a simulation, all agents are active and placed in the queue. If an agent reaches the market and trades his unit, he becomes inactive for the rest of the simulation. Otherwise, he is in one of two states: either he has an order on the book (because he is active and the queue has already reached him), or he is still queueing for a chance to act. An important detail in the design of an experiment is the set of events that triggers the formation of a new queue, reshuffling the state of active agents. For instance, the full resampling assumption in Gode and Sunder (1993) makes each transaction a trigger event that sends all traders with an order on the book back to the end of the queue.

2.2.3 The Protocol

The implementation of the continuous double auction in Gode and Sunder (1993) is based on several rules. Some of them are not stated explicitly in the paper, but may be gathered by a joint reading of other related papers; see in particular Gode and Sunder (1993a, 2004). For completeness and ease of reference, we collect here all the ingredients we found necessary to replicate their results.

The first three rules correspond to the assumptions cited above. We begin with the first and the third. The *single unit* rule states that all quotes and prices refer to one unit of the good. A standard rule of *precedence* decides the transaction price: when two quotes cross, the price is set by the earlier quote. We maintain both the single unit and the precedence rules, because they entail no loss of generality.

The second of the three assumptions put forth in Gode and Sunder (1993) as "simplifications" states that the book is cleared after each transaction. By itself, this rule is surprising because tossing away the book at any opportunity seems to run contrary to the obvious purpose of storing past orders and make them available to future traders. In Gode and Sunder's design, moreover, this rule triggers a refreshing of the queue: after each transaction, all recorded orders are deleted and their owners are given a new chance to act. When a ZI agent goes back to the queue and comes up again, he randomly issues a new quote. Hence, the real consequence of tossing away the book is to free up the agents' past quotes and force them to issue novel ones. That is, after each trade, all active agents who have placed an order since the former transaction are resampled. This is the reason for calling their assumption *full resampling*. Section 2.3 shows that full resampling is crucial for Gode and Sunder's results and hence cannot be dismissed as a mere "simplification". In fact, one of the motivations for this paper is to study the import of this neglected assumption.

There are other rules that need to be made explicit. *No retrading* states that buyers and sellers can never exchange roles: a buyer (seller) who acquires (transfers) a unit is not allowed to sell (buy) it later to other traders. The intuition that, given sufficient retrading, a market populated with ZI agents should reach full allocative efficiency is proven in Hurwicz, Radner and Reiter (1975). Therefore, no retrading is necessary to avoid trivialities. Gode and Sunder (2004) provide further comments on the role and plausibility of this assumption.

The *uniform sequencing* of agents within a simulation arranges them in a queue according to an exogenously given order, which is independently drawn from the uniform distribution over all permutations of agents. As explained in Gode and Sunder (2004), in their simulations the queue of traders is sampled *without replacement*. That is, when the execution of a transaction triggers a refreshing of the queue, the agents who have a quote stored on the book re-enter it behind the traders still waiting in the original queue. The no replacement assumption is a sensible simplification that allows for faster and more efficient coding. However, since this rule violates anonymity, its practical implementation requires either additional information processing (when control is centralized) or some traders' coordination (under decentralization). Therefore, in the interest of simplicity and realism, we maintain the uniform sequencing rule but we switch to sampling with replacement: when an event triggers the formation of a queue, we simply apply uniform sequencing over all active agents.

Finally, the *halting* rule mandates when a trading session is over. In Gode and Sunder (1993) a trading session is a period of fixed duration that lasts 30 seconds for each computational simulation. Traders are put in a queue and asked to provide a quote. If all the queued agents have issued a quote and no transaction has occurred, the books are cleared and a new queue is started until time is over. Given that robot players are remarkably fast, this implies that an agent is likely to be asked to issue a quote several times. (We have been unable to determine how often queues are restarted in Gode and Sunder (1993) only because the time limit has not been reached.) Unfortunately, given that hardware (and software) vary in processing power (and efficiency), a halting rule based on a fixed duration is not sufficient to ensure comparable results when simulations are run on different machines. Therefore, we choose a different halting rule that allows for full comparability: a trading session is over when the queue of traders waiting to place an order is exhausted. An additional advantage of this assumption is that it biases our simulations in the proper direction: *ceteris paribus*, we resample traders less often because our halting rule is more stringent. This makes allocative efficiency harder to attain.

For the reader's convenience, we recap here the rules of the continuous double auction protocol used in all the simulations discussed in this paper. We use single unit trading, set the transaction price by precedence, exclude retrading, and apply uniform sequencing. Differently from Gode and Sunder (1993), we put traders back in the queue with replacement and use a more restrictive halting rule.

2.3 The Resampling Assumption

We test the import of the resampling assumption for the allocative efficiency of the continuous double auction (as implemented by our protocol). As usual, we define allocative efficiency as the ratio between the realized gains from the trade and the maximum feasible gains from trade, which can be formally defined as done in Zhan et al. (2002, p.678). This measure is adimensional, facilitating comparisons throughout the paper.

2.3.1 Resampling is Necessary for Allocative Efficiency

We contrast full resampling against no resampling under zero intelligence trading. *Full resampling* mandates that after each transaction the book is cleared and active traders with an order on the book are sent back to the waiting queue. *No resampling* postulates that submitted orders stay on the book until the end of the trading session (unless they are used up for a transaction); e.g. see Maslov (2000).

The difference between full and no resampling is stark. A ZI agent acts only when its turn in the waiting queue comes up. Under no resampling, each agent is given only one chance to act by sending a random quote to the book. Under full resampling, on the other hand, until an agent completes a trade and becomes inactive, any refresh of the waiting queue following a transaction may give him a new chance to act and generate another random quote. Therefore, the number of opportunities for actions is much greater under full resampling, and this should increase allocative efficiency.

The datapoints on the left-hand side of Fig. 2.1 represent the allocative efficiencies under *full resampling* for 500 different runs with $n = 200$ agents. The data match Gode and Sunder's (1993) results, confirming that the impact of our (more stringent) halting rule on allocative efficiency is negligible. The right-hand side provides analogous information for the case of *no resampling*. The y-axes use the same scale, so that a direct comparison by visual inspection is immediate: the higher the level, the higher the allocative efficiency.

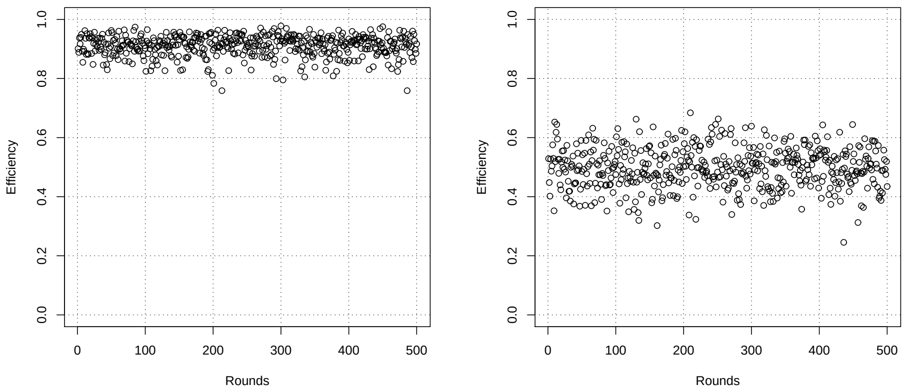


Figure 2.1: Allocative efficiency under full (left) or no resampling (right).

The difference in performance under full or no resampling is remarkably substantial. The mean (median) allocative efficiency is 0.910 (0.916) with full resampling and 0.497 (0.498) with no resampling. (All statistics reported in this paper are rounded to the closest third decimal digit.) The min–max range (standard deviation) for the allocative efficiency is [0.759,0.978] (0.036) with full resampling and [0.246,0.684] (0.070) with no resampling. Within our sample, the worst allocative efficiency with $n = 200$ agents under full resampling (0.759) is much higher than the best allocative efficiency under no resampling (0.684). Visual inspection strongly suggests that the distribution of the allocative efficiency under full resampling stochastically dominates the distribution under no resampling.[2] More modestly, we claim that the expected value of the allocative efficiency under full resampling is higher. In fact, the Wilcoxon signed-rank test rejects the hypothesis that the means are equal at a level of significance of 10^{-3}. (Throughout the rest of the paper, unless otherwise noted, we use the Wilcoxon signed-rank test to compare means and we require a p-value lower than 10^{-3} to claim statistical significance.)

Similar effects occur for different values of n, but a larger number of agents tends to improve allocative efficiency. Thus, when comparing data for a different number of agents, we should take into account a fixed size effect. We believe that $n = 200$ is a representative case, but for comparability Table 2.1 lists the main statistics for $n = 200/5 = 40$ and $n = 200 \times 5 = 1000$.

Based on the relative size of the agents' pool, we say that the market is *thin* ($n = 40$), *thick* ($n = 200$), or *crowded* ($n = 1000$). Each column summarizes 500 distinct simulation rounds.

It is apparent that no resampling may be calamitous in a thin market, because an agent who happens to issue a "wrong" quote is given no later chance to remedy. Analogously, a few "lucky" trades may shoot allocative efficiency up. Hence, the dispersion of the allocative efficiency is

[2] LiCalzi and Pellizzari (2008) document a similar effect over four different trading protocols.

Table 2.1: Summary statistics for the allocative efficiency.

	full resampling			no resampling		
	$n = 40$	$n = 200$	$n = 1000$	$n = 40$	$n = 200$	$n = 1000$
mean	0.735	0.910	0.949	0.441	0.497	0.517
median	0.765	0.916	0.951	0.456	0.498	0.518
minimum	0.053	0.759	0.911	0.000	0.246	0.405
maximum	1.000	0.978	0.971	0.933	0.684	0.609
std. dev.	0.169	0.036	0.009	0.157	0.070	0.032

much higher in a thin market. Such effects are washed out in a crowded market. Overall, an increase in n has a positive effect on allocative efficiency under either resampling assumption. But the effect is sharper under full resampling, because this rule gives traders more chances to trade.

Our experiment shows that, *ceteris paribus*, full resampling yields a much higher allocative efficiency than no resampling. Speaking figuratively, no resampling switches off the ability of a protocol to help ZI agents capture most of the available gains from trade. We conclude that (at least some) resampling is a necessary condition for allocative efficiency. This reduces the scope of Gode and Sunder's (1993) results about the ability of a market to substitute for agents' lack of rationality: an effective protocol for ZI agents must include rules that ensure an adequate amount of resampling.

On the other hand, our results do not invalidate their claim that it is possible to design markets that may overcome agents' cognitive limitations. To the contrary, they suggest that the use of a (partial) resampling rule may be a particularly clever design choice for fostering allocative efficiency in exchange markets. Section 2.4 sets out to examine a continuum of alternative rules that enforce different degrees of resampling in this respect. We find that less than full resampling is sufficient to reach high levels of efficiency.

2.3.2 Efficiency and Full Resampling

Before moving to issues of market engineering, there are two hanging questions to address. First: why does full resampling lead to higher allocative efficiency than no resampling? Second: where does the efficiency loss go?

We begin with the second question, whose answer leads naturally to the first one. Gode and Sunder (1997, p. 605) point out that in general there are "three causes of inefficiency: (1) traders participate in unprofitable trades; (2) traders fail to negotiate profitable trades' and (3) extramarginal traders displace intramarginal traders." Since individual rationality rules out the first source of inefficiency, we need being concerned only with the other two. They can be measured; e.g., see Zhan and Friedman (2007) who also provide formal definitions.

Let p^* be the market-clearing price. (There may be an interval of market-clearing prices. We assume that p^* is the midpoint.) Individually rational traders who would transact at p^* are called *intramarginal*; all other traders are *extramarginal*. If at the end of a trading session an intramarginal trader i has failed to trade, this creates a loss of total surplus equal to $v_i - p^*$ if he is a buyer and $p^* - c_i$ if he is a seller. The sum of these losses corresponds to (2) above: we call it MT, as a mnemonic for the inefficiency caused by *missed trades*. The third case comes about when a transaction involves an extramarginal trader, causing a loss equal to his profit at p^*. The sum of such losses corresponds to (3) above: we call it EM, as a mnemonic for the inefficiency due to *extramarginal* trades. As discussed in Zhan and Friedman (2007), the allocative efficiency decomposes as $AE = 1 - MT - EM$; or, equivalently, $MT + EM = 1 - AE$ measures the allocative inefficiency. Table 2.2 provides a breakdown of the efficiency loss for thin, thick and crowded markets by listing mean values over 500 distinct simulation rounds. Values may not add up to 1 because of rounding effects.

There are two observations to be made. The first one is that the efficiency loss (MT) attributable to missed trades is decreasing in the thickness of the market, because thicker markets facilitate the search for a matching quote. Moreover, trading under no resampling terminates too soon: most of the efficiency loss comes from missed trades. (The difference between the mean values for MT under full or no resampling is statistically significant.) The reason for a high allocative efficiency under full resampling is elementary: this rule is of course more effective in prolonging the trading session, and hence gives traders enough chances to find their right match. This suggests that an effective market protocol should offer agents an adequate number of matching opportunities to keep the MT component of the efficiency loss under control.

The second observation points out a shortcoming of full resampling. The average value of EM is higher under such rule. (The difference between the means is once again statistically significant.) This is not difficult to explain: by the precedence rule, the best outstanding bid and ask in the book bound the price of the next transaction. The narrower the spread, the more difficult is to steal a deal for an extramarginal trader. Storing earlier quotes in the book provides (intramarginal) traders with some price protection and makes them less exploitable by extramarginal agents. As the full resampling rule tosses away the book after each transaction, it renounces such protection all too frequently (compared to no resampling). This is apparent by a straightforward comparison: the average spread (sampled before a trader places an order) is 0.152 with no resampling and 0.327 with full resampling when $n = 200$. (Corresponding values are 0.267 and 0.398 for $n = 40$; 0.113 and 0.268 for $n = 1000$.) The differences between the mean values are statistically significant. Since the zero intelligence assumption prevents traders from adjusting their quotes based on the state of the book, full resampling is a rule more favorable to extramarginal traders than no resampling. This suggests that an effective market protocol should incorporate some form of price protection to keep the EM component of the efficiency loss under control.

2.4 Randomized Resampling

Section 2.3.1 established that the resampling rule is crucial to reach allocative efficiency under zero intelligence. To evaluate its impact, this section begins by looking at a continuum of resampling rules that generalize the simple dichotomy between no and full resampling. We emphasize that these rules are chosen to compare and understand how resampling affects the trading protocol. Like engineers, we are searching for improvements and tweaks over a basic design.

2.4.1 Full Resampling is Not Necessary for Allocative Efficiency

A simple way to conceptualize the distinction between no and full resampling is to note that these two rules react differently to the same event; namely, the occurrence of a transaction. When two

Table 2.2: A breakdown of the efficiency loss.

	full resampling			no resampling		
	$n = 40$	$n = 200$	$n = 1000$	$n = 40$	$n = 200$	$n = 1000$
AE	0.735	0.910	0.950	0.441	0.497	0.517
MT	0.241	0.055	0.012	0.548	0.495	0.477
EM	0.025	0.035	0.037	0.011	0.008	0.006

orders cross, full resampling clears the book with probability one whereas no resampling does so with probability zero. This naturally suggests to consider a family of randomized resampling rules that clear the book with probability π in $[0,1]$ whenever there is a transaction. This set embeds full resampling for $\pi = 1$ and no resampling for $\pi = 0$.

The right-hand side of Fig. 2.2 shows the allocative efficiency under π-*resampling* with $n = 200$ agents. The graph is obtained as follows. We choose the 21 equispaced points $\{0, 0.05, 0.10, \ldots, 0.90, 0.95, 1\}$ in the $[0,1]$ interval. For each of these π-values, we run 500 distinct simulations.

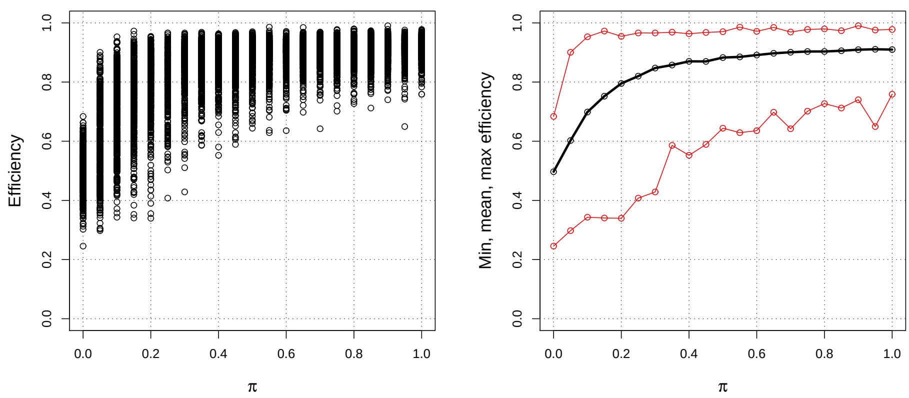


Figure 2.2: Allocative efficiency under π-resampling.

The allocative efficiencies obtained over these $21 \times 500 = 10500$ simulations are plotted as datapoints on the left-hand side of Fig. 2.2. We summarize these data by the mean allocative efficiency for each π. (The difference between a mean and the corresponding median is never greater than 0.022.) The 21 sample averages are joined using segments to obtain the thicker central piecewise linear representation.[3] The two external thin graphs are similarly obtained by joining respectively the minimum and maximum values obtained for the allocative efficiency at a given value of the resampling probability π. We emphasize that the resulting band is not a confidence interval but the actual range of efficiencies obtained under our simulations: its main purpose is to provide a simple visual diagnostic for the dispersion of the data around their central tendency. We adopt the usual $[0,1]$-scale for the y-axis.

The graph on the right of Fig. 2.2 is easily interpreted. As expected, allocative efficiency is on average increasing in the probability π that a transaction triggers a clearing of the book. Under zero intelligence, the frequency with which resampling takes place has a direct effect on the ability of the protocol to reap high levels of efficiency. On the other hand, the graph shows also that full resampling ($\pi = 1$) is not necessary: the (average) allocative efficiency in our simulations is more than 90% for $\pi \geq 0.7$ with a (statistically insignificant) peak of 91.08% at $\pi = 0.95$; the standard deviations are never greater than 0.132. There is an upper bound on the allocative efficiency that can be attained but a sufficiently large π is enough to approach it.

Similar results hold for thin and crowded markets: when $n = 40$, $AE \geq 67\%$ for $\pi \geq 0.7$ with a peak of 73.48% at $\pi = 1$ and standard deviations never greater than 0.209; when $n = 1000$, $AE \geq 93\%$ for $\pi \geq 0.2$ with a (statistically insignificant) peak of 95.12% at $\pi = 0.85$ and standard deviations never greater than 0.080. The thickness of the market affects the upper bound on the allocative efficiency but, in general, there is a whole range of resampling probabilities that achieve comparably high levels of allocative efficiency under zero intelligence.

[3] We consistently apply this approach to construct the graphs for this paper: a broken line joins 21 points, each of which represents a statistic over 500 distinct simulations for a fixed value of a parameter such as π.

Our conclusion is that full resampling is not necessary for allocative efficiency. Full resampling sets $\pi = 1$ and tosses the book away after each transaction: this yields a high allocative efficiency under zero intelligence, but it is also an extreme assumption that is likely to be unpalatable for human traders in real markets. As it turns out, we can temper the strength of full resampling at the mere cost of a tiny reduction (if any) in allocative efficiency.

This leads naturally to frame the choice of a resampling rule as a tradeoff between its allocative benefits and its implementation costs. On the part of the market designer, there are obvious costs to continuously monitor and update the state of the book. Similarly, traders who are forced to check whether their past orders have been voided are likely to resist frequent cancellations. Intuitively, when the costs of full resampling are not trivial, we expect partial resampling ($0 < \pi < 1$) to be preferable. The rest of this section fleshes up this argument. Section 2.5 takes up a related question and examines a different family of resampling rules to find out whether they perform better than π-resampling.

2.4.2 Where is the Best π?

Let us take stock of the starting point we have reached so far. First, given the thickness of the market, there is an upper bound on the (mean) allocative efficiency that can be attained using π-resampling. Second, the set of π-values for which the protocol reaches comparably high levels of efficiency is an interval. Thus, we need to look at additional performance criteria in order to pinpoint a smaller interval for the choice for π.

We do not claim that it is possible to find the *best* π and reduce such interval to a singleton, because the zero intelligence assumption provides at best a lower bound for the evaluation of a protocol. More modestly, we can define plausible performance criteria and measure them for different values of π under zero intelligence trading. Clearly, this procedure cannot provide a final verdict for the performance of the protocol with human subjects. Hence, the aim of this section is to carry out an engineering exercise and derive a robust choice: what is the range of π for which performance under zero intelligence is better, and why?

We consider two simple criteria. (Others are of course possible, and we take up a third major one in Sect. 2.4.3.) The first criterion deals with the basic requirement that an effective market protocol should offer some guidance to traders' choice in the form of a price signal. The closer the outstanding bid and ask straddle the (competitive) equilibrium price, the stronger the information that they provide. It is obvious that zero intelligence makes no use of this information: therefore, the object of our investigation is the ability of the protocol to provide an effective price signal *independently* of traders' behavior.

We measure it by the (mean) spread on the market: the closer the spread, the stronger the signal. The average is taken by sampling data when a trader arrives and places an order (as opposed to just before a transaction occurs), because we are interested in the state of the book found by a generic agent reaching the market. As it turns out, our environment is sufficiently regular that the best bid and the best ask are (on average) symmetric around the equilibrium price p^*. Hence, the outstanding spread is a sufficient statistic for such purpose. The left-hand side of Fig. 2.3 shows the (mean) outstanding bid and ask under π-resampling. The y-axis is truncated to $[0.3, 0.7]$ to enhance readability.

Unsurprisingly, the spread is on average increasing in π. When resampling is more frequent, the book is cleared more often and hence is more likely to have both fewer quotes and a larger spread. For $n = 200$, the average (median) spread in our simulations increases monotonically from 0.152 (0.124) at $\pi = 0$ to a peak of 0.327 (0.232) at $\pi = 1$; the standard deviations are never greater than 0.033. Qualitatively similar results hold for $n = 40$ and $n = 1000$, and spreads are smaller in thicker markets. This leads to the following general piece of advice. Suppose that, conditional on achieving comparable levels of allocative efficiency, a market designer prefers

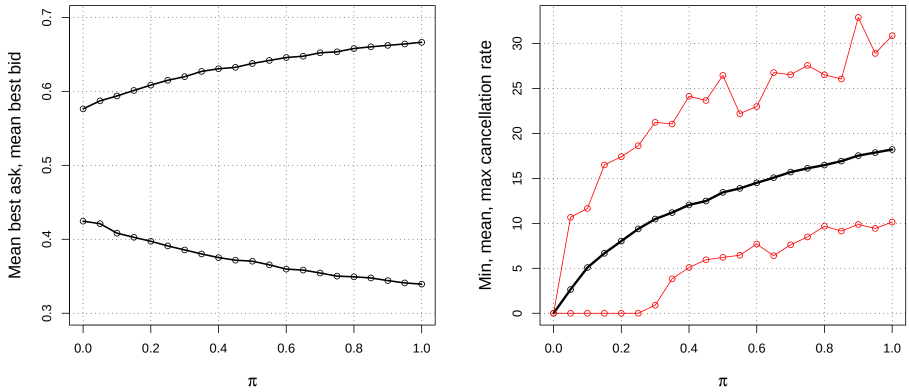


Figure 2.3: Mean spreads and cancellation rates under π-resampling.

narrower spreads. Then he should aim towards choosing a level of π that is bounded away from zero (to achieve efficiency) as well as from one (to obtain smaller spreads). The thicker the market, the weaker the need to stay away from one.

A second simple criterion has to do with the number of cancellations imposed on traders. (Recall that traders cannot cancel their orders.) The benefit of a cancellation is to offer a new chance for action to the trader. On the other hand, in general there are costs associated with the inconvenience of monitoring the state of an order or placing a new one. Therefore, when the allocative efficiency of two protocols are similar, it is reasonable to expect that the one leading to fewer cancellations should be preferred. We measure the cancellation rate as the average of the ratio between the number of orders canceled over the number of transactions completed over each of our 500 simulated trading sessions. Clearly, allocative efficiency is strongly correlated with volume; hence, the higher the ratio, the higher the cost of redundant cancellations. The right-hand side of Fig. 2.3 depicts the (mean) cancellation ratio with $n = 200$ agents. As usual, we report the mean values as a thick black line surrounded by thinner red lines that correspond to the minima and maxima.

Similarly to the spread, the cancellation rate is on average increasing in π because the amount of resampling directly correlates with the number of canceled orders. For $n = 200$, the mean (and standard deviation) of the cancellation rate go up[4] from 2.643 (2.498) at $\pi = 0.05$ to a peak of 18.22 (3.002) at $\pi = 1$; the standard deviations are never greater than 3.124. Similar results hold for $n = 40$ and $n = 1000$, and mean cancellation rates are higher in thicker markets. The conclusion we draw is similar to the earlier one. Suppose that, conditional on achieving comparable levels of allocative efficiency, a market designer prefers a lower cancellation rate. Then an optimal π should be bounded away from zero (for efficiency) as well as from one (for a lower rate). The thicker the market, the stronger the need to stay away from one.

2.4.3 Traders' Protection

The last performance criterion that we consider in this paper is directly inspired by Stigler (1964), who pioneered the use of simulations to address issues of market engineering. He put down a clear statement: "The paramount goal of the regulations in the security markets is to protect the innocent (but avaricious) investor" (p. 120). While his paper is concerned with security markets, the conditions for achieving this goal should also be investigated for exchange markets.

[4] We start from $\pi = 0.05$ because the cancellation rate at $\pi = 0$ is zero by assumption.

Curiously, the literature on zero intelligence has so far neglected this issue to the point that there is not even an agreed convention on the exact meaning of protection.

This section provides a measurable criterion for traders' protection in an exchange market, and then applies it to the evaluation of the π-resampling rule. Ideally, in a competitive equilibrium, all[5] the intramarginal traders exchange the good at the same equilibrium price p^*: nobody pays (or is paid) differently from the others. On the other hand, a continuous double auction offers neither of these guarantees: first, an intramarginal trader may fail to close a deal; second, the price at which a trade occurs may be different from the price agreed for another trade. Both of these events deny the competitive outcome to the intramarginal trader. When a market protocol hold such events under control, it manages to offer *traders' protection*.

Clearly, allocative efficiency does not measure traders' protection: since it focuses on the gains from trade that are realized, it fails to register at what terms these gains materialize. We need a more sophisticated measure that takes into account the price at which a transaction is carried out, and hence touches on the distribution of gains. To this purpose, we define the *competitive share* of a trader as the (positive part of the) profit he would make by transacting at the competitive equilibrium price. Given an equilibrium price p^*, the competitive share of a buyer with valuation v is $(v - p^*)^+$ and that of a seller with cost c is $(p^* - c)^+$. Clearly, the competitive share of any extramarginal trader is zero.

The *realized competitive share* is the portion of his competitive share realized by an agent. (Extramarginal traders are entitled to no competitive share.) If an agent fails to trade, this portion is zero. If a trade occurs at price p, the realized competitive share is $v - \max\{p, p^*\}$ for an (intramarginal) buyer and $\min\{p, p^*\} - c$ for an (intramarginal) seller.

The realized competitive share is concerned only with measuring whether a trader gets its due, and ignores any additional gains that he may be able to reap. The profit realized by an intramarginal trader may be greater than his realized competitive share if he manages to secure terms of trade more favorable than p^*; similarly, any extramarginal agent who completes a trade makes positive profits by individual rationality, but his realized competitive share remains zero.

Note that the sum of all the competitive shares equals the maximum feasible gains from trade. In analogy with allocative efficiency (AE), we define the traders' protection (for short, TP) offered by a market protocol as the ratio of the realized competitive shares and the sum of all the competitive shares. This measure is adimensional and takes values in $[0, 1]$.

The left-hand side of Fig. 2.4 shows the traders' protection under π-resampling with $n = 200$ agents. As usual, we report the mean values surrounded by minima and maxima. The right-hand side superimposes AE and TP to allow for a direct comparison: the black line corresponding to TP is the same visible on the left, while the red line depicting AE corresponds to the inner black line from the right-hand side of Fig. 2.2.

In general, traders' protection is not increasing in π. For $n = 200$, the mean protection starts at 0.431 in $\pi = 0$, peaks at 0.718 in $\pi = 0.7$ and then declines to 0.709 in $\pi = 1$ (with two local maxima of 0.711 at $\pi = 0.4$ and 0.715 at $\pi = 0.9$); the standard deviations are never greater than 0.111. Qualitatively similar results hold for crowded and thin markets. When $n = 1000$, TP is 0.461 in $\pi = 0$, peaks at 0.791 in $\pi = 0.2$ and then declines to 0.744 in $\pi = 1$ with no other local maxima and standard deviations never greater than 0.070. For $n = 40$, TP is 0.355 in $\pi = 0$ and peaks at 0.563 in $\pi = 1$, with four more local maxima in between and standard deviations never greater than 0.172.

Here, the thickness of the market has a very strong effect on the range of the best value for π: the more crowded the market, the smaller the resampling rate that provides the best protection. The overall conclusion is similar to the above, with a strong word of caution as regards the thickness of the market. Suppose that, conditional on achieving comparable levels of allocative

[5] When the number of intramarginal traders is odd, one of them will not trade for lack of a partner.

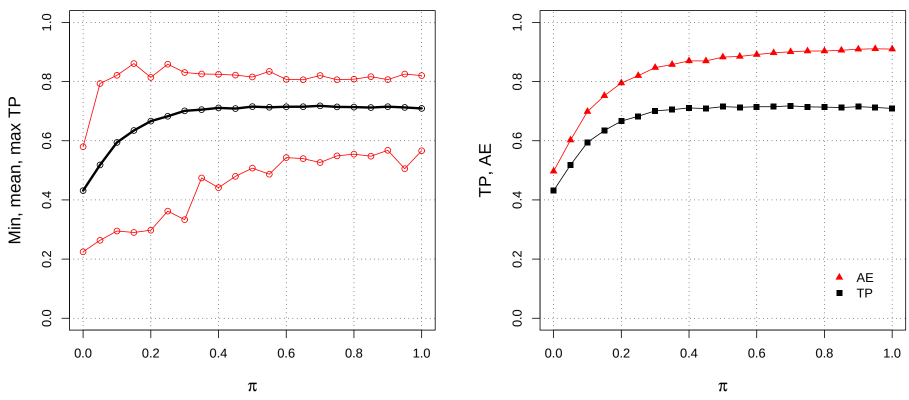


Figure 2.4: Traders' protection (left), superimposed to allocative efficiency (right).

efficiency, a market designer prefers to offer a higher traders' protection. Then an optimal π should be bounded away from zero (for efficiency) as well as from one (for protection). When the market gets thicker, however, the need to stay away from one is remarkably higher.

To sum it up, we have considered three criteria based respectively on spread, cancellation rate, and traders' protection. To a different extent, they all support a choice of π from the interior of the interval $[0, 1]$: at the cost of a nominal reduction in allocative efficiency, it is possible to have lower spreads, fewer cancellations, and higher traders' protection. It is clear that both the relative importance of these criteria to the market designer as well as the thickness of the market matter for the exact choice of π. However, generally speaking, all of our performance criteria strongly suggest that full resampling is unlikely to be a defensible choice.

2.5 Resampling Outside of a Price Band

Section 2.4 has studied randomized resampling, but it is obvious that there exist many other rules. It may be impossible to pick a best one, but we can compare the performance of different resampling techniques. This section considers a different rule that shares a few basic properties with π-resampling. First, it depends on a single parameter γ in $[0, 1]$. Second, it implies an average resampling rate that is increasing in the parameter. Third, it embeds the two extreme cases of full and no resampling for $\gamma = 1$ and $\gamma = 0$. Fourth, it requires minimal information and thus imposes very little burden on the market protocol or the cognitive abilities of the traders.

The γ-resampling rule is the following. After a trade carries out at price p, the protocol cancels all outstanding orders that fall outside the *price band* $[\gamma p, \gamma p + (1-\gamma)]$; moreover, in the special case $\gamma = 1$, we require the protocol to erase even the outstanding orders at price p so that it clears the book entirely.[6] (This specification is necessary to embed full resampling, because the book might contain orders with price p but lower time priority.) It is useful to keep in mind that π is the probability with which the book is cleared after a transaction, while $(1-\gamma)$ is the width of the price band within which orders are *not* deleted after a transaction.

Like π-resampling, the γ-resampling rule is triggered whenever a transaction occurs. Differently from it, its application implies that traders whose orders are deleted may infer a one-sided bound for the last transaction price. For instance, given γ, when a buyer sees that his past order at price p has been canceled, he can deduce that the last transaction price must have been strictly greater than p/γ. We do not view this a significant limitation, since it is seems

[6]See Appendix B2.

highly plausible that all agents would be given public access to such information. On the other hand, since it makes no use of the best outstanding bid and ask, the γ-resampling rule does not require to divulge this kind of information. This may be an additional advantage in view of the results in Arifovic and Ledyard (2007), who consider a sequence of call markets and show that the closed book design[7] brings about a higher allocative efficiency than an open book in environments populated with human subjects or (non ZI) simulated agents. If similar results should suggest adopting a closed book design for the continuous double auction, both π- and γ-resampling are compatible. For the current study, it suffices to say that the ZI assumption precludes a direct comparison between closed and open book design, because it prevents agents from making use of any disclosed information.

The γ-resampling rule may be easily adapted in other dimensions. For instance, our definition embeds a symmetry assumption that may be removed. We choose the endpoints of the price band at the same distance from the extremes of the price range: the left endpoint is a convex combination between the last transaction price p and the minimum possible price, while the right endpoint is a convex combination between p and the maximum possible price. Clearly, this choice requires the implicit assumption that we know that p lies in the interval $[0, 1]$. More generally, when no bounds for the price are known, it suffices to set the price band to be the interval $[\gamma p, (1/\gamma)p]$ for γ in $[0, 1]$ or other analogous formulations.

Figure 2.5 shows the allocative efficiency under π-resampling (on the left) and γ-resampling (on the right) with $n = 200$ agents.

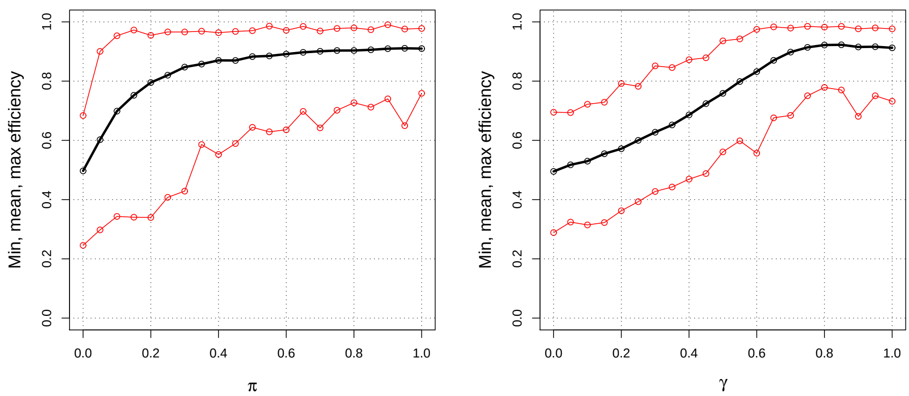


Figure 2.5: Allocative efficiency under π- and γ-resampling.

The graph on the left is the same as in Fig. 2.2. The graph on the right is the analog for γ-resampling: a thick inner black line joins the mean values, and two thin outer red lines join the corresponding minima and maxima. The directionality of the graphs is aligned because they depict two resampling rules that coincide for $\pi = \gamma = 0$ and $\pi = \gamma = 1$.

Both resampling rules are on average increasing in the corresponding parameter. However, the qualitative behavior is different. Under π-resampling, allocative efficiency picks up fast and rapidly settles on a plateau: for $n = 200$, the sample average is greater than 0.90 for $\pi \geq 0.7$. As already discussed, even moderate levels of π suffice to attain an adequate level of efficiency. On the other hand, efficiency under γ-resampling grows up more slowly and, quite interestingly, peaks at $\gamma < 1$: for $n = 200$, the sample average is greater than 0.90 for $\pi \geq 0.75$ and peaks at 0.923 for $\gamma = 0.85$; the standard deviations are never greater than 0.073.

[7] In a closed book, traders learn only the clearing price after each call; in an open book, they are also told the quotes processed in that call.

Table 2.3: Maximum allocative efficiency under π- and γ-resampling.

	π-resampling			γ-resampling		
	$n = 40$	$n = 200$	$n = 1000$	$n = 40$	$n = 200$	$n = 1000$
maximum AE	0.735	0.911	0.951	0.743	0.923	0.960
maximizer (π, γ)	1.000	0.950	0.850	0.900	0.850	0.800

Similar results hold for the case of thin and crowded markets. For $n = 40$, the sample average is greater than 0.67 for $\gamma \geq 0.65$ and peaks at 0.743 for $\gamma = 0.9$, with standard deviations never greater than 0.197; for $n = 1000$, the sample average is greater than 0.94 for $\gamma \geq 0.7$ and peaks at 0.96 for $\gamma = 0.8$ with standard deviations never greater than 0.033. Thicker markets exhibit a superior allocative performance for lower values of γ but the overall conclusion is the same: a narrow (but not empty) price band is a necessary condition to attain sufficiently high levels of efficiency.

2.6 A Comparison of Alternative Rules

This section compares the performance of the protocol when adopting π-resampling versus γ-resampling over four different criteria: allocative efficiency (AE), mean spread, cancellation rate, and traders' protection (TP).

Table 2.3 compares the allocative efficiency under π- and γ-resampling for thin, thick, and crowded markets.

For each combination of n and resampling rule, we list the highest mean values obtained. These are slightly higher under γ-resampling, but we would not stake big claims over tiny differences that are subject to sampling errors. (However, they are statistically significant for $n = 200$ and $n = 1000$.) We prefer to conclude that there is no clear winner over AE: both resampling rules can be tuned to attain comparably high levels of allocative efficiency .

The second performance criterion is the mean spread. Figure 2.6 shows the best bid and ask under both π-resampling (on the left) and γ-resampling (on the right) with $n = 200$ agents. The y-axes are truncated to $[0.3, 0.7]$ to enhance readability.

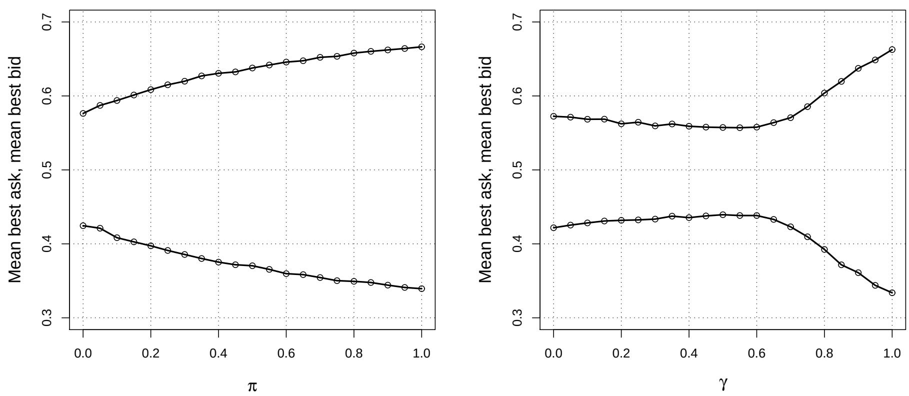


Figure 2.6: Mean spread under π- and γ-resampling.

Predictably, as a mere visual inspection confirms, the clear winner is the γ-resampling rule that is based on an explicit form of price control. Table 2.4 validates this conjecture by listing the lowest mean spread obtained under π- and γ-resampling for thin, thick, and crowded markets. The difference between the mean values is statistically significant for each choice of n.

Conditional on choosing the right parameter, the mean spread with γ-resampling is remarkably smaller. However, note that the best performances of both π- and γ-resampling with regard to the mean spread require a choice of parameters that are far from being optimal for allocative efficiency. This is easily seen by comparing the second rows from Table 2.3 and Table 2.4. Therefore, while it is clear that γ-resampling yields a lower mean spread than π-resampling under ideal conditions, we need a further test to find out whether it is still superior once we take into account both efficiency and mean spread.

This test is provided on the left-hand side of Fig. 2.7, where we plot the average outstanding bid and ask under both π-resampling (in red) and γ-resampling (in black) with $n = 200$ agents. This graph combines information about the two resampling rules. For each level of the (mean) allocative efficiency attained under either rule, we plot the corresponding average values of the best bid and ask and then join the datapoints using broken lines. Since the two rules attain different (mean) efficiencies, the datapoints are not vertically aligned. The left-hand picture shows clearly that, for comparable levels of allocative efficiency, γ-resampling leads to smaller (mean) spreads that π-resampling. In other words, a resampling rule based on a price band tends to produce a smaller spread than a rule based on a full clearing of the book, without sacrificing allocative efficiency.

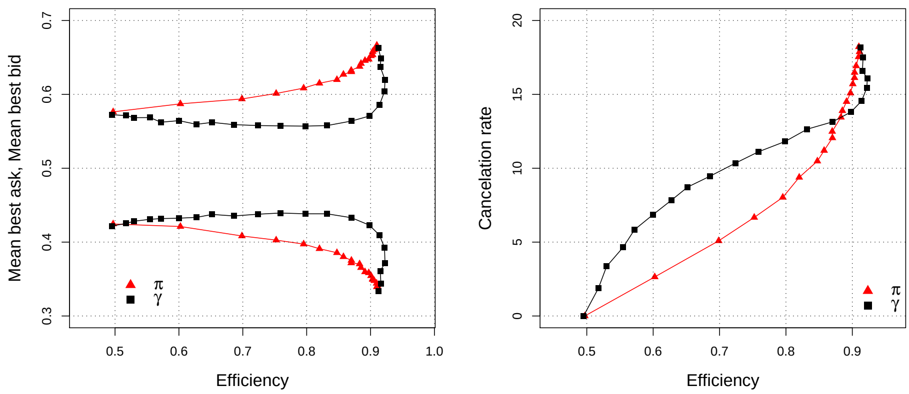


Figure 2.7: Bid-ask spreads and cancellation rates versus allocative efficiency.

Our third performance criterion is the cancellation rate. As is the case for π-resampling, this rate in on average increasing in γ because the width of the price band inversely correlates with the number of canceled orders. For $n = 200$, the mean (and standard deviation) of the cancellation rate go up from 1.868 (0.561) at $\gamma = 0.05$ (it is zero for $\gamma = 0$) to 18.16 (3.211) at $\gamma = 1$; the standard deviations are never greater than 3.212. A direct comparison shows that the range of attainable values for the cancellation rate is virtually identical under π- and γ-resampling. Similar results hold for $n = 40$ and $n = 1000$. Taken by itself, therefore, a criterion based on the cancellation rate is not conclusive.

Table 2.4: Mean spread under π- and γ-resampling.

	π-resampling			γ-resampling		
	$n = 40$	$n = 200$	$n = 1000$	$n = 40$	$n = 200$	$n = 1000$
minimum (mean) spread	0.267	0.152	0.113	0.228	0.118	0.086
minimizer (π, γ)	0.000	0.000	0.000	0.450	0.500	0.600

As for the mean spread, however, we can compare the combined performance of either resampling rule with respect to allocative efficiency and cancellation rates. The right-hand side of Fig. 2.7 plots the (average) cancellation rates for each level of the (mean) allocative efficiency attained under either rule. For a large range of (lower) allocative efficiencies, π-resampling has a substantially lower cancellation rate; for high values, γ-resampling comes out better by a thin margin. (We do not report the graphs for different values of n, but increasing n makes this conclusion sharper.) Hence, whenever the market designer views the cancellation rate as ancillary to the allocative performance, he should prefer a resampling rule based on the price band.

The last (and in our opinion, more important) criterion is traders' protection. Table 2.5 compares the performance of π- and γ-resampling in thin, thick, and crowded markets. Similarly to Table 2.3, we list the highest mean values obtained for each combination of n and resampling rule. For $n = 200$ or $n = 1000$, the differences between the mean values are statistically significant. (For $n = 40$, this holds at the 1% significance level.) Conditional on choosing the right parameter, traders' protection is higher using γ-resampling. Note also that the optimal values of π and γ are decreasing in the thickness of the market, but this effect is much stronger for π-resampling. Therefore, when the exact size of the market is not known, the choice of the parameter under γ-resampling is more robust.

This superiority carries over when traders' protection is ancillary to allocative efficiency. The left-hand side of Fig. 2.8 superimposes the usual graphs of the mean values for AE and TP under γ-resampling for $n = 200$. The equivalent representation for π-resampling is on the right-hand side of Fig. 2.4. In general, γ-resampling delivers a higher traders' protection than π-resampling for any given level of allocative efficiency. This is shown on the right-hand side of Fig. 2.8, where we report the (mean) traders' protection offered by the two resampling rules with respect to their (mean) allocative efficiency. The γ-resampling frontier on the AE–TP plane dominates the π-resampling frontier.

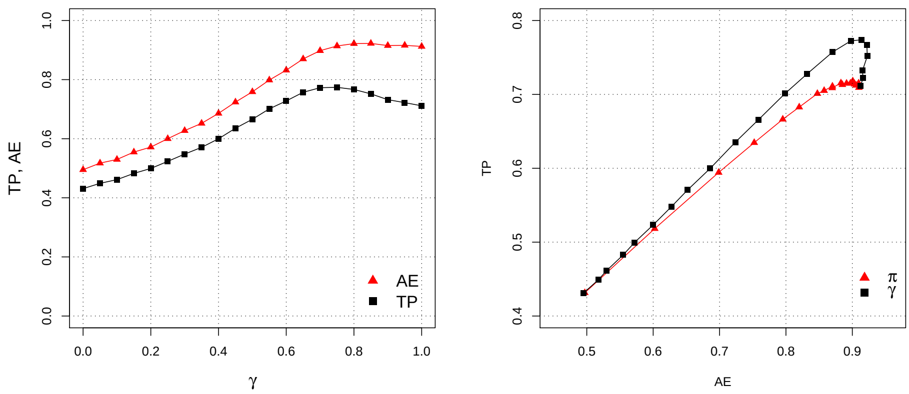


Figure 2.8: Traders' protection and allocative efficiency for π- and γ-resampling.

Table 2.5: Maximum traders' protection under π- and γ-resampling.

	π-resampling			γ-resampling		
	$n = 40$	$n = 200$	$n = 1000$	$n = 40$	$n = 200$	$n = 1000$
maximum TP	0.563	0.718	0.791	0.589	0.774	0.833
maximizer (π, γ)	1.000	0.700	0.200	0.800	0.750	0.700

2.7 Conclusions

We have studied the continuous double auction from the point of view of market engineering, tweaking the trading protocol in search of improved designs. Our starting point has been the rules for exchange adopted by Gode and Sunder (1993) for experiments with human agents and simulations with robot traders. We have disassembled their trading protocol into several component rules, and focused attention on resampling. We have assumed zero intelligence trading as a lower bound for more robust behavioral rules in order to elucidate the consequences of different resampling techniques.

Like Gode and Sunder (1993) and most of the subsequent literature, we look first at allocative efficiency. Their trading protocol makes an extreme assumption that we call full resampling. We show that full resampling is especially favorable to allocative efficiency, biasing Gode and Sunder's results about the ability of the market to substitute for the lack of traders' intelligence. (A second negligible bias may come from their halting rule.) On the other hand, we demonstrate that partial resampling may be sufficient for the purpose of attaining a high allocative efficiency.

Based on this, we have devised a family of rules that includes as extreme cases both Gode and Sunder's full resampling and the opposite assumption of no resampling. This class of rules is parameterized by the probability π of clearing the book after a transaction occurs. We find that there is a large range of values for which π-resampling can attain a high allocative efficiency. In order to discriminate among such π's, we introduce three subordinate performance criteria: spread, cancellation rate, and traders' protection. The spread criterion measures the capacity of the protocol to provide a useful price signal. The cancellation rate looks at the inconvenience created by over-resampling. Finally, traders' protection measures the ability of a protocol to help agents capture their share of the competitive equilibrium profits. This latter criterion, patterned after the usual measure of allocative efficiency, is (to the best of our knowledge) new to the literature: we argue that ignoring it neglects one of the paramount goals of designing a market protocol.

We then introduce a different family of rules, based on the idea to delete only those quotes that fall outside of a price band parameterized by γ. We find that from the point of view of allocative efficiency, the optimized versions of either resampling rule are virtually indistinguishable. However, several differences emerge when we study their performance with respect to the other three criteria. In particular, when we consider a pair of criteria where the first one is allocative efficiency and the second one is any of the other three, we find that it is always the case that (at least for high efficiencies) γ-resampling dominates π-resampling. We then conclude that a resampling rule based on the price band is superior.

Chapter 3

Learning Cancellation Strategies in a Continuous Double Auction Market

3.1 Introduction

"The individual is foolish; the multitude, for the moment is foolish,
when they act without deliberation;
but the species is wise, and, when time is given to it,
as a species it always acts right"
Edmund Burke (1782)

Recent literature has devoted increasing attention to the study of the dynamics of a limit order market. This attention is primarily due to the fact that the most part of the stock exchange markets are governed using continuous double auction protocol's rules (e.g., the Paris Bourse, the Spanish Stock Exchange, the ASX, the NYSE, the Nasdaq and many others). Results on different issues are mixed and no unique answer is provided to many questions.

In this work, we deal with two different key issues: the search for equilibrium trading strategies in a dynamic limit order market and the role played by the order cancellation option. Let us introduce them one at a time.

We cite Edmund Burke: "the individual is foolish [...] but the species is wise". The context in which this citation was made is very different from the one that is treated here but we can adapt this citation and interpret its meaning as follow: an individual agent might be not able to really strategically act as a maximizer; it is limited in his own computational abilities and makes mistakes. However, he behaves as a species and takes his time (to learn - the species has to survive) he acts in the right direction. This is our first point at issue. Assume that agents are not able to maximize their expected profits in a continuous double auction; they are only able to learn from their past experience and from their own interactions with the other agents. Are they able to behave in the right way? Do they learn the (same) optimal strategy? We implement a genetic algorithm that drives the evolution of the species and analyze the relevance and the properties of the achieved (equilibrium) results.

As a second stage, and maintaining this evolutionary approach, we deal with the cancellation issue. This is a typical feature over which analytical modelling loses in tractability. In fact, in the last few decades, many authors tried to introduce it in their mathematical models. The results is an abuse of assumptions or a simplification of the cancellation rule itself treated and modelled as an exogenous random variable. The research questions that we try to answer are the following: from the point of view of a market designer, does the introduction of a cancellation rule in a continuous double auction protocol (i.e., allowing traders to cancel a submitted order and place a new offer) have any effect on order aggressiveness? Does it help to increase the performance of the market according to specific criteria such as average spread, allocative efficiency, volume and individual average profits?

We conduct both these studies using a computational approach and running simulations. The advantage of using an agent-based model for this analysis is to computationally overcome limitations that make impossible to analytically reach a closed form solution for the equilibrium result in a simpler model.

This report is organized as follows. Section 2 describes the benchmark model. Section 3 illustrates and implements a genetic algorithm and compares its findings with the equilibrium results. A cancellation chance is introduced in the protocol in Sect. 4 and Sect. 5 compares the two hypothesis of a market with and without cancellation according to different performance criteria and testable implications. Section 6 summarizes future work and Sect. 7 concludes.

3.2 The Model

With the aim to study the dynamics of a limit order market, we initially base our simulation on the analytical model proposed in Foucault et al. (2005). In this section we describe that model; following Smith (1982), we distinguish three components of their exchange market: the rules that govern exchange (institutional structure), the agents' tastes and endowments (market environment) and the specified trading strategies (agents' behavior).

Institutional Structure. They consider a limit order market in which agents can trade at most one unit at a time. There is a specified range of admissible prices (determined by the latent information about the security value) and offers and spreads are placed on a discrete grid governed by the tick size $\triangle > 0$[1]. Traders' arrivals follow a Poisson process with parameter $\lambda > 0$. *(Ass.1) Each trader arrives only once, submits a market or a limit order and exits. Submitted orders cannot be canceled or modified. (Ass.3) Buyers and sellers alternate with certainty. The first is a buyer with probability* 1/2.

Market Environment. There is an economy of traders equally divided between buyers and sellers. Valuations and costs lie outside the range of admissible prices[2]. Both buyers and sellers are allowed to be of two types (patients-P and impatients-I) that differ by the magnitude of their waiting costs ($0 \leqslant \delta_P \leqslant \delta_I$). Proportions of buyers (sellers) of a certain type in the population are given by θ_P and θ_I (for patient and impatient traders respectively), where $\theta_P = 1 - \theta_I$.

Agents' Behavior. Bids and asks are determined as distances (J, expressed by the number of tick size) from the best outstanding offer on the opposite side of the book (a and b), given the actual spread (s):

$$\text{bid}_i = a - J$$
$$\text{ask}_j = b + J$$

with $J \in \{0, ..., s-1\}$ *(Ass.2) Limit orders must be price improving, that is, narrow the spread by at least one tick.*[3]

[1] Prices are discretized in the Paris Bourse (empirical evidence).

[2] We assume valuations (costs) to be equal across buyers (sellers), without loss of generality, to favor computational tractability and comparison.

[3] The choice to introduce a spread improvement rule in the model may appear as too much restrictive; however, this rule is applied in many real stock market exchanges (eg, NYSE). At the same time, empirical evidence is this direction exists even when that rule is not imposed by the market: "*A large fraction of the order placements improves upon the best bid or ask quote. Such improve- ments on one side of the quotes tend to occur in succession(undercutting),which reflects competition in the supply of liquidity.*" (Bias et al. 1994, p. 4)

Profits from trade are given by the following expressions:

$$\pi_i = v - p - \delta_i T$$
$$\pi_j = p - c - \delta_j T$$

where $v(c)$ is the valuation (cost) of the buyer (seller), p is the transaction price, $\delta_{i,j}$ is a measure for the waiting costs and T is the time elapsed between the submission of the offer and its cancellation from the book because of trade. At the end of the trading session, agents that do not trade make a negative profits equal to $-\delta_{i,j}T$.

3.2.1 Main Findings and Validation of the Agent-Based Model (ABM)

The first step was to replicate the model in Foucault et al. (2005). The aim of this exercise is twofold. On one hand, we test their equilibrium result by simulation; on the other hand, the code's validation encourages us to safely use this framework as a baseline for modifications and/or extensions related to the scopes of our analysis.

This section summarizes results in this direction. They are presented accordingly to the main findings of the paper. The authors claim that the key determinants of the market dynamics are the percentage of patient traders and the order arrival rate; we divide this section in two parts using the same categorization.

In order to favor a direct comparison, all the simulations are conducted according to the numerical examples in the original paper. We report the values of the parameters of the model in Table 3.1. Agents play as stated by the equilibrium strategies. Payoffs and time between arrivals[4] are realized values (not expected as in the analytical model) in the spirit of agent-based simulations.

	N	λ	θ_P	δ_P	δ_I	Range of admissible prices
Case (1)	1000	1	**0.55**	0.05	0.125	[20, 21.25]
Case (2)	1000	1	**0.45**	0.05	0.125	[20, 21.25]

Table 3.1: Parameters of the Model

However, the use of an ABM implies an important adjustment: the original model assumes a stream of agents over an infinite horizon; we need the number of traders to be finite (both for computational reasons and to allow agents to learn)[5]. This necessitates the introduction of a halting rule: the trading session ends when all traders had their chance to submit an order and no more trades are possible.

The percentage of patients traders

Book Dynamics and Resiliency. Figure 3.1 shows the book dynamics in two markets characterized by different bidding behaviors. It reports (left) two numerical example contained in the original paper and replicates them (right) by running simulations. The two scenarios differ

[4]We extract time between arrivals from a negative exponential distribution with parameter λ, accordingly to the Poisson process.

[5]This choice has an effect on the ability of the system to reach the suggested equilibrium result. Details in Sect 2.1.3.

only in the percentage of patient traders in the market (55% in the first line, 45% in the second line). Figures are rescaled to favor direct comparison. The competitive spread is reached quite fast in a market with a majority of patient traders; in a market characterized by the opposite composition the quoted spread remains quite large during all the trading sessions.

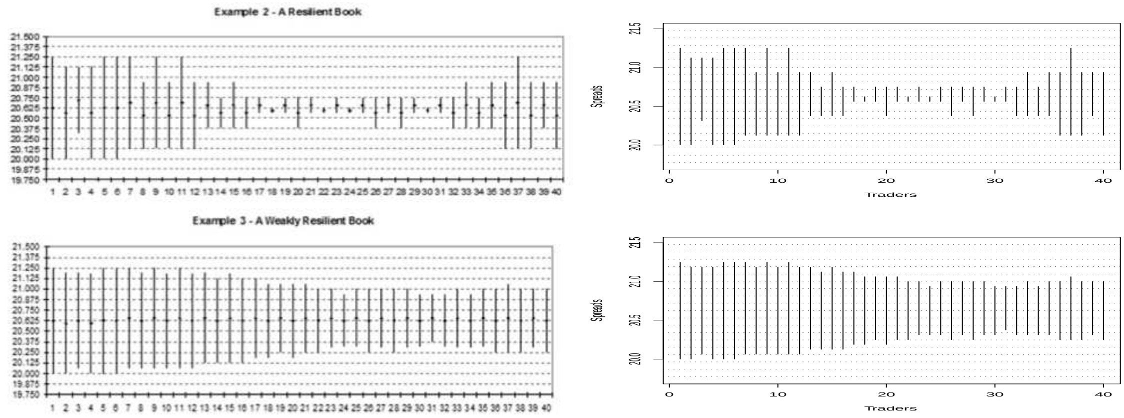


Figure 3.1: Source: Foucault et al. (2005), Fig. 3 p.1190 (*left*), Replication by ABM (*right*)

Result 1. The market appears much more resilient when the percentage of patient traders in the market is higher than the percentage of impatient traders.

This result is imputed by the authors to the bidding behavior: the equilibrium strategy suggested (and analytically derived) for markets in which patient traders prevail force them to submit more aggressive orders to speed up execution. The authors use the same sequence of traders' types in each example to be able to disentangle this effect; in fact, they claim that when realizations of traders' types differ, a market dominated by impatient traders shows small spreads less frequently due to an additional reason: the likelihood of a market order is higher.

Clearly, in a framework based on a finite horizon (and hence, on a finite number of traders N) the use of the same sequence of traders' types does not reflect differences in the composition of the market.

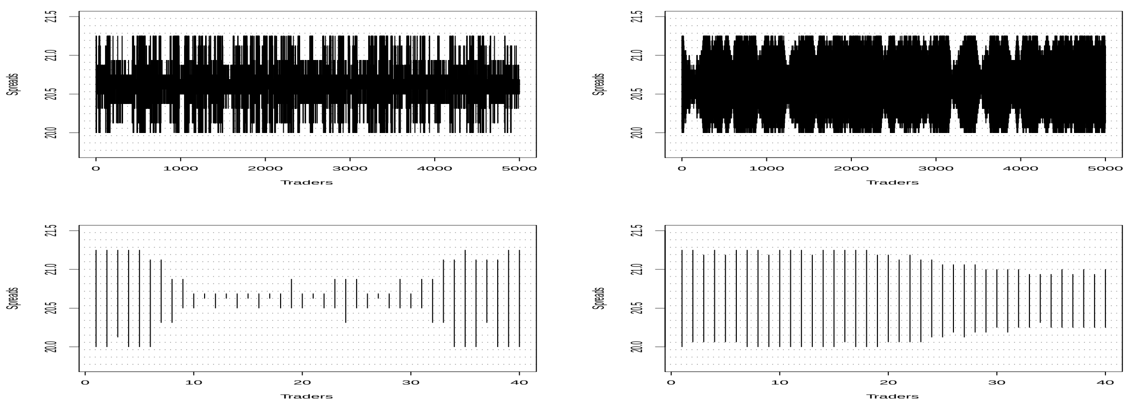


Figure 3.2: Book Dynamics (random sequence)

The use of two different (randomly chosen) sequences respecting the percentage of patient and impatient traders does not significantly affect the result. Figure 3.2 reports the book dynamics for markets populated by $N = 10000$ traders, distributed accordingly to the different percentage of patient traders in the two examples. The average spreads in the top-left market is 0.529, while the average spread in the top-right market is 1.005. Results are confirmed.

The first line shows the book dynamics of the first five-thousand submissions. The second line shows the book dynamics of the first forty submissions (to favour direct visual inspection). Even if the sequence of traders differs from the one used in Fig. 3.1, the book dynamics appear very similar and consistent with the finding in the paper.

Distribution of spreads. Figure 3.3 presents the stationary distribution of spreads in the two markets described above. As usual, we report and compare original and simulated results.

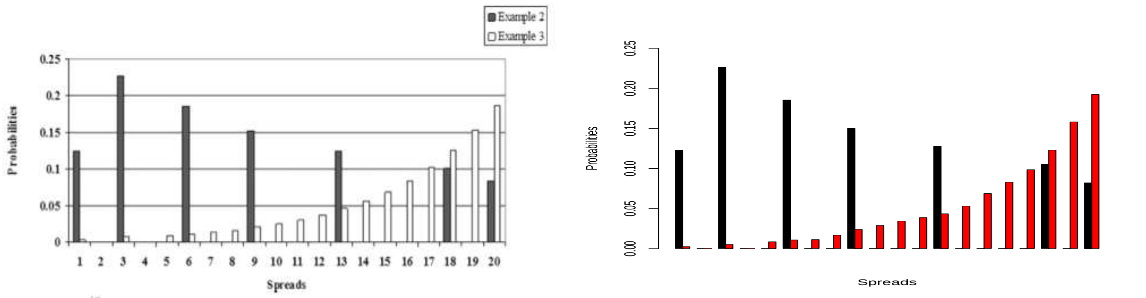


Figure 3.3: Source: Foucault et al. (2005), Fig. 4 p.1192 (*left*), Replication by ABM (*right*)

Result 2. The distribution of spreads becomes skewed toward low (high) spreads when the percentage of patient (impatient) traders is higher. The average spread is smaller in markets dominated by patient traders (more resilient) since small spreads are more frequent in this case.

State of the book and order aggressiveness. Spread improvements are a measure of order aggressiveness. They describe the amount (in number of ticks) by which the agent reduce the spread placing his offer. For example, given a spread of twenty ticks an optimal strategy for a patient agent is to place a limit order with $J^* = 18$; since J^* represents the new spread created by the offer itself, this means that the original spread is reduced by two ticks (namely, 20-18=2). Figure 3.4 reports the (optimal) spread improvements: as in the previous picture, results from a market dominated by patient (impatient) traders are shown in black (red).

Result 3. Spreads improvements increase (decrease) with the size of the spread when the market is dominated by patient (impatient) traders.

Measures of trading activities

The market composition determines resiliency; then, it is indirectly related to some measures of trading activities, namely (a) the order arrival rate and (b) the time between trades. We are interested in the former, since it is a key determinant of the book dynamics. Foucault et al. (2005) analyzes this relationship leading to the conclusions described below.

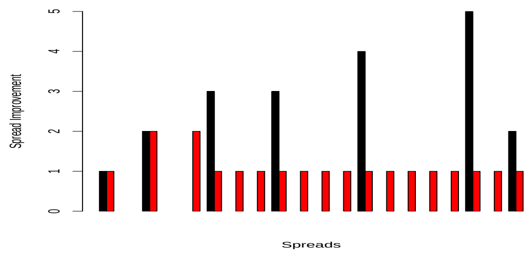


Figure 3.4: Spread Improvements by ABM

The order arrival rate. The authors identify two different effects with opposite impact on the average spread. On one hand, the average spread tends to be inversely related to the order arrival rate *(Corollary 2.1, p.1193) The support of possible spreads in the fast market is shifted to the left compared to the support of possible spreads in the slow market.* Figure 3.5 shows the spread distributions for fast (black, $\lambda = 1$) and slow (red, $\lambda = 0.2$) markets dominated by patient (left) and impatient (right) traders. Agents maximize following optimal strategies in Foucault et al. (2005)[6].

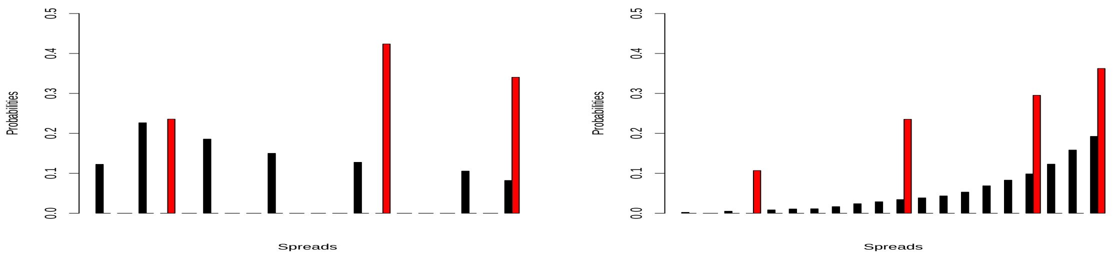


Figure 3.5: Distribution of Spreads by ABM model: Fast market vs Slow market

As stated in Foucault et al. (2005), higher spreads appear with higher probability in a slow market.

On the other hand, market resiliency is inversely related to the order arrival rate *(Corollary 2.2, p.1193) The slow market is more resilient than the fast market.*

Figure 3.6 shows the spread improvements. Traders are more aggressive in a slow market (red), i.e., the spread is reduced more than in the fast market (black). This holds for both markets dominated by patient traders (left) and dominated by impatient traders (right), and leads to more resilient books.

Given two different values of λ (that characterize "slow" and "fast" markets), which market exhibits the smaller spread on average depends on which effect prevails. Foucault et al. (2005) conclude by computation that, for a large set of parameters' values, the former effect (Corollary 2.1) dominates the latter; hence, in most cases there is a negative correlation between the order

[6]Computations for equilibrium order placement strategies in slow markets with $\lambda = 0.2$ are reported in appendix.

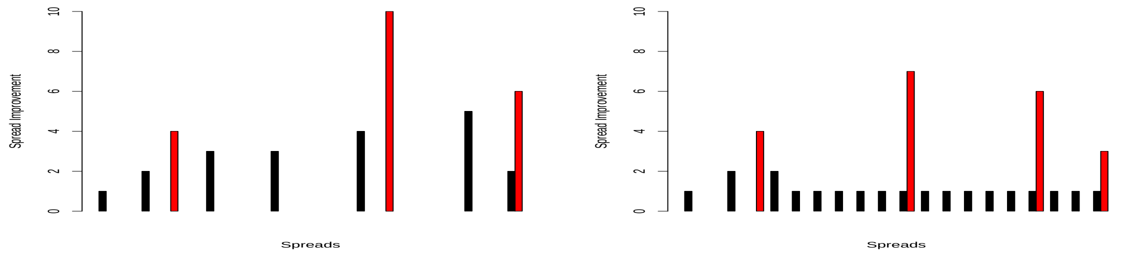


Figure 3.6: Spreads Improvements by ABM model: Fast market vs Slow market

arrival rate (λ) and the average spread.

Result 4. Fast market exhibits lower spreads with respect to slow markets (i.e., higher values of λ decrease the average spread).

Figure 3.7 shows the book dynamics for the first fourty traders in fast (black) and slow (red) markets characterized by different percentage of patient traders ($\theta_P = 0.55$ in the first line, $\theta_P = 0.45$ in the second line). Table 3.2 reports average spreads in fast and slow markets populated by $N = 10.000$ traders. Results 4 in Foucault et al. (2005) is confirmed by simulation for markets dominated by patient traders but not always for markets that exhibit the opposite composition of traders' types. This conclusion is not surprisingly due to the (previously mentioned) fact that the likelihood of a market order is higher in the latter markets and this implies that small spreads are shown less frequently. This effect tends to be amplified by a higher order arrival rate (i.e., in a fast market).

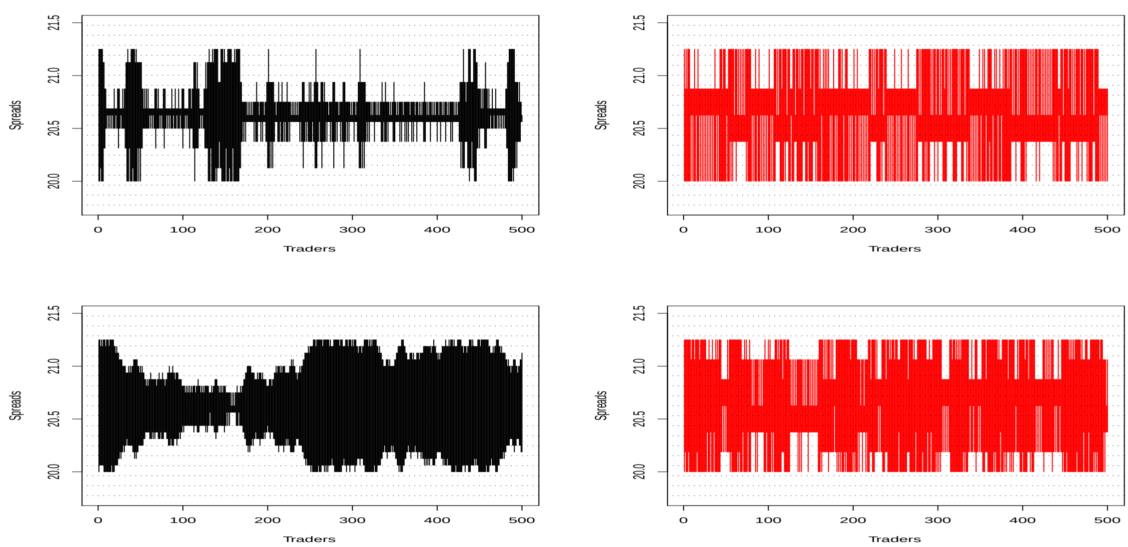


Figure 3.7: Market Resiliency by ABM model: Fast market vs Slow market

Table 3.2: Average Spreads in Fast/Slow Markets populated by different percentage of patient traders (N=10000)

	Fast Market ($\lambda = 1$)	**Slow Market** ($\lambda = 0.2$)
$\theta_P = 0.55$	0.53045	0.858325
$\theta_P = 0.45$	0.9996625	0.9537125

Equilibrium Strategies

Result 6. In equilibrium, patient traders tend to submit limit orders, whereas impatient traders submit market orders.

Definition 1. *An equilibrium of the trading game is a pair of order placement strategies, $\sigma_P^*(.)$ and $\sigma_I^*(.)$, such that the orders prescribed by the strategies solve*

$$\max_{j \in \{0,\dots,s-1\}} \pi_i(j) \cong j\triangle - \delta_i T^*(j) \tag{3.1}$$

when the expected waiting time $T^*(.)$ is computed assuming that traders follow strategies $\sigma_P^*(.)$ and $\sigma_I^*(.)$.

We use a test to check if the equilibrium result is confirmed by simulation. The baseline setup replicates the parameters (of the model) used in the numerical example in Foucault et al. (2005) for both market dominated by patient traders and market dominated by impatient traders. The suggested strategy-profile to be tested (according to Eq. 3.1) are the following:

	spread																				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	composition of the market
J (patient)	0	1	1	3	3	3	6	6	6	9	9	9	9	13	13	13	13	13	18	18	
J (impatient)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\theta_P = 0.55$
J (patient)	0	1	1	3	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
J (impatient)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\theta_P = 0.45$

Table 3.3: Equilibrium order placement strategies, Table 2, p. 1187 (Foucault et al., 2005). Parameter values: $\lambda = 1$, $\triangle = 0.0625$, $\delta_P = 0.05$, $\delta_I = 0.125$

A randomly selected agent play a certain number of independent rounds (R); at each round he can submit his offer only once, choosing a random strategy in the feasible set given the actual spread. All the other agents play accordingly to the strategy reported in Table 3.3. Average payoffs are computed for strategies on and off the equilibrium path (i.e., for every possible deviation), conditional on spread. Then, the maximizing strategy is identified and correspondence with the equilibrium strategy in Foucault et al. (2005) is checked.

We run two simulations for each market, one for each type of players (patient or impatient). By symmetry, the difference in role (i.e., buyers or sellers) need not be considered.

We consider three different scenarios. The benchmark is a market populated by $N = 1000$ traders that play for 1.000.000 consecutive and independent trading sessions; in the other two cases, the number of agents in the market and the number of rounds over which average payoffs are computed are (alternatively) reduced to test their role in reaching the expected equilibrium. Table 3.4 reports results for patient traders in a market dominated by patient traders. Results are similar for impatient traders.

	Equilibrium Spreads						
	1	3	6	9	13	18	20
(a)	0	1	3	6	9	13	18
(b)	0	1	1	6	9	13	18
(c)	0	1	1	6	12	6	18
Foucault	0	1	3	6	9	13	18

Table 3.4: Test on Equilibrium, $\theta_P = 0.55$, maximizing strategies for patient traders.

As it can be easily seen by direct visual inspection, the equilibrium result is confirmed: given a sufficiently large number of traders and rounds over which average payoffs are computed, the strategy suggested in Foucault et al. (2005) maximizes the payoffs of patient (who place limit-orders) and impatient (who place market orders) traders in both markets. The choice of a high N is crucial. In fact, as previously mentioned, Foucault et al. (2005) construct a model over an infinite horizon, considering a stream of traders who continuously submit an order. If we redo this exercise with a lower N the same equilibrium result is no longer obtained.

At the same time, the choice of R (i.e., number of rounds over which average payoffs are computed) is even more decisive. In fact, the size of the range of admissible prices is not particularly large compared with the number of traders in the market, valuations and costs do not differ across buyers and sellers and are chosen outside the interval in which agents can submit their offers. As a joint effect of these facts, differences in payoffs are minimal and a large number of observations is needed to figure out which strategy is really the optimizing one.

3.3 Is this (unique) equilibrium learnable?

Foucault et al. (2005) claim that Eq. 3.1 describe the equilibrium order placement strategies and this equilibrium is unique by construction (Foucault et al. 2005, p.1179). However, the recent behavioral literature strongly suggest that agents are not able to behave as a profit-maximizer due to their own computational limitations. Since we expect that individual agents participate in the market to exploit as much as possible from transactions, a first important issue we deal is to try to answer the following question: are agents able to learn the suggested equilibrium strategies adjusting their trading behavior on the basis of their own past histories? From a practical point of view and with the aim to provide some market-design policy implications we consider this an important point at issue.

The recent literature has provided many learning models and there is no clear (definitive) reason to choose one of them instead of another. Given the recognized ability of evolutionary techniques to discover maximizing results, we decide to implement a genetic algorithm that will be described in detail in the next section[7].

3.3.1 The Design of the Evolutionary Algorithms

Following Ashlock (2002, 2006), we describe how trading strategies evolve answering five different questions.

What data structure will you use (*gene*)? The population of N traders is initially divided into two subgroups accordingly to the parameter θ_P; namely, NP (patients traders) and NI

[7]We have also tried to implement a reinforcement learning algorithm; it leads to similar conclusions. Reinforcement learning is a well-known and commonly applied algorithm in framework in which people learn using only data available from their own past histories; it does not imply any other additional source of information.

(impatient traders). Each of these groups represents a population that evolves autonomously. In fact, we are dealing with a kind of learning that is known in the literature as *type-learning*: "agents of several distinct types interact with each other, but they only learn from successful agents of their own type" (Vriend, 2000). Each trader is endowed with a string that describes a strategy to be implemented: for each possible spread faced (s) it indicates which strategy the trader has to play (J). Unfeasible strategies are ruled out. Only pure strategies are considered.

What fitness function will you use? Learning is driven by the average profit of a profile of trading strategies. It is computed over an evaluation window defined by a fixed number of trading sessions (R); strategies are updated using a genetic algorithm every R consecutive days.

What crossover and mutation operators (i.e., *variation operators*) will you use? Children are expected to preserve some part of the parents' structure; mimicking sexual reproduction, the crossover operator maps parents' genes onto children's genes. We choose a *uniform crossover* in order to avoid representational bias[8]: "this crossover operator flips a coin for each locus in the gene to decide which parent contributes its genetic value to which child" (computational expensive). The mutation operator makes small changes in a gene with the aim to (a) be sure that all the possible profiles of strategies have a chance to be exploited and (b) "good" profiles that perform bad by chance and are randomly substituted have a chance to re-enter the game. Inspired by Lettau (1997) we apply mutation as follow: with probability decreasing in time (as measured by the cumulative number of revision, i.e. with probability 1/n. of revisions by GA) we substitute one of the siblings' actions with a pure action selected with uniform probability among all the feasible strategies.

How will you select parents from the population and how will you insert children into the population (i.e., *Model of Evolution*)? Parents are selected using the *double tournament selection* method; a subgroup of size n is picked and the fittest one is taken as a parent. The procedure is repeated in order to choose the second parent; a new different subgroup of n traders is randomly drawn from the same initial pool of N with replacement, i.e. the same parent can be picked twice. Children are placed in the population using the *absolute fitness replacement* method; "we replace the least fit members of the population with the children".

What termination condition will end your algorithm? The simulation ends after a fixed number of trials (= $R\times$ n. of revisions by GA).

Figure 3.8 illustrates the pseudo code of a standard simple evolutionary algorithm. Details concerning the implementation of the genetic algorithm in our simulation are fully described in appendix.

3.3.2 (ϵ-)Equilibrium

The genetic algorithm implemented leads to an "equilibrium" profile as it can be learned by agents. This equilibrium differs from the one discussed in Foucault et al. (2005) for many reasons. Firstly, an agent-based simulation, consistently with what happens in real markets, considers a finite number of traders for each trading session; as a consequence, the equilibrium suggested by the authors cannot be reached unless we use a quite high number of traders and reviews of strategies are based on average profits over a large evaluation window. Secondly, the

[8]Ashlock (p.39) highlights a problem with single point crossover: "loci near one another in the representation used in the evolutionary algorithm are kept together with a much higher probability than those that are farther apart."

```
Create an initial population.
Evaluate the fitness of the population.
Repeat
    Select pairs from the population to be parents, with a fitness bias.
    Copy the parents to make children.
    Perform crossover on the children (optional).
    Mutate the resulting children (probabilistic).
    Place the children in the population.
    Evaluate the fitness of the children.
Until Done.
```

Figure 3.8: A Simple Evolutionary Algorithm [source: Ashlock 2002, p. 31]

reached result in terms of strategies chosen by the agent is not unique; learning takes time and is strictly related to both the initial conditions (i.e., random inizialization of the agents' strings that describe the strategy), the kind of mutation implemented, the parameters of the model and the path of learning itself. So, the learning process may leads to different profile of strategies.

We have to deal with two different issues. First, we need to test if we actually obtain an equilibrium strategy profile. To do that, we provide a quality control for the GA strategies. As a second step, we want to test if the main predictions in Foucault et al. (2005) still hold. If this is the case, we may claim that, even if the results formally differ, the main properties of the model itself remain valid and its explanatory power remains unchanged. We will test Foucault's predictions using one of the "equilibrium" profiles we end up with; even if the learning process leads to different profiles of strategies we argue that they all replicate the same features of the model. Even if they cannot be considered as strategically equivalent, we can claim outcome equivalence.

Quality control of the GA strategies

Definition 2. *Given $\epsilon > 0$, a (possibly randomized) strategy profile σ is an ϵ-equilibrium if*

$$u_i(\sigma_i, \sigma_{-i}) \geqslant u_i(s_i, \sigma_{-i}) - \epsilon, \ \forall i, s_i \in S_i$$

When $\epsilon = 0$, we go back to the standard definition of a Nash Equilibrium.

We can consider ϵ as a measure of "how much" the strategy profile that we are going to test approximates an equilibrium; the lower the ϵ, the lower is the temptation for an agent to deviate (deviation is weakly profitable), and the closer we are to an equilibrium.

Two computational problems arise given our setup: (a) our genetic algorithm (as behavior of learning processes in general) does not lead to the same equilibrium strategy profile and (b) agents of the same type (namely, *patients* and *impatients*) does not converge to a unique strategy in each GA-implementation. To deal with these issues, we set up a quality control as described below.

We consider 10 different profile of strategies, obtained by specific GA simulations. For each of them, we randomly pick up a specific number of agents (namely, $N/10$ - where N is the total number of agents in the market). Then, a simulation is run for all traders under examination (separately); taking as fixed the strategies of all the other players, deviations from the original strategy to each feasible alternative pure strategy are experimented. We list average profits gained by the tested agent over the number of rounds played for each possible alternative strategy.

Finally, we compare the original strategy to be tested with the one that comes out choosing for each actually faced spread the pure strategy that leads to the maximum (average) profit (ceteris paribus). Since spreads might be faced with different frequencies, expected profits are weighted according to these frequencies. The difference between the maximum expected profits and the expected profits given the original strategies are computed; they represents our ϵ(s). We also compute the median and the mean value of ϵ for each scenario (when scenarios differ by the number of traders in the market). This gives an intuition of how ϵ might be distributed.

N	number of picked up agents	number of simulations	ϵ_{max} (type)	ϵ_{median}	ϵ_{mean}
1000	100	1000	0.05 (P)	0.0312	0.0289

Table 3.5: ϵ-equilibrium test

Table 3.5 summarizes the results. Results do not seem to be too far from an equilibrium: ϵ is sufficiently small. ϵ_{median} and ϵ_{mean} do not differ significantly; this suggests that ϵ_{max} cannot be considered an exceptional result but it represents quite well a worst-case benchmark[9].

3.3.3 Results

All the results presented in this report are (not surprisingly) quite sensible to the chosen parameters. To favor direct comparison (and "validate" our GA-model), we replicate the setup as in the numerical examples by Foucault et al. (2005). Table 3.1 reports the parameters used in the first round of simulations.

Results are shown with respect to a specific outcome of the genetic algorithm procedure that is conducted using an evaluation window of $R = 1000$ trading sessions and allowing agents to have (at least) 2500 possibility to revise the adopted strategy (i.e., the GA algorithm is implemented 2500 times). Once this learning procedure is completed, we save the genes that represent the strategies chosen for each spread by each traders in both populations. Then, keeping the strategy profile fixed, we run 200 independent trading sessions over which we compute average values of outcome variables.

This is not the only possible scenario and, possibly, it might be not the best. However, we claim that the results shown are without loss of generality since all the realizations are outcome equivalent even if not strategically equivalent.

In order to answer the research question "*is this equilibrium learnable?*", we conduct two different simulations.

In the first one, we construct an initial setup in which the 70% of the population of each type play accordingly to the Foucault equilibrium strategies in Table 3.3[10]. The aim of this exercise is to figure out if, with a sufficiently high number of players whose gene replicates the equilibrium choices, the equilibrium result is achieved (i.e., all players converge to the same strategy as in Foucault et al. 2005).

[9] We also checked markets with different size. As expected, the number of traders in the market seems to be a key-determinant the value of ϵ; it is quite intuitive: higher N, finer is the learning process and, as a consequence, the better is the equilibrium's approximation.

[10] It is important to note that also players endowed with the strategy in Foucault are allowed to learn and switch to a different choice; this setup concerns only the initial conditions

The percentage of patient traders

The distribution of spreads[11] in market dominated by patient traders (black) is very similar to the distribution of spreads in Foucault even if a positive probability is attached to low spreads out of equilibrium path. In the opposite case (red, $\theta_P = 0.45$) the probability to face low spreads (namely, $s = 1$ and $s = 2$) is significantly higher than in the benchmark case in which all players play accordingly to the equilibrium strategy (see Fig. 3.3 for a quick direct comparison).

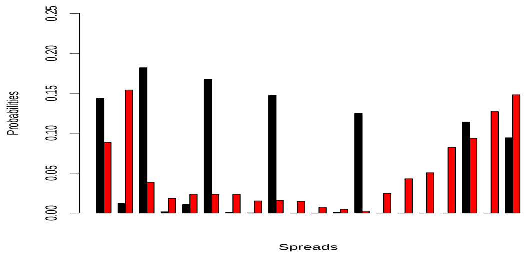


Figure 3.9: Distribution of Spreads

A possible explanation for this finding is that (due to the spread improvement assumption) the probability to choose a lower j is higher than the probability to choose a high j for the percentage of traders who initially choose randomly this own strategies; as a consequence, small spreads appear more frequently. This effect is amplified in markets dominated by impatient traders since this type of players usually behave more aggressively and their behavior contributes to narrow the spread.

Results about order placement strategies are shown in Fig. 3.10 (left: $\theta_P = 0.55$, right: $\theta_I = 0.45$) only for spreads that appear with sufficiently high probability (namely, probability greater than 0.012).

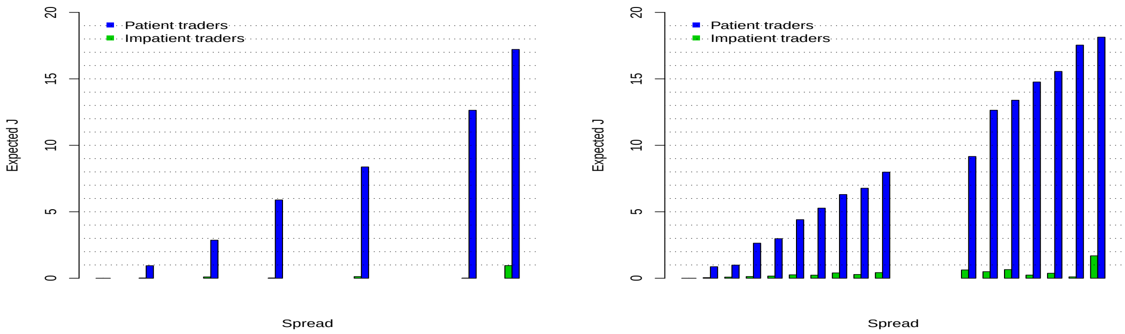


Figure 3.10: Expected Js given spread, $J \in [0, 19]$ - left: $\theta_P = 0.55$, right: $\theta_P = 0.45$

[11] We choose to plot average probabilities. This means that for each spread, the reported value is the mean of the probabilities with which that spread was faced over 200 rounds. This implies that probabilities might not sum up to one. However, there is no significant difference between this picture and the same plot constructed using data from each of the 200 rounds in isolation.

Expected Js are computed looking at the share of population who choose a specific j, conditional on the spread. For example, given a current spread of two ticks, by the improvement rule the only feasible choices for an agent who have to submit an offer are to place (1) a j-limit order with $j = 1$ or (2) a market order (i.e., choose $j = 0$). If the 80% of the population make the first choice and the rest adopt the residual strategy, then the expected j will be $0 \times 0.2 + 1 \times 0.8 = 0.8$.

A lower (higher) probability to face high (low) spreads has a direct consequence also on trading behavior. In fact, as a general result, equilibrium strategies given high spreads are more difficult to be learned and the distance between the outcome from the learning process and the equilibrium strategies in Foucault et al. (2005) is increasing in the size of the spread.

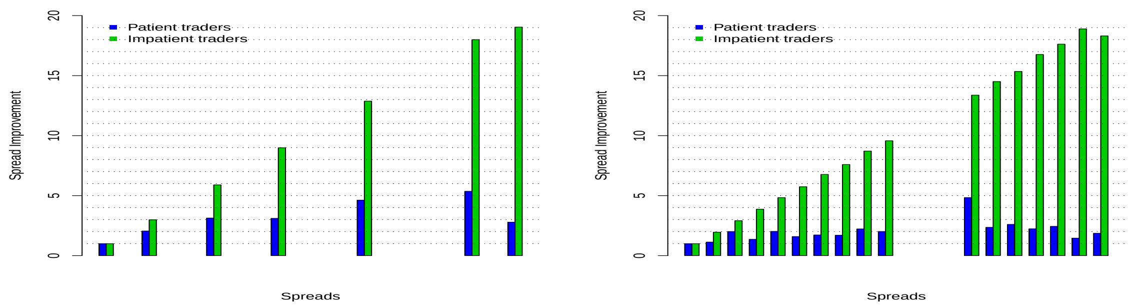


Figure 3.11: Spread Improvements - left: $\theta_P = 0.55$, right: $\theta_P = 0.45$

Trading behavior in markets dominated by impatient traders is always less aggressive for both patient and impatient traders with respect to markets dominated by patients due to the joint effect of a higher probability to face low spreads[12] and a general more aggressive behavior of impatient traders compared to patients. Impatient traders, in fact, are more aggressive in both types of markets. This can be easily seen by visual inspection of Figs. 3.10 and 3.11 where spread improvements are reported.

It is interesting (and not surprising) to see that the trading behavior for which impatient traders maximize their choice submitting a market order is more difficult to learn in markets dominated by impatient traders. However, coherently with the result in Foucault we can state that impatient traders tend to go market (their expected j is lower than 1) with a significative exception when they face the maximum possible spread given the range of admissible prices.

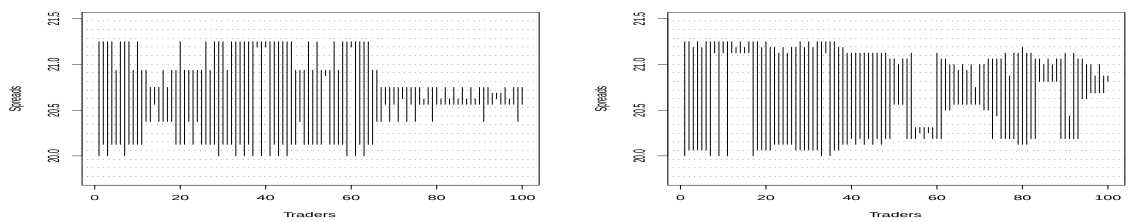


Figure 3.12: Book Dynamics and resiliency

[12]Remember also that lower spreads increase the probability to trade.

The described bidding behavior strategies lead to a book dynamics that appears more resilient in markets dominated by patient traders than in markets with the opposite distribution of types, coherently with findings in Foucault. Average spreads are 0.543 (Fig. 3.12 - *left*) and 0.741 (Fig. 3.12-*right*) respectively.

Measures of trading activities. The order arrival rate.

In this subsection we analyze the impact on learning outcome of the other key determinant of the book dynamics introduced by Foucault et al. (2005): the order arrival rate.

In particular, we compare results in the previous section (simulated using $\lambda = 1$) with the performance of a slower market characterized by a Poisson arrival process with parameter $\lambda = 0.2$ (i.e., the average time between arrivals is $1/\lambda_2 = 0.2 = 5 > 1 = 1/\lambda_1 = 1/1$). This allow us to refer to the Foucault equilibrium strategies computed in Sect. 2 and reported in appendix.

Analizing the slow market with different composition of types in isolation, all the observations made in the previous section still hold (see figures in the next page for a direct comparison).

Table 3.6: Average Spreads in Fast/Slow Markets populated by different percentage of patient traders (N=1000)

	Fast Market ($\lambda = 1$)	**Slow Market** ($\lambda = 0.2$)
$\theta_P = 0.55$	0.543135	0.763289375
$\theta_P = 0.45$	0.7409490625	0.7913225

Let us compare slow and fast markets. We have discussed in Sect. 2 the two opposite effects that make difficult to determine which market displays a lower average spread. The main finding in Foucault is confirmed by our simulation: "*Fast markets exhibits lower spreads than slow markets*" (for a large set of parameters). However, we claim that the ambiguity in the sign of inequality is more relevant in markets dominated by impatient traders probably because of the fact that impatient traders place market orders more frequently. Results are shown in Table 3.6.

A final observation concerns analogy with the findings in Foucault et al. (2005) concerning the order arrival rate. First of all, the distribution of spreads is skewed towards the right in slow markets (Fig.16), coherently with Corollary 2.1., p.1193 in the original paper. Secondly, the trading behavior is more aggressive in slow markets (Figs. 13, 14, 15) and this leads to a more resilent market, as stated by Corollary 2.2., p.1193. This suggest and confirm that both effects play a role in determining the average spread in the market and mixed results are possible and justified.

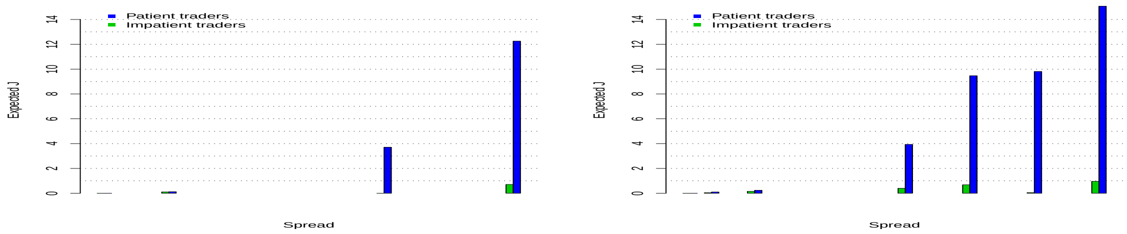


Figure 3.13: Expected J

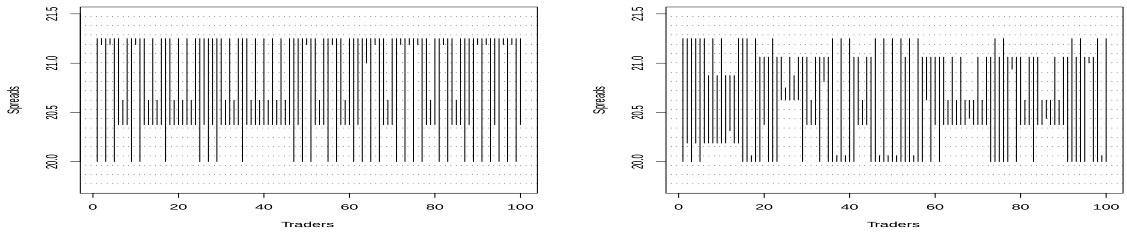


Figure 3.14: Book Dynamics

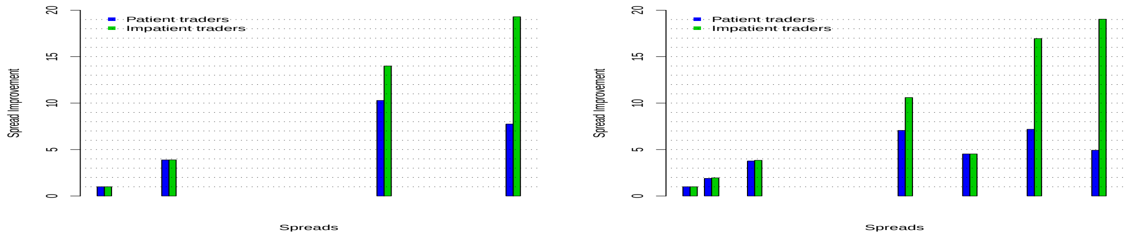


Figure 3.15: Spread Improvements

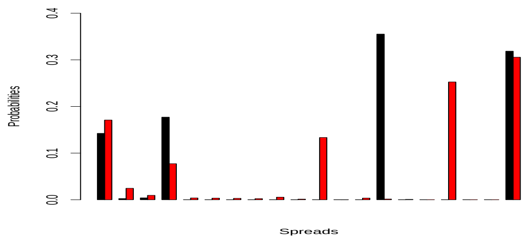


Figure 3.16: Distribution of Spreads

Increasing (initial) randomness

We conduct a second simulation increasing the level of randomness in the initial setup. In particular, we reduce the percentage of traders that initially play Foucault's equilibrium strategies to 30%[13]. Let us concentrate on the main differences in results due to the use of this alternative framework.

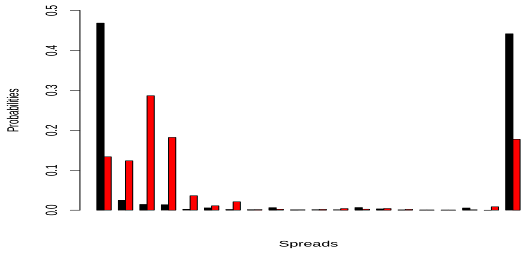


Figure 3.17: Distribution of Spreads

The more interesting observation concerns the distribution of spreads. As it can be easily seen by visual inspection in Fig. 17, lower spreads are faced with an even higher probability and medium spreads have probability zero (or close to zero) to be faced. This is a direct effect of the increase in initial randomness that is related with an higher probability to choose a low J since lower values are feasible for almost all the possible spreads. This effect seems to be more relevant in markets dominated by patient traders probably as a consequence of a decrease (on average) in aggressiveness for impatient traders.

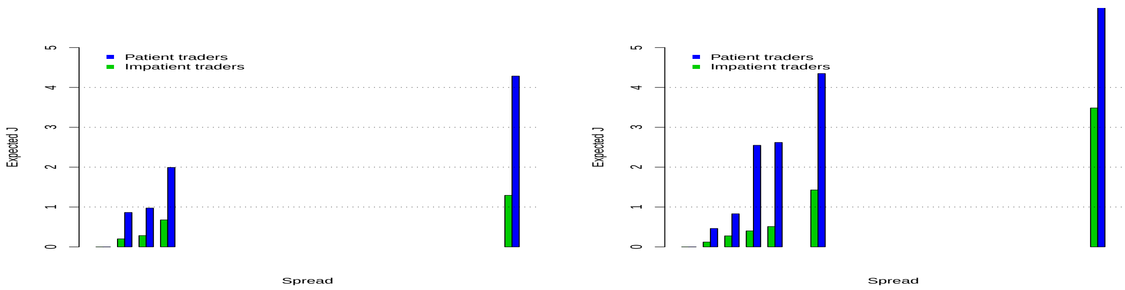


Figure 3.18: Expected J (order placement strategies)

Note that for high spreads (namely, $s = 20$) also patient traders play aggressively, see Fig. 19, first line. This suggests that the joint effect of the aggressive behavior and the high probability to face low spreads may force the system to converge to a situation in which a loop is generated: for long periods only marginal spreads (i.e, the maximum spread $s = 20$ and the minimum spread $s = 1$) are faced and trades occur in succession[14].

There is a direct consequence of this also on the average spreads, see Table 3.7.

As a consequence of the described loop, we observe a lower average spread in fast market dominated by impatient traders. In fact, the average spread in market characterized by the

[13]We test also a completely random initial setup and results turn out to be equivalent.

[14]As we will see in Sect. 5, this will have an important effect on the efficiency of the market.

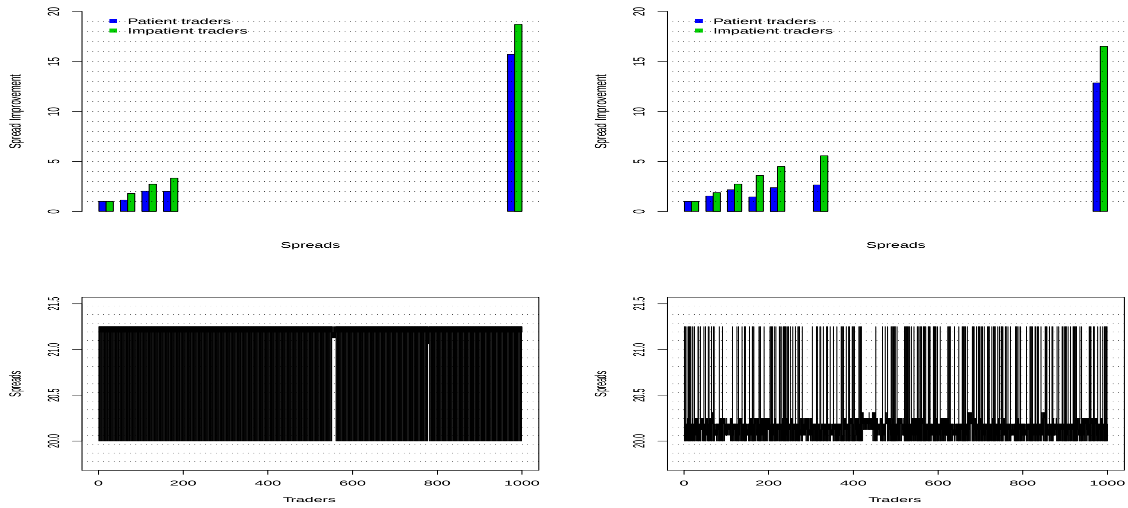


Figure 3.19: Spread Improvements and Book Dynamics

Table 3.7: Average Spreads in Fast/Slow Markets populated by different percentage of patient traders (N=1000)

	Fast Market ($\lambda = 1$)	**Slow Market** ($\lambda = 0.2$)
$\theta_P = 0.55$	0.6163025	0.6550984375
$\theta_P = 0.45$	0.395001875	0.877490625

opposite distribution of types ($\theta_P = 0.55$) is slightly lower than the half-difference between the maximum and the minimum feasible spreads (namely, $20\triangle/2$=1.25/2=0.625) while the average spread when the market is dominated by impatient traders is quite low due to the fact that (1) lower spreads are faced with higher probability, (2) medium spreads are faced with positive probability (in particular $s = 5$ and $s = 7$) and (3) traders react less aggressively to $s = 20$.

This effect is mitigated in slow markets due to the fact that, as before, distribution of spreads is skewed towards the right, low spreads are faced with probability close to zero and patient and impatient traders behave aggressively, see figures in appendix[15].

We can conclude that the ϵ-equilibrium profiles of strategies obtained by simulation implementing our GA algorithm is able to replicate the main features of the model in Foucault et al. (2005) even if with some discussed differences. Moreover, if we randomize the initial adopted strategies, equilibrium turns out to be very difficult to be learned.

Since they can be considered outcome equivalent, we will use it as a benchmark searching for (possibly) better protocols.

[15]For completeness, figures that refer to order placement strategies, spread distribution, spread improvements and book dynamics in slow market with increasing (initial) randomness are reported in appendix.

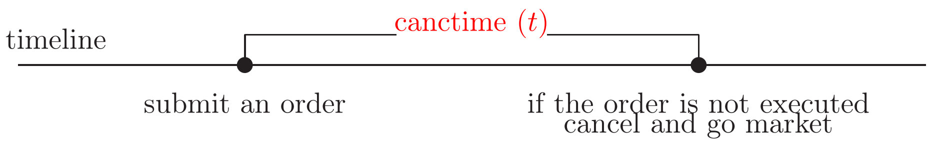
3.4 Adding Cancellation

The second contribution of this paper is to study what happens when the restrictive assumptions for which orders might not be cancelled is relaxed. Cancellation is a key issue in the recent literature and it turns out to be very difficult to analyze analytically. In fact, the introduction of some cancellation rules causes a significative increment in the level of sophistication of the model and makes it much more difficult to figure out a closed form solution for the equilibrium strategies.

We allow traders to cancel their orders under the condition that the "cancellation chance" might be taken only once at a time (i.e., if you decide to retract an order, then you cancel your outstanding offers and immediately go market). This seems to be (still) a restrictive assumption but it is the easiest one, since you only need to decide your willingness to wait (measured by t), and it may be justified by the fact that in a context in which (a) there are no many traders and (b) the spread-improvement rule applies, if you place an order and it does not pass after a "reasonable" amount of time, it might be convenient to cancel it and there is no room, both in terms of time and profitability, for placing another limit order.

Let us describe the main changes in the model:

- *Strategies*. Differently from before, each agent choses a pair $(j, t|s)$ where j is the distance from the best offer on the opposite side of the book that determines his/her own bid/ask and t is the interval of time after which, if the order is not executed, the trader decides to cancel the previous offer and place a (new) market order. t is computed from the point in time in which the offer is placed and is chosen on a discrete grid. Taking into account the value of the grid ($\triangle = 0.0625$), the size of range of admissible prices ($K\triangle = 1.25$) and the number of traders in the market ($N = 1000$), we define the set of admissible $t \in \{2, 4, 8, 16, 32\}$.



- *Halting rule*. A session closes when (a) all traders have placed their offers, (b) no more trades are possible, and (c) no more cancellations are in line to be activated given the chosen strategy (j, t).

Both from a point of view of market evaluation and market design, it is interesting to understand what happens when cancellation is introduced. We apply the same genetic algorithm with the only difference that now two genes for each trader evolve simultaneously: the first one indicates which j the agent will choose, given the current spread s (exactly as before); the second associates a specific t in the feasible set for each admissible j. The process of learning, then, is not more complicated (apart from the increasing computational effort) but it takes more time to converge towards a clear pattern. In this section we summarize the main results. As in the previous section, we first show conclusions for a market in which 70% of the population is initialized using Foucault's strategies; then, we compare results with a second simulation in which we increase the level of randomness in the initial setup. It has to be observed that this exercise has (partially) a different meaning from the one performed before. In fact, analyzing learning we wanted to see if there was or not convergence to the (unique and known) equilibrium. Here, changing the market structure due to the introduction of a chance to cancel an offer affects also the equilibrium strategies. Trying to compute them analytically is exactly what is difficult in a context close to reality (and the motivation for which we conduct a computational analysis). We do not expect convergence to a unique strategy profile but we want to compare situations with the same initial setup to be able to isolate the effect of learning combined with the cancellation issue.

3.4.1 The percentage of patient traders

We initially comment on the distributions of spreads shown in Fig. 3.20 where we compare scenarios with and without cancellation. As it can be easily seen from direct visual inspection, distribution of spreads is skewed towards the right; this means that higher (lower) spreads are faced with higher (lower) probability. At the same time, deviations from spreads on the path described in Foucault (that previously was an equilibrium) is more important than in the case without cancellation.

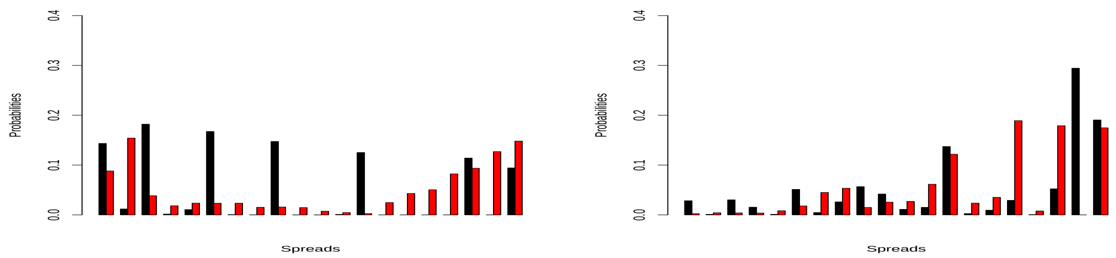


Figure 3.20: Distribution of Spreads without (left) and with (right) cancellation

A first reason for that is a direct consequence of the cancellation rule. In fact, when an order is removed by cancellation the current spreads that will be faced by the next trader in line enlarges. However, we claim that this is not the only motivation. To explain this point, have a look at Fig. 3.21.

One of the most important conclusion in Foucault et al. (2005) is that in a market dominated by patient traders, impatient traders do not submit limit orders. Such a conclusion partially still holds when we consider the result of the learning process, as discussed in the previous sections. The introduction of a cancellation chance modify significantly this statement. If we compare (expected) order placement strategies with (first line) and without (second line) cancellation we can observe that, on average, impatient traders tend to submit higher J-limit order in both markets under this second scenario. Also, patient traders on average place orders that reduce less the spread. Hence, when I get the chance to retract an order, aggressiveness decreases since my willingness to sustain the risk of no-execution is higher (the risk itself is mitigated by the possibility to go market as a second choice).

This lower aggressiveness in trading behavior is confirmed by lower values of spread improvements as reported in Fig. 3.22. Note that aggressiveness is reduced more for impatient traders than for patient traders (on average); this leads to the following observation. On one side, cancellation gives to both types of players the opportunity to revise their choice; this contributes to increases the j placed. At the same time, as a consequence of this fact, impatient traders becomes more "competitive" for trade in the market than in the previous scenario. Then, patients reduce their spread improvements but less than their opponents to partially mitigate this effect

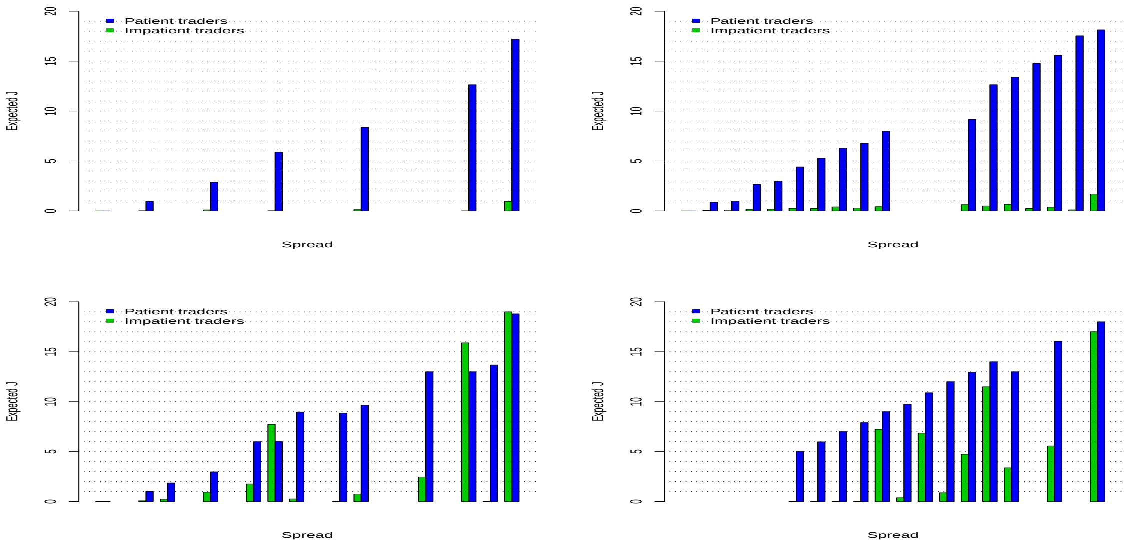


Figure 3.21: Expected J - Order placement strategies - left: $\theta_P = 0.55$, right: $\theta_P = 0.45$

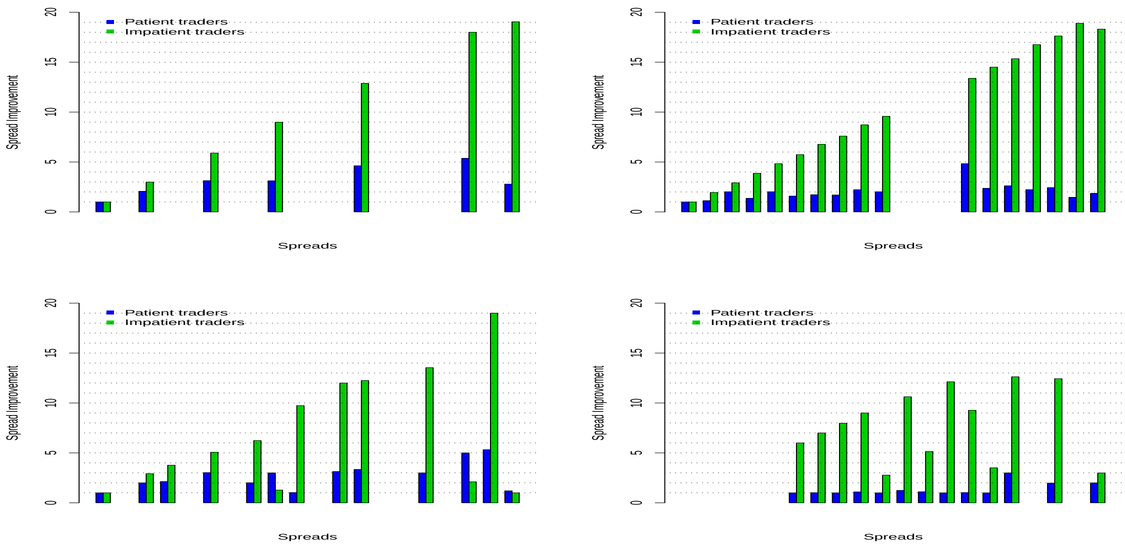


Figure 3.22: Spread Improvements without (top) and with (bottom) cancellation

and sustain the reduced probability to conclude a transaction at a favorable price.

Finally, not surprisingly, since higher spreads are faced with higher probability the book appears to be less resilient with no cancellation, see Fig. 3.23.

3.4.2 Measure of trading activities. The order arrival rate.

As we have done for the learning process, in this section we analyze the impact on the joint effect of learning and cancellation of variation in the Poisson arrival parameter λ.

Again, analyzing the slow market in isolation all the observations done above still holds with

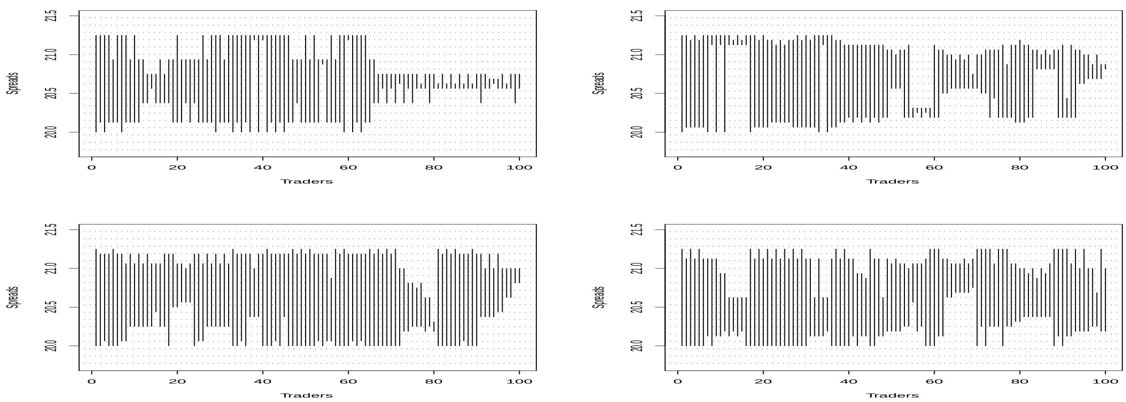


Figure 3.23: Book dynamics without (top) and with (bottom) cancellation

only one interesting exception: in slow markets impatient traders tend to submit market orders, then their order aggressiveness increases (see figures in the next page).

If we concentrate our attention on the comparison between fast and slow markets, results on average spreads are reported in Table 3.8. All the conclusions in Sect. 3.2.4 still holds.

As an additional observation, note that average spreads are always greater under cancellation. This can be seen as a direct consequence of the higher average time between arrivals. In fact, using the same timeline, in slow markets people have to wait longer and there is more room for cancellation.

Table 3.8: Average Spreads in Fast/Slow Markets populated by different percentage of patient traders (N=1000)

	Fast Market ($\lambda = 1$)	**Slow Market** ($\lambda = 0.2$)
$\theta_P = 0.55$	0.922603125	0.94617
$\theta_P = 0.45$	0.9173509375	1.0313659375

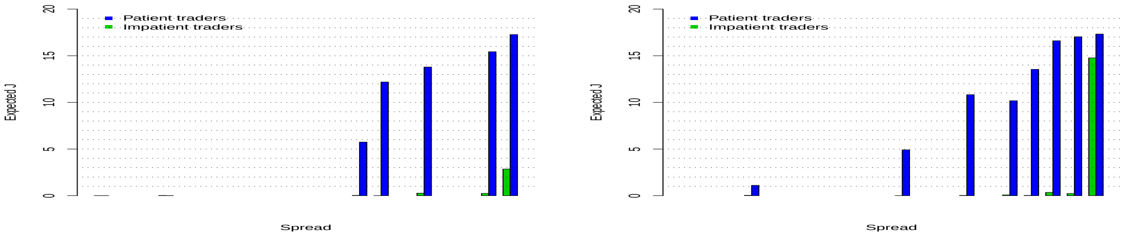


Figure 3.24: Expected J

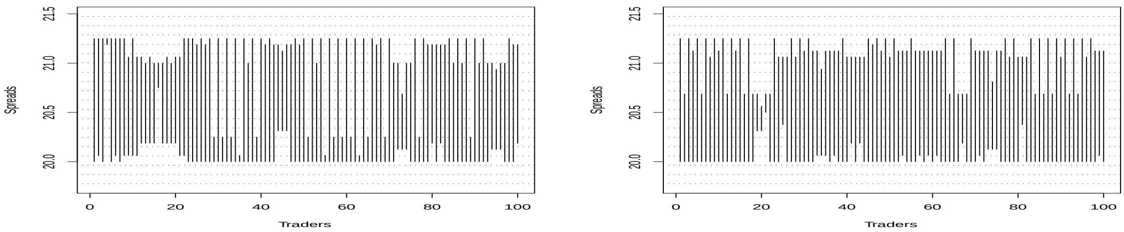


Figure 3.25: Book Dynamics

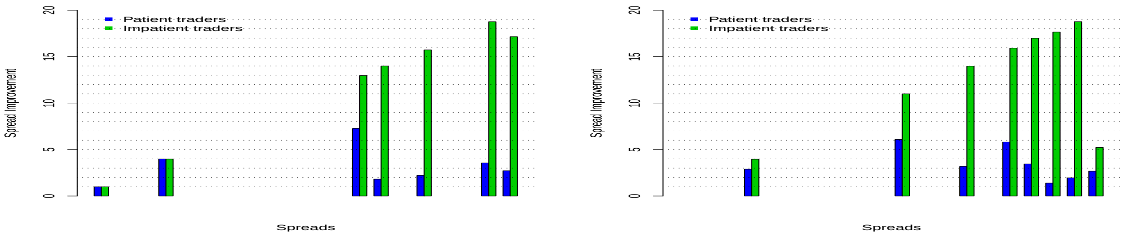


Figure 3.26: Spread Improvements

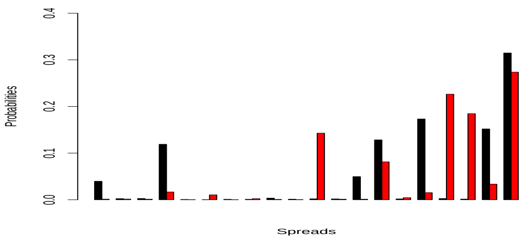
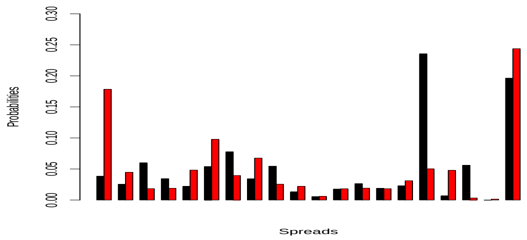


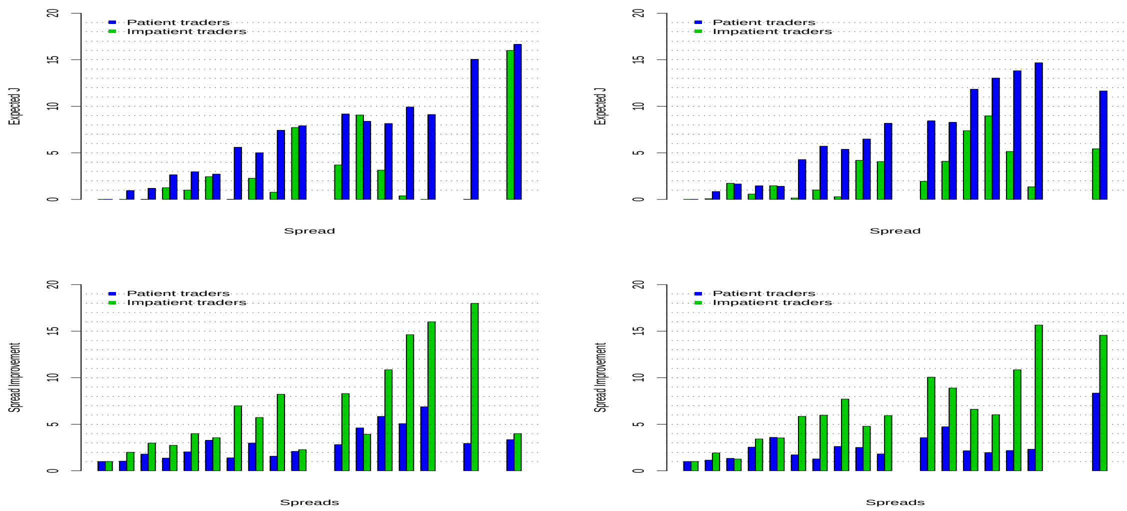
Figure 3.27: Distribution of Spreads

3.4.3 Increasing (initial) randomness

Similarly to what observed for learning, if we increase the share of the population that initially play choosing a random j in the feasible set, then the distribution of spreads move towards the left. This means that lower spreads are faced more frequently but, under cancellation, this fact does not lead to a loop in which only extreme spreads are played. In fact, almost no spread is faced with probability close to zero.



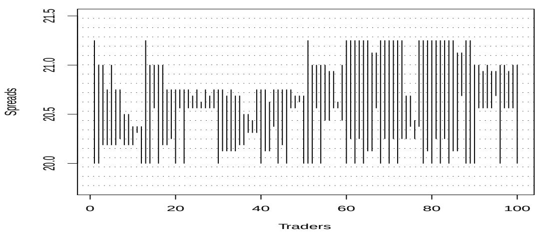
The order placement strategies are roughly increasing with the size of the spread for patient traders in both markets and the average spreads, as in the previous section, are higher in slow markets independently from the composition of the market.

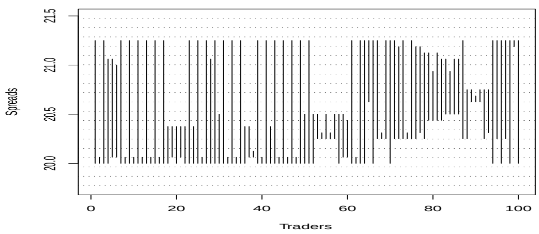


The main difference with what we found in the learning section consists in the fact that in slow markets the average spread is lower when market is dominated by impatient traders; this is (at least partially) justified by the fact that the likelihood to face low spreads is not as high as before and also medium and high spreads appear with positive probabilities.

Table 3.9: Average Spreads in Fast/Slow Markets populated by different percentage of patient traders (N=1000)

	Fast Market ($\lambda = 1$)	**Slow Market** ($\lambda = 0.2$)
$\theta_P = 0.55$	0.77033125	1.052458125
$\theta_P = 0.45$	0.64518625	0.8530928125





3.5 Does cancellation matter?

In this paper, accordingly to the rest of the thesis, we perform our analysis on two different levels of investigation. Our first goal is to *evaluate* the performance of the market. This is a preliminary and necessary step with the final aim to give some indications of *market design* that answer the question "if and how is it possible to modify a protocol in order to achieve desired and/or better results".

We will conduct this analysis looking at three different criteria to evaluate the market outcome. Section 3.5.1 report results about the number of realized transactions (volume); the higher they are, the better is the performance of the market. However, this is not enough. In order to determine if the introduction of cancellation might be beneficial and how good is the market performance, we need to analyze how these transactions are realized; namely, who exploit the most from trade and how the traders in the market perform as a whole. Section 3.5.3 answers the first question, sect. 3.5.2 deals with the second issue.

3.5.1 Volume

In general, we can observe that volumes are quite high independently on the percentage of patient traders in the market, the average time between arrivals and the introduction of the cancellation chance. In fact, we are considering a market with $N = 1000$ traders equally divided between buyers and sellers; then, the maximum number of executable transactions is 500. Table 3.10 reports the average number of transactions for each case under consideration and the total number of patient and imaptient traders that are involved in a trade. As it can be easily seen, the average number of trades is never lower than 492: only four trades are missing!

	without cancellation				with cancellation			
	$N = 1000$, $\theta_P = 0.55$		$N = 1000$, $\theta_P = 0.45$		$N = 1000$, $\theta_P = 0.55$		$N = 1000$, $\theta_P = 0.45$	
	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)
Total number of transactions	498.215	499.295	498.485	497.355	498.16	496.73	492.17	496.45
Number of patient traders involved in a trade	546.97	549.27	447.765	449.09	547.22	545.785	437.805	446.705
Number of impatient traders involved in a trade	449.46	449.32	549.205	545.62	449.1	447.675	546.535	546.195

Table 3.10: Average Volumes

The reason for that is quite trivial. Independently from everything else, one assumption of our model is given by the spread improvement rule: every trader needs to reduce the actual spread of at least one tick. This force not only the spread to be narrowed but also avoid the possibility to choose a limit order when no more reduction is possible.

Volumes are on average lower when the cancellation chance is introduced. In particular, this holds for fast markets that are dominated by impatient traders. We can find a reason for that in the fact that impatient traders (in particular) are much less aggressive; when there is no possibility to retract an order they place almost always market orders. Coherently with this observation, we can note that is the number of patient traders involved in a trade that decreases (more than the number of impatient traders) since their offers are no more picked up with the same frequency, see Fig. 21.

This is a peculiarity of fast markets; as it can be seen in appendix, the order placement strategies for impatient traders are much more aggressive for lower values of the parameter λ.

Increasing (initial) randomness. When a higher percentage of initial randomness is introduced results are summarized in Table 3.11.

	without cancellation				with cancellation			
	$N = 1000$, $\theta_P = 0.55$		$N = 1000$, $\theta_P = 0.45$		$N = 1000$, $\theta_P = 0.55$		$N = 1000$, $\theta_P = 0.45$	
	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)
Total number of transactions	499.265	499.485	498.85	497.525	489.27	416.465	495.77	497.3
Number of patient traders involved in a trade	549.255	549.6	448.67	449.74	539.29	546.88	444.76	446.27
Number of impatient traders involved in a trade	449.275	449.37	549.03	545.31	439.25	286.05	546.78	548.33

Table 3.11: Average Volumes

The level of randomness seems not to play a decisive role in determining the number of transactions in a market in which cancellation is not possible; this is in line with what explained above.

By the contrary, cancellation performs worst when markets are dominated by patient traders, especially in slow markets where impatient traders significantly fail to trade; namely, only 286 impatient traders over a total of 450 conclude an exchange. This happens even if in slow markets with cancellation impatient traders tend to go market. A partial explanation can be found in the fact that, in general, in slower markets there is more room to cancel since time between arrivals is higher.

In the opposite case, cancellation performs better (but results are still worst than in the no-cancellation scenario).

3.5.2 Allocative Efficiency

One of the most common criteria used to evaluate the perfomance of a market is given by the notion of *allocative efficiency*. In Foucault et al. (2005), the authors devote an entire paragraph to make some observations concerning efficiency, defined as the sum of the expected payoffs[16]. Their first claim is that efficiency cannot be fully reached due to waiting costs that constitute a dead-weight loss; then, the aim become to minimize such a cost.

[16]As usual, we deal with ABM, then measure of efficiency is based on realized payoffs.

As we will see, it happens when two conditions are fullfilled: (1) waiting time is minimized (no more than one period) and (2) all limit orders generate the competitive spreads (=1 in our examples). Note that Foucault et al. (2005) assumes that all impatient traders go market; then, $EC = \delta_P/\lambda$. In our framework this is not necessarily true and computations about the maximum level of allocative efficiency achievable will be updated in Definition 4.

The benchmark paper states that (in general) efficiency is not achieved in equilibrium due to the fact that the model in Foucault et al. (2005) do not uncover two source of inefficiency: (a) impatient traders sometimes submit limit orders and (b) patient traders post spreads larger than the competitive spreads. Both of these sources of inefficiency will be taken into account in what follows.

The maximum level of efficiency achievable by the market as a whole is usually measured as the competitive outcome (thereon, CO) defined as the difference between values and costs for intramarginal traders[17]:

$$CO = \sum_{i=1}^{q^*}(v_i - c_i) \tag{3.2}$$

where v_i (c_i) are the ith highest (lowest) value (cost) and q^* is the competitive quantity. Hence, the allocative efficiency measure is defined as follow[18]:

Definition 3. *The allocative efficiency can be defined as the total profit actually earned by all traders divided by the maximum total profit that could have been earned by all traders:*

$$AE = \frac{\sum(v_j - p) + (p - c_k)}{CO} \tag{3.3}$$

where v_j (c_k) is the value (cost) of a buyer j (a seller k) involved in a trade and p is the transaction price for that specific exchange [19]*.*

However, in our model there is something more to be taken into account: the waiting time has a cost that depends on agents' type and this cost has the effect to decrease the maximum payoff achievable.

Definition 4. *The maximum level of allocative efficiency attainable in a market with waiting costs in which buyers (sellers) have the same value, v (costs, c) and alternate with certainty (namely, the first player is a buyer with probability $1/2$) can be defined by the following equation:*

$$AE_{max} = v - c - \frac{1}{\lambda}[\theta_P\delta_P + (1-\theta_P)\delta_I] \tag{3.4}$$

where θ_P is the share of patients in the population, δ_P (δ_I) is the waiting cost for patient (impatient) traders and λ is the parameter of the Poisson arrival process.

Proof. Let us assume the case in which the first player is a buyer (wp 1/2).
If the following seller will place a market order, then the profit of the buyer will be $\pi_b = v - p - [\theta_P(\delta_P T) + (1-\theta_P)(\delta_I T)]$, where p is the transaction price. Since, on average, the interarrival

[17]Intramarginal traders are buyers (sellers) with their values (costs) higher (lower) than the equilibrium price. In this implementation we have assumed simemtric reservation values with respect to the range of admissible price; hence, the equilibrium price coincides with the medium point of this interval

[18]See Zhan et al. (2002) for definitions of competitive outcome and allocative efficiency; for the latter, refer also to Smith (1982).

[19]Given our set of parameters, according to the Foucault's numerical example, we have $v_i = 22, \forall i$ and $c_j = 19.25, \forall j$. Hence, $AE_{max} = N \cdot (22 - 19.25)$

time of a Poisson process is $1/\lambda$, then we can substitute $T = 1/\lambda$. The seller profit can be written as $\pi_s = p - c$. Then, the total gain from transaction will be $\pi = v - c - [\theta_P(\delta_P T) + (1-\theta_P)(\delta_I T)]$. By symmetry, the same happens if the first player is a seller (with complementary probability 1/2) from which the result follows. □

With the aim to compare the level of efficiency for the two markets under consideration (namely, with and without cancellation), we can use as a reference point (technically, the denominator in the AE definition equation) alternatively the CO or the AE_{max} as described above. This does not affect the direction of our results.

	without cancellation				with cancellation			
	$N = 1000$, $\theta_P = 0.55$		$N = 1000$, $\theta_P = 0.45$		$N = 1000$, $\theta_P = 0.55$		$N = 1000$, $\theta_P = 0.45$	
	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)
AE (mean)	0.8841	0.8021	0.8775	0.7501	0.8793	0.7224	0.8637	0.6519
AE (median)	0.8839	0.8023	0.8793	0.7507	0.8792	0.7235	0.864	0.6535
st dev	0.0061	0.0082	0.0189	0.0129	0.0111	0.0126	0.0082	0.0168

Table 3.12: Average Allocative Efficiency

The main results are summarized in Table 3.12 and Fig. 3.28.

In general, allocative efficiency is lower under cancellation; this is primarily due to the less aggressive behavior of impatient traders (motivation (1) stated above).

In fast markets ($\lambda = 1$) this difference is marginal. An increase in the percentage of impatient traders does not change significantly the result when cancellation is not possible[20] but in the alternative scenario it plays a role in (slightly) lowering the efficiency achieved by the market (see top-left and top-right of Fig.28, respectively). In slow markets the presence of impatient traders make the average allocative efficiency achieved lower in both cases, with and without cancellation, in accordance with the previous finding (see bottom-left and bottom-right of Fig. 3.28, respectively).

Coherently with findings in Foucault et al. (2005), we are far from the maximum value of allocative efficiency achievable, represented by the red line in Fig. 3.8.

Increasing (initial) randomness. Increasing the level of randomness in the initial setup something really interesting happens. In fact, the loop described in the previous section describes exactly the conditions under which the maximum level of allocative efficiency can be achieved. See Fig. 29. The only case in which this not happens, due to the order placement strategies described in Sect.3, is a fast market dominated by impatient traders.

Numerical results are summarized in Table 3. 13.

[20] However, note that in this case the standard deviation is significantly higher.

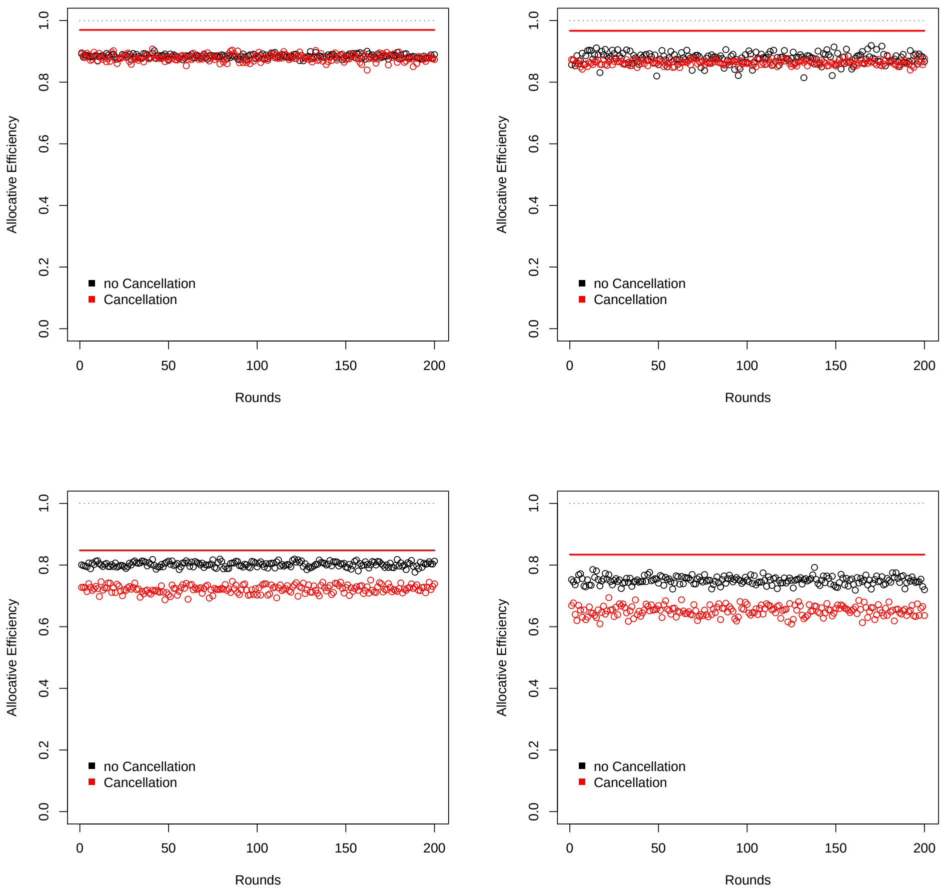


Figure 3.28: Allocative Efficiency - market dominated by patient (impatient) traders on the left (right), fast (slow) market on the top (bottom).

	without cancellation				with cancellation			
	$N = 1000,\ \theta_P = 0.55$		$N = 1000,\ \theta_P = 0.45$		$N = 1000,\ \theta_P = 0.55$		$N = 1000,\ \theta_P = 0.45$	
	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)
AE (mean)	0.9547	0.8428	0.9175	0.822	0.8808	0.6008	0.9042	0.7463
AE (median)	0.9626	0.8431	0.9241	0.8222	0.8809	0.6008	0.905	0.746
st dev	0.0203	0.0086	0.0191	0.008	0.0078	0.0139	0.0107	0.0119

Table 3.13: Average Allocative Efficiency

We can separately consider three cases:

- (a) Fast market dominated by patient traders (top-left of Fig. 3.29)
  Due to the loop cited above and in the previous section, in a scenario with no-cancellation the maximum attainable level of allocative efficiency is achieved. Cancellation performs worst.

- (b) Fast market dominated by impatient traders (top-right of Fig. 3.29)
  Under both scenarios we are not able to reach the maximum level of allocative efficiency achievable. However, no-cancellation still (slightly) underperforms cancellation.

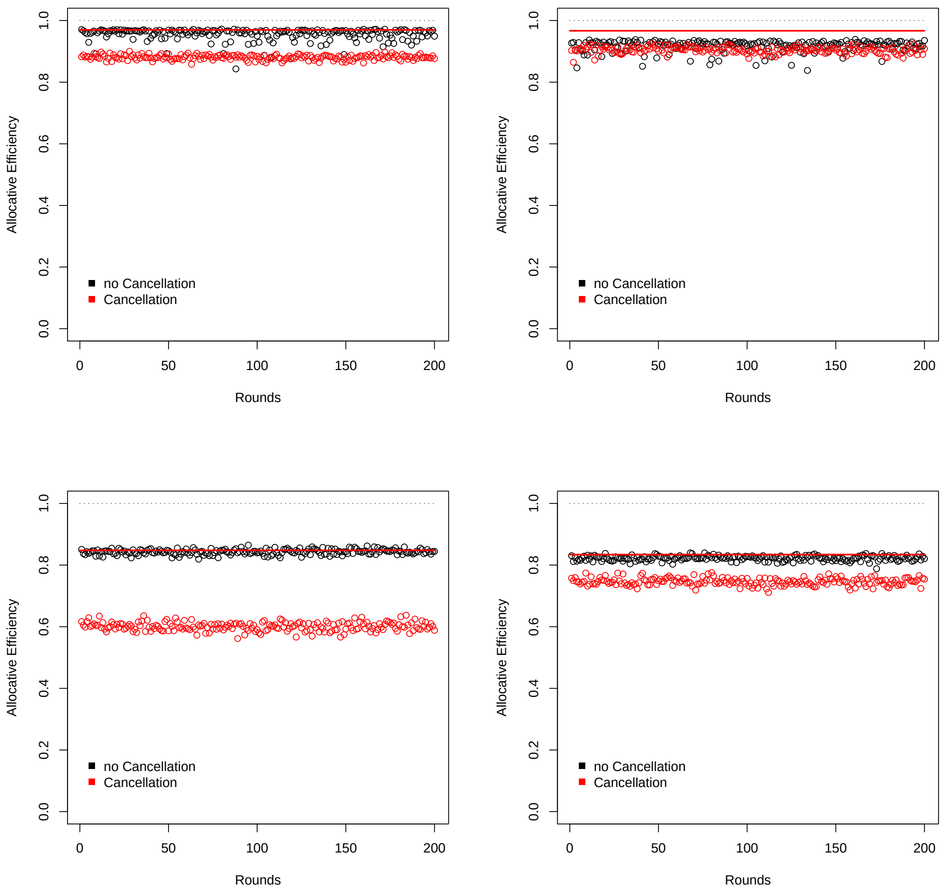


Figure 3.29: Allocative Efficiency - market dominated by patient (impatient) traders on the left (right), fast (slow) market on the top (bottom).

- (c) Slow markets (second line Fig. 3.29)
  Coherently with previous findings, no-cancellation performs at best and cancellation behaves even worst than in fast markets (i.e., too many trades are cancelled, also trades potentially profitable).

3.5.3 Individual Average Profits

From an individual point of view, it could be interesting who has the chance, in teh two different scenarios under evaluation, to exploit the most from transactions. In fact, the main force that drives agents' trading behavior choices is the desire to maximize their own profits.

For that reason, we summarize in Table 3.14 all the results concerning average individual profits. First of all an overall picture is provided. Then, the average profits is displayed for both types of players (patients and impatients, accordingly with their percentage in the market); they take into account both the gains from trade and the loss from no trade. Finally profits from trade, computed as the total payoffs per type from trade over the total number of agents of that type that conclude a transaction, are isolated.

The main interesting result is that in fast markets dominated by patient traders cancella-

	without cancellation				with cancellation			
	$N = 1000,\ \theta_P = 0.55$		$N = 1000,\ \theta_P = 0.45$		$N = 1000,\ \theta_P = 0.55$		$N = 1000,\ \theta_P = 0.45$	
	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)
Average profits overall	1.2125	1.1026	1.2037	1.0308	1.2083	0.9926	1.1867	0.8956
Profits overall (P)	1.2487	1.1969	1.4017	1.1302	1.3075	1.1093	1.3487	1.1028
Profits overall (I)	1.1683	0.9873	1.0417	0.9494	1.0872	0.8501	1.0542	0.726
Profits from trade (P)	1.2613	1.1991	1.4148	1.134	1.3146	1.119	1.3867	1.1116
Profits from trade (I)	1.1697	0.9888	1.0434	0.957	1.0904	0.8545	1.0621	0.732

Table 3.14: Average Profits

tion is beneficial for patient traders. This leads to higher average payoffs from trade and (also) higher average payoffs overall. This means that the average loss from no trade is no larger than that difference. Under cancellation, there is room for patient traders to exploit the most from transaction. The same story does not apply to impatient traders.

As usual, in slow market the bad effects attached to the cancellation chance are amplified.

Increasing (initial) randomness. The main finding described above partially still holds even if its effects are less powerful. In fact, in fast market dominated by patient traders there is no significant difference in average profits from trade for patient traders. However, the overall profits of this type of players, accordingly to overall result, is lower under cancellation. We can conclude that they face a higher loss from no trade.

Results are reported in Table 3.15.

	without cancellation				with cancellation			
	$N = 1000,\ \theta_P = 0.55$		$N = 1000,\ \theta_P = 0.45$		$N = 1000,\ \theta_P = 0.55$		$N = 1000,\ \theta_P = 0.45$	
	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)	$\lambda = 1$ (fast)	$\lambda = 0.2$ (slow)
Average profits overall	1.3099	1.1589	1.2512	1.1302	1.2106	0.8256	1.2429	1.0258
Profits overall (P)	1.3236	1.2406	1.2805	1.4952	1.3045	0.9976	1.3485	1.2281
Profits overall (I)	1.2931	1.059	1.2271	0.8316	1.0958	0.6155	1.1564	0.8603
Profits from trade (P)	1.33	1.2415	1.2991	1.4961	1.3309	1.0042	1.3649	1.2391
Profits from trade (I)	1.2958	1.0605	1.2362	0.8388	1.1232	0.9682	1.1635	0.8629

Table 3.15: Average Profits

3.6 Conclusion

In this paper we try to answer two different research questions.

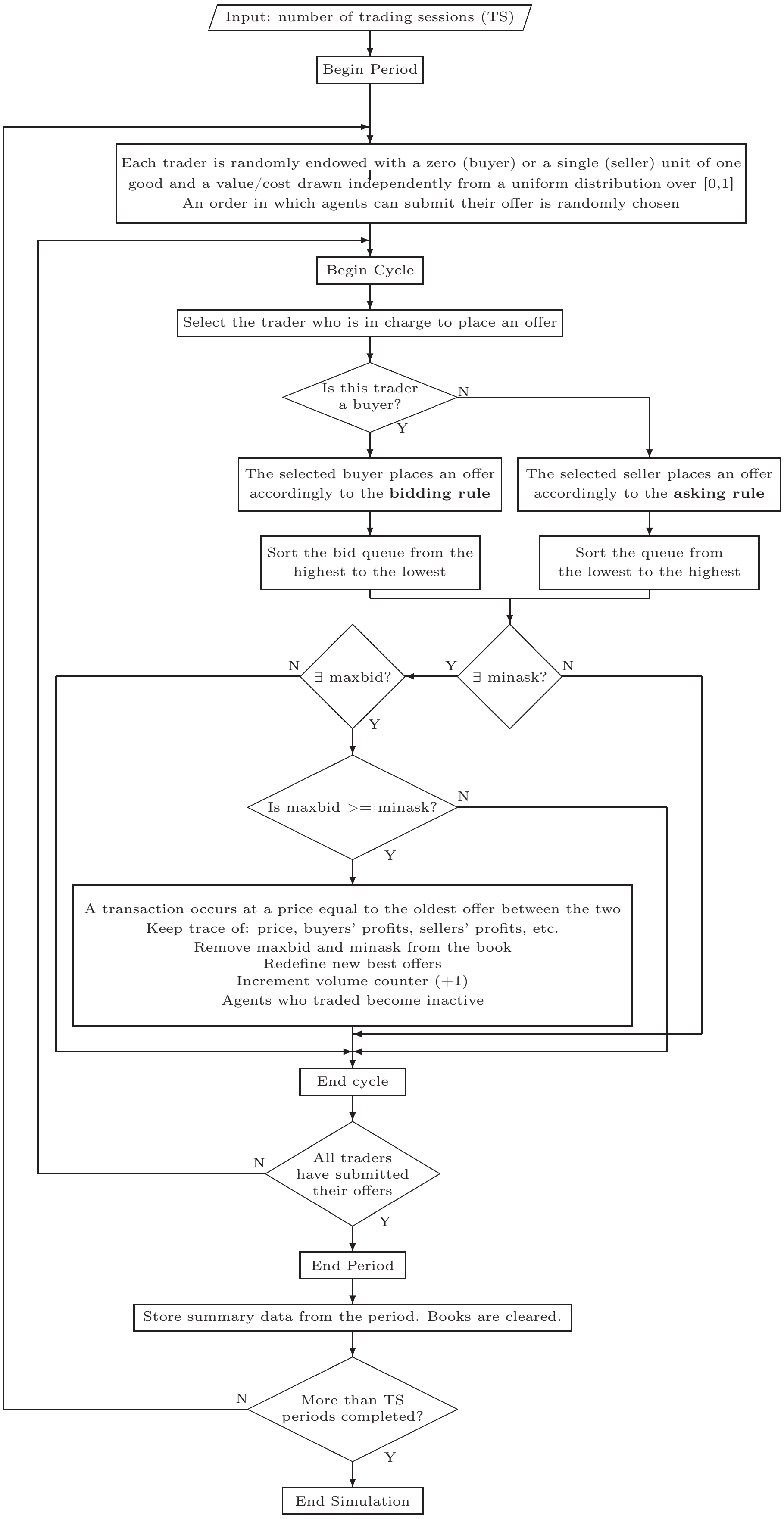
Firstly, we propose an evolutionary algorithm with the aim to test if learning agents' trading strategies converge to the equilibrium strategies proposed in Foucault et al. (2005). For many reasons (details in Sect.3) the profile of strategies achieved is not exactly the same; the learning process is affected by the initial conditions and the ABM necessarily slightly differs from the original benchmark since it has to be developed over a finite horizon. However, even if equilibrium is difficult to be learned, the achieved result in terms of strategy is close to an equilibrium (the "ϵ-equilibrium hypothesis" is tested) and is able to reproduce the most part of the main findings of the original model in Foucault et al. (2005).

Secondly, we relax the assumption for which it is not possible to cancel a limit-order and evaluate the performance of the market with the introduction of a cancellation rule that force agents to submit a market order when they decide to retract a previous offer. Then, we evaluate the performance of a market dominated by patient traders according to different performance criteria. As an empirically testable implication and suggestion for market designers, we find that cancellation does not lead to better results under almost all the performance criteria analyzed (i.e., verage spreads, volumes, allocative efficiency, individual average profits). The force that really drives the result is the joint effect of the distribution of traders' types in the market, the order arrival rate and the trading behavior. In Foucault et al. (2005) this last key determinant of book dynamics is not analyzed since they assume that traders adopt optimizing strategies. However, if this is not the case (or, even simpler, if this is not the case for everybody) results are ambiguous.

Appendix A

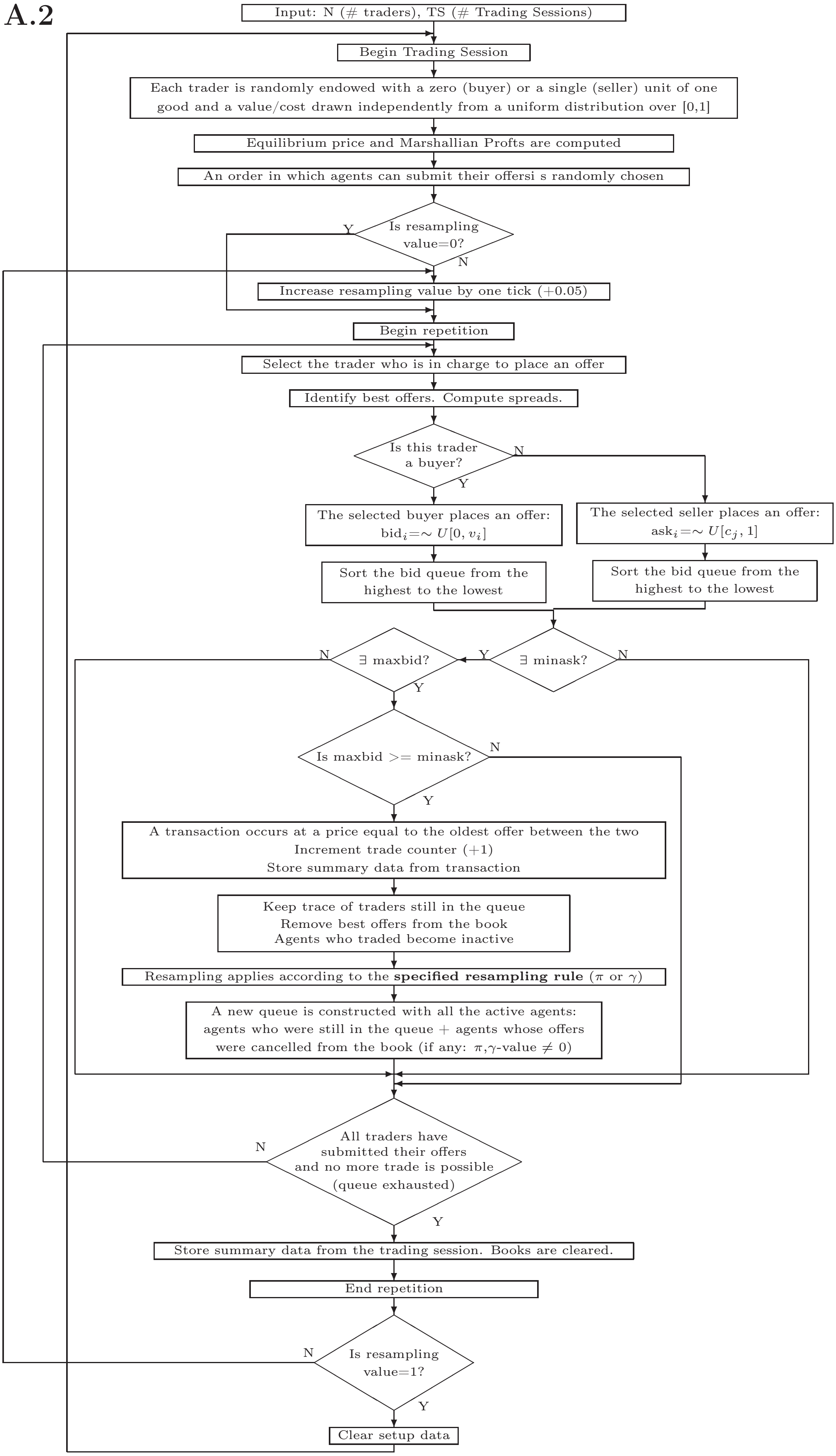
Flow Charts

A.1



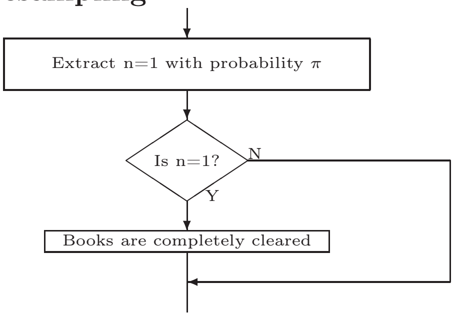
Input: number of trading sessions (TS)
Begin Period
Each trader is randomly endowed with a zero (buyer) or a single (seller) unit of one good and a value/cost drawn independently from a uniform distribution over [0,1]
An order in which agents can submit their offer is randomly chosen
Begin Cycle
Select the trader who is in charge to place an offer
Is this trader a buyer?
N
Y
The selected buyer places an offer accordingly to the **bidding rule**
The selected seller places an offer accordingly to the **asking rule**
Sort the bid queue from the highest to the lowest
Sort the queue from the lowest to the highest
∃ maxbid?
∃ minask?
N
Y
Y
N
Is maxbid >= minask?
N
Y
A transaction occurs at a price equal to the oldest offer between the two
Keep trace of: price, buyers' profits, sellers' profits, etc.
Remove maxbid and minask from the book
Redefine new best offers
Increment volume counter (+1)
Agents who traded become inactive
End cycle
All traders have submitted their offers
N
Y
End Period
Store summary data from the period. Books are cleared.
More than TS periods completed?
N
Y
End Simulation

A.2

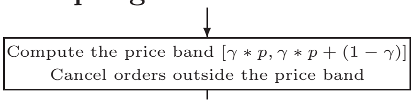


Input: N (# traders), TS (# Trading Sessions)
Begin Trading Session
Each trader is randomly endowed with a zero (buyer) or a single (seller) unit of one good and a value/cost drawn independently from a uniform distribution over [0,1]
Equilibrium price and Marshallian Profts are computed
An order in which agents can submit their offersi s randomly chosen
Is resampling value=0?
Y
N
Increase resampling value by one tick (+0.05)
Begin repetition
Select the trader who is in charge to place an offer
Identify best offers. Compute spreads.
Is this trader a buyer?
N
Y
The selected buyer places an offer: $\text{bid}_i = \sim U[0, v_i]$
The selected seller places an offer: $\text{ask}_i = \sim U[c_j, 1]$
Sort the bid queue from the highest to the lowest
Sort the bid queue from the highest to the lowest
∃ maxbid?
∃ minask?
N
Y
N
Y
Is maxbid >= minask?
N
Y
A transaction occurs at a price equal to the oldest offer between the two
Increment trade counter (+1)
Store summary data from transaction
Keep trace of traders still in the queue
Remove best offers from the book
Agents who traded become inactive
Resampling applies according to the **specified resampling rule** (π or γ)
A new queue is constructed with all the active agents: agents who were still in the queue + agents whose offers were cancelled from the book (if any: π,γ-value $\neq 0$)
All traders have submitted their offers and no more trade is possible (queue exhausted)
N
Y
Store summary data from the trading session. Books are cleared.
End repetition
Is resampling value=1?
N
Y
Clear setup data

π-resampling



γ-resampling



Appendix B

An example of γ-reampling rule:
The Australian Stock Exchange (ASX)

The Australian Stock Exchange is an example of market in which the orders might be centrally inactived following some price purging rules based on a price-band width[1]. A **Purge Price Level (ppl)** (60%) and a **Purge Price Threshold (ppt)** (20 cents) are given. The former is used to compute the **Calculated Purge Price (cpp)**, the latter to define which orders should be purged.

Example.

- At the end of the day a spread (given by the outstanding offers) is faced and the Calculated Purge Price is computed as follow:

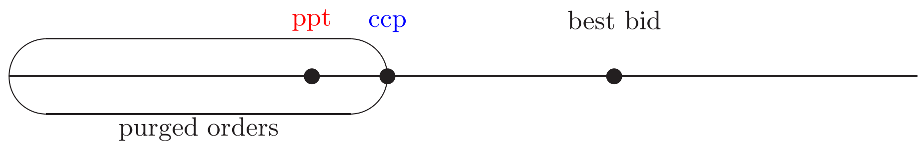
$$cpp = \begin{cases} 0.4(\text{best bid}) & [\text{on the buy side}] \\ 0.4(\text{best ask}) & [\text{on the ask side}] \end{cases}$$

[1]The mechanism is similar in spirit to the one analyzed in the paper (γ-resampling). Here the event at which cancellation follows is the end of the trading-day instead of a single transaction.

- How Cancellation works:

<u>Book of bids</u>

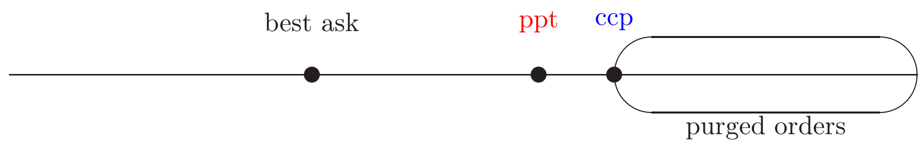
  - If the Calculated Purge Price is higher than or equal to the Price Purge Threshold, buy orders with a price less than the Calculated Purge Price are purged.



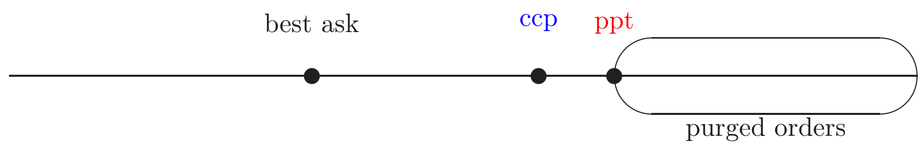
  - If the Calculated Purge Price is lower than the Price Purge Threshold, no orders are purged on the buy side.

<u>Book of asks</u>

  - If the Calculated Purge Price is higher than or equal to the Price Purge Threshold, sell orders with a price higher than the Calculated Purge Price are purged.



  - If the Calculated Purge Price is lower than the Price Purge Threshold, sell orders with a price higher than or equal to the purge price threshold will be purged.



Appendix C

Appendixes Chapter 3

C.1 Validation of the ABM model - The order arrival rate ($\lambda = 0.2$)

Since the condition $s_c = K$ (i.e., the cutoff spread for which impatient traders submit market orders is equal to the maximum spread) holds given both the parameter sets in Tables C.1 and C.2, *impatient traders* optimal placement strategy is always to place a (market) order with $J^* = 0$.

Equilibrium strategies for *patient traders* derive from proposition 5, p.1183.

Proposition 5.
The set of equilibrium spreads is given by:
$n_1 = j_P^*$, $n_q = K$
$n_h = n_1 + \sum_{k=2}^{h} \Psi_k$, $h = 2, ..., q-1$
where $\Psi_k = CF(2\rho^{k-1}\dfrac{\delta_P}{\lambda\triangle})$
and q is the smallest integer such that $j_P^* + \sum_{k=2}^{q} \Psi_k \geqslant K$

Note: CF is the ceiling function; it returns the smaller integer greater than or equal to the object.

Case (1)

θ_P	δ_P	δ_I	$\triangle$	λ
0.55	0.05	0.125	0.0625	0.2

Table C.1: Parameters set - market dominated by patient traders.

$$\begin{cases} j_P^* = CF(\dfrac{\delta_P}{\lambda\triangle}) \Rightarrow j_P^* = 4 \\ \rho = \dfrac{\theta_P}{\theta_I} = 1.22 \\ \\ \Psi_2 = CF(9.76) = 10 \\ \Psi_3 = CF(11.9072) = 12 \\ \\ n_1 = j_P^* = 4 \\ n_2 = n_1 + \Psi_2 = 4 + 10 = 14 \\ n_3 = n_1 + \Psi_2 + \Psi_3 = 4 + 10 + 12 = 26 > 20 = K \Rightarrow q = 3 \end{cases}$$

Case (2)

θ_P	δ_P	δ_I	$\triangle$	λ
0.45	0.05	0.125	0.0625	0.2

Table C.2: Parameters set - market dominated by patient traders.

$$\begin{cases} j_P^* = CF(\frac{\delta_P}{\lambda \triangle}) \Rightarrow j_P^* = 4 \\ \rho = \frac{\theta_P}{\theta_I} = 0.8\bar{1} \\ \\ \Psi_2 = CF(6.48) = 7 \\ \Psi_3 = CF(5.2488) = 6 \\ \Psi_4 = CF(4.25) = 5 \\ \\ n_1 = j_P^* = 4 \\ n_2 = n_1 + \Psi_2 = 4 + 7 = 11 \\ n_3 = n_1 + \Psi_2 + \Psi_3 = 4 + 7 + 6 = 17 \\ n_4 = n_1 + \Psi_2 + \Psi_3 + \Psi_4 = 4 + 7 + 6 + 5 > 20 = K \Rightarrow q = 4 \end{cases}$$

	Equilibrium strategies (J^*) - patient traders -	
spread faced	Caso (1)	Caso (2)
1	0	0
2	0	0
3	0	0
4	0	0
5	4	4
6	4	4
7	4	4
8	4	4
9	4	4
10	4	4
11	4	4
12	4	11
13	4	11
14	4	11
15	14	11
16	14	11
17	14	11
18	14	17
19	14	17
20	14	17

Table C.3: Equilibrium order placement strategies for patient traders. In red strategies on equilibrium path.

C.2 Increasing (initial) randomness without cancellation The order arrival rate.

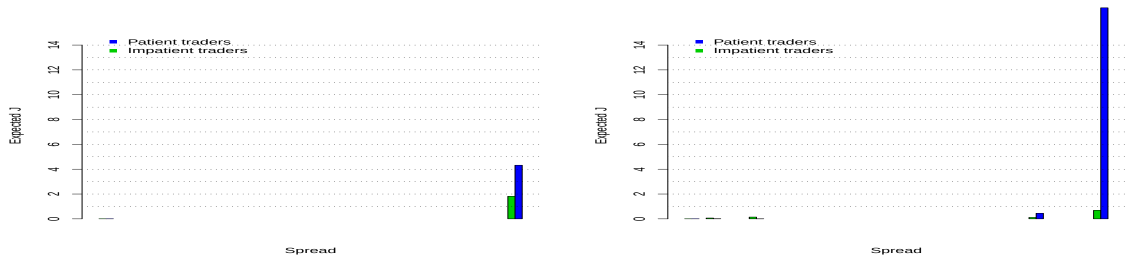


Figure C.1: Order placement strategies - Expected J - slow markets ($\lambda = 0.2$)

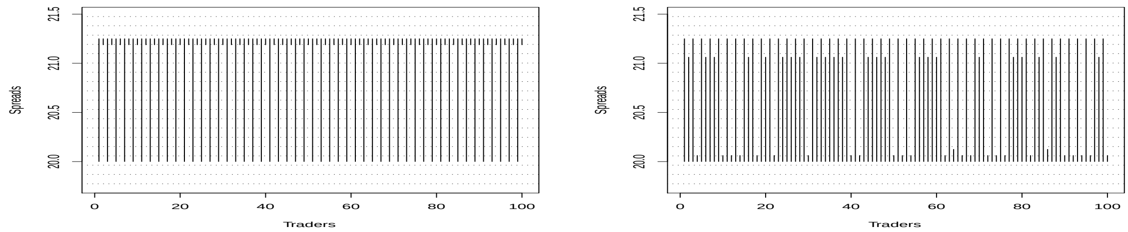


Figure C.2: Book Dynamics - slow markets ($\lambda = 0.2$)

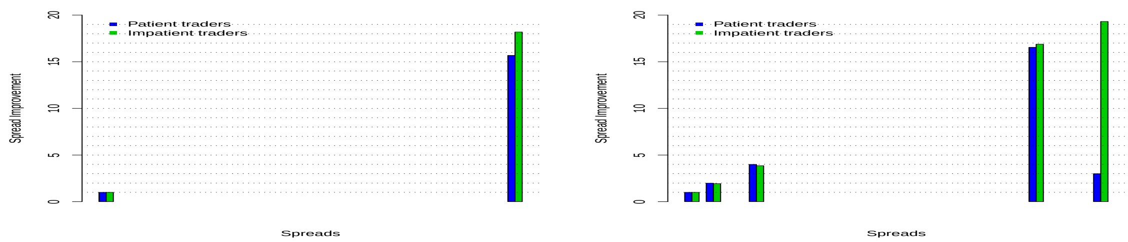


Figure C.3: Spread Improvements - slow markets ($\lambda = 0.2$)

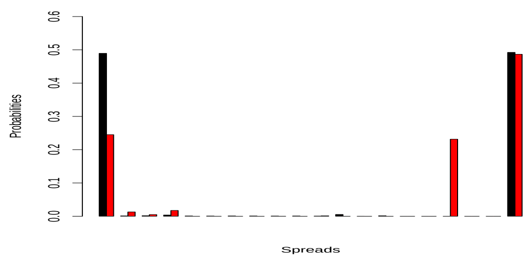


Figure C.4: Distribution of Spreads - slow markets ($\lambda = 0.2$)

C.3 Increasing (initial) randomness with cancellation The order arrival rate.

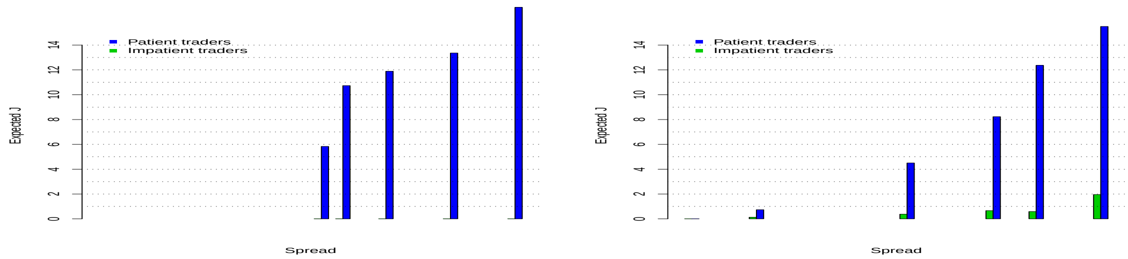


Figure C.5: Order placement strategies - Expected J - slow markets ($\lambda = 0.2$)

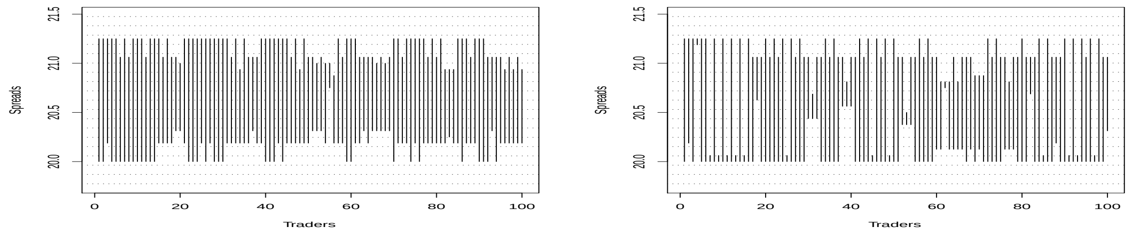


Figure C.6: Book Dynamics - slow markets ($\lambda = 0.2$)

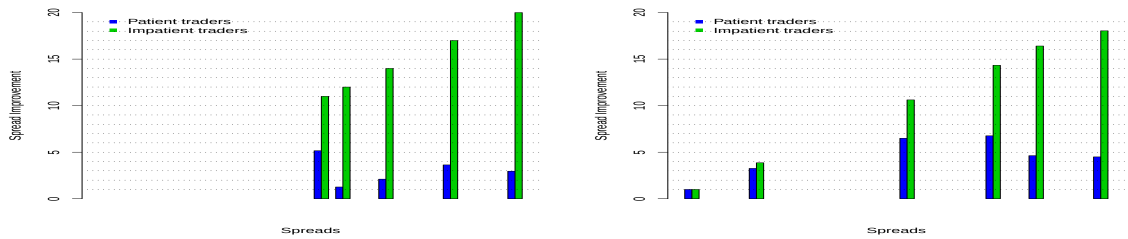


Figure C.7: Spread Improvements - slow markets ($\lambda = 0.2$)

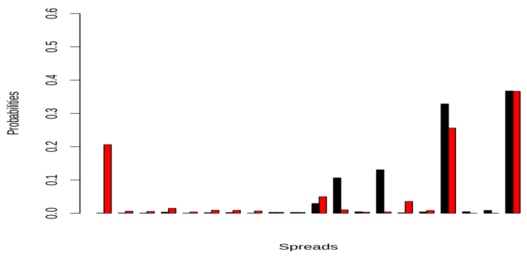


Figure C.8: Distribution of Spreads - slow markets ($\lambda = 0.2$)

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