Università
Ca' Foscari
Venezia

Dottorato di ricerca
in Economia
Scuola di dottorato in Economia
Ciclo XXIII
(A.A. 2010-2011)

Three essays on asymmetric information in financial markets

SETTORE SCIENTIFICO DISCIPLINARE DI AFFERENZA: SECS-P/01

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ACKNOWLEDGEMENTS

To my family

ABSTRACT

In Chapter 1 we study how a "bubble" can emerge in a financial market, in which trade is intermediated, i.e., in which only "specialized" agents can deal both with sellers and buyers of an asset. We analyze how second-order uncertainty, i.e., uncertainty about other agents' beliefs about the dividend from the asset, can "fuel" bubbles, under the assumptions that the supply of asset is limited and short-sales are not allowed. Our setting allows both for agents sharing the same first-order prior beliefs and for agents having different first-order priors. We show that, differently from previous model on bubbles under rational expectations and asymmetric information, second-order uncertainty can generate a bubble, even if agents share the same first-order priors and there is no agency problem between intermediaries and other agents.

In Chapter 2 we examine how asymmetric information and liquidity (or wealth) can affect the acquisition of information in financial markets. We analyze a general setting, in which some agents, which we name sellers, can both "verify the quality" of and invest in an asset, where by "verify the quality" of the asset we mean to acquire information about the future value of, or cash-flow from, the asset. We show that, in such a setting, the presence of potential buyers, who have no liquidity (or wealth) constraint and who cannot verify the quality of the asset, induces sellers not to acquire information about the future value of asset, before investing in it.

In Chapter 3 we survey theoretical literature concerned with conditions under which bubbles can emerge in financial markets, in which traders do not have the same information or do not behave on the base of the same information.

1. BUBBLES IN AN INTERMEDIATED FINANCIAL MARKET

1.1 Introduction

In popular, and also academic, accounts of relatively sharp - and sustained over some time - increases in asset prices followed by rapid declines the word "bubble" has sometimes been used to describe such phenomena. [1]

The rise of the housing prices in U.S. in the first half of the 2000s and their subsequent decline after 2006 has also been termed a bubble. The word bubble has been used to denote also the inflow of credit into the U.S. housing market, which accompanied the increase in the housing prices. [2] Credit did not flow into the U.S. housing market only in the form of mortgages, but also in the form of assets, whose returns were dependent on those of the mortgages in the housing sector, and which financial institutions, such as investment banks, were active both in creating and in trading. [3] It has been pointed out that financial institutions apparently did not correctly assess the risk of such derivative securities and therefore suffered losses as a consequence of homeowners defaulting on their mortgages.

In the present work, we analyze a model of a bubble, in the sense attributed to such a word in the academic literature. The model is aimed at explaining how a bubble could emerge in an asset market, in which financial firms act as intermediaries between sellers and buyers. It could also be used to interpret the behavior of financial institutions, who invested in mortgage-backed securities (MBS) and derivatives linked to such securities, during the above-mentioned appreciation of housing prices.

Our model describes the market for an asset, in which a bubble is "fueled" by

1. presence of intermediaries,
2. uncertainty about other agents' beliefs about the dividend (or cash flow) from the asset, and

[1] See Garber (1990).
[2] See Hong and Sraer (2011).
[3] See Brunnermeier (2009) and Gorton (2010).

3. limited supply of the asset relative to the demand.

In our setting agents cannot short-sell the asset. Even if we do not explicitly discuss the role of short-sale constraints, we believe that, in absence of such constraints, a bubble would not emerge.

The model mainly builds on and bring some contributions to the literature about bubbles under rational expectations and asymmetric information.

1.2 Related literature

In the economic literature, the word bubble is used to describe a situation, in which the price of an asset is higher than the asset fundamental value. The fundamental value of an asset (or, simply, fundamental) at a given point in time can be defined as the highest among the prices, which agents are willing to pay in order to buy the asset at that point in time and hold it forever. In the theoretical literature on bubbles, the fundamental equals different values according to different assumptions about agents' preferences and beliefs. [4]

We briefly discuss how the value, which the fundamental equals, differs from one type of model to another, according to different assumptions about agents' beliefs. We focus exclusively on settings, in which all agents are risk-neutral, since in many models about bubbles (and possibly in most of them), as well as in our model, agents are assumed to be risk-neutral (see section 3.1 about general assumptions in our model). In settings, in which there is no uncertainty about the future cash flow from the asset, the fundamental equals the discounted cash flow. In settings, in which all agents share the same beliefs, i.e., expectations, about an uncertain cash flow, the fundamental equals the expected discounted cash flow. [5]

Agents' willingness to pay more than their expectations (or valuations) in order to buy the asset is explained by assuming that they have the possibility to resell the asset, once they have bought it, and expect to be able to do so at a price, which surpasses their valuations. In other words, the asset embodies an option to resell it, which agents find valuable and are willing to pay for on the top of their valuations.

In models on bubbles under rational expectations and asymmetric information, a bubble emerges, since agents do not know that other agents also know that the price of the asset is higher than the fundamental, given certain necessary conditions. Allen et al. (1993) show that necessary conditions for a bubble

[4] See Allen et al. (1993), Allen and Gorton (1993), Brunnermeier (2008 and 2001).
[5] See Allen et al. (1993), Brunnermeier (2008 and 2001).

under rational expectations and asymmetric information are that prices cannot be fully revealing, agents have to be short-sale constrained, and they have to think that there are gains from trade, i.e., it cannot be common knowledge that the initial allocation is interim Pareto efficient. Allen et al. (1993) propose four examples of ways, which lead to gains from trade: 1) heterogeneous first-order prior beliefs, i.e., heterogeneous beliefs about the dividend from the asset, due to agents having different partitions over the set of states of the world, with each state being characterized by the dividend paid by the asset in that state (even if this could be considered at odds with the notion of all agents having rational expectations), 2) state-dependent utility functions, 3) random endowments and identical concave utility functions, and 4) identical endowments and different concave utility functions. In Allen and Gorton (1993) and Allen and Gale (2000), asymmetric information between and an agency problem between "direct" investors, i.e., investors with direct access to assets or projects (e.g., banks or portfolio managers) and "normal" investors, i.e., investors, who cannot invest directly in assets or projects, and, in order to do so, have to lend their money to "direct" investors. In such models direct investors "excessively" buy risky assets, bringing their price above the fundamental values. Given the impossibility for "normal" investors to write contracts conditional on the type and the quantities of assets, in which "direct" investors invest, and given other features of the contracts between investors of one type and the other, limited liability allows direct investors to make profits in "good" states of the world, in which the dividend from the asset is high, but limits their losses in "bad" states of the world, in which the dividend is low.

In the present work, we depart from previous models on bubbles under rational expectations and asymmetric information, by showing that, if trade is intermediated and there is scarcity of asset relative to the demand, a bubble can emerge even if agents share the same first-order priors and there is no agency problem between agents, simply because of second-order uncertainty, i.e., uncertainty about other agents' first-order beliefs. In other words we propose differences in second-order beliefs, even if first-order priors are common among all agents and there is no agency problem between agents, as a mechanism, which generates gains from trade, allowing a bubble to emerge.

1.3 The market

1.3.1 General assumptions and notation

General assumptions

The model features three periods $t = 0, 1$, and 2. There is a risky asset and a consumption good, which does not depreciate. In period 2 the risky asset delivers a cash-flow of consumption good or wealth, which can be used for consumption. [6] The prices of the risky asset are denoted in terms of consumption good.

There are $n = 1, \ldots, N$ agents. All of them are risk-neutral and have no time-preferences, i.e., they do not discount consumption. Therefore we can "safely" assume that consumption takes place in period 2 and that each agent cares only about his own final-period expected consumption. There are three groups of agents: $s = 1, \ldots, S$ original owners of the asset (or sellers), $i = 1, \ldots, I$ intermediaries, and $b = 1, \ldots, B$ other agents (or buyers). Each agent belongs to one, and only one, group. Therefore $S + I + B = N$. Agents differ in their endowments. Each seller is endowed with a quantity of asset, which, without loss of generality, we normalize to 1. Both intermediaries and buyers are endowed with consumption good and they are not wealth-constrained. For simplicity, we assume that each of them is endowed with infinite wealth. Moreover sellers cannot sell directly to buyers and buyers cannot buy directly from sellers, i.e., only intermediaries can trade both with sellers and with buyers. Markets are open both in period 0 and in period 1. In period 0 only sellers and intermediaries are allowed to trade, while in period 1 trade can take place only between intermediaries and buyers. The structure of trade protocols is further described in section 3.2.

The assumption that trade in a financial market can take place only through intermediaries is not new in the economic literature. A stream of theoretical literature investigates financial markets' behavior under a conceptually very similar assumption, i.e., that some agents can invest in asset only "indirectly", i.e., by giving (part of) their endowments to intermediaries - named also as, e.g., banks, experts, or portfolio managers - who have direct access to assets. Examples of such literature are the models by Diamond and Dybvig (1983), Diamond (1984), Allen and Gorton (1993), Holmstrom and Tirole (1997), Allen and Gale (2000), and, more recently, by He and Krishnamurthy (2010a,b), Brunnermeier and Sannikov (2011), and Myerson (2011). Moreover many of the papers, which

[6] The consumption good can be also thought as a riskless asset, whose return is normalized to 1, i.e., q units of asset deliver q units of consumption good or wealth in period 2.

are usually considered part of the so-called literature on market microstructure, model financial markets, in which agents can trade only by submitting orders to "specialists" or "market makers", who set prices: see, e.g., Copeland and Galai (1983), Glosten and Milgrom, Kyle(1985), and Dennert(1993).

The structure of the uncertainty is the following one. There exists second-order uncertainty. In other words, agents are uncertain about other agent's first-order beliefs, i.e., about the amount of consumption good, which other agents think that the asset will deliver in period 2. Such uncertainty is aggregate within the three groups of agents - sellers, intermediaries, and buyers - and idiosyncratic across such groups. We assume that each group receives a signal about its first-order beliefs and that each group perfectly observes the signal, which it receives, but does not observe the ones received by the other two groups. More formally, we let each agent receive signal $v_n \in \{v|v \in \{L, M, H\}\}$, $n = 1, \dots, N$. Let also $\Pr(v_n = v) = \pi^v$, $\forall n$, with $v \in \{L, M, H\}$. L (low), M (medium), and H (high) denote different (expected) cash-flow, which the asset could deliver, i.e., any agent n expects $q > 0$ units of asset to deliver $v_n q$ units of wealth in $t = 2$. We assume that $L < M < H$, and, without loss of generality, that $L \geq 0$. Let also $v_s = v_S$, $\forall s$, $v_i = v_I$, $\forall i$, and $v_b = v_B$, $\forall b$. This means that each agent in each group observes the signal received by the group, to which he belongs, i.e., the signal is the same for all agents in any group and is common knowledge among all agents in that group. However agents do not observe the signals received by the groups, to which they do not belong.

Second-order uncertainty does not rule out the possibility that agents have common first-order prior beliefs. The uncertainty structure, which we assume in the present work, does not imply that the assumption of common first-order priors about the asset cash-flow is discarded. Our analysis is aimed mainly at finding (both necessary and sufficient conditions) for equilibrium bubbles, if agents share the same first-order priors, i.e., if $v_S = v_I = v_B = v$, with $v \in \{L, M, H\}$.

The assumption of second-order uncertainty, if accompanied by the assumption of first-order priors does not also prevent to interpret the model as analyzing a financial market, in which each asset (one in our case) has one fundamental value (equal to L in our case). Second-order uncertainty can be simply interpreted as agents being uncertain that other agents are also aware of the fundamental value of the asset. [7]

[7] Under different assumptions about, e.g., agents' utility functions or technologies, uncertainty could refer to such aspects of the model, rather than to beliefs about the asset cash-flow. If we assumed, for instance, that agents discounted consumption or could invest consumption good in projects and that either discount factors or returns on projects were the same within

The assumption that first-order priors about the asset cash-flow be common knowledge within groups but not among groups of agents should not be surprising. We believe not to be unrealistic that agents, who have a given "role" in the market (or "relation" with the asset)—i.e., in our analysis, original owners, intermediaries, and final buyers—can be "quite certain" about the expectations of agents having a similar (or the same) "role" in the market (or "relation" with the asset) and can be "less certain" about the expectations of those having a different "relation" with the asset (or "role" in the market). Moreover we believe that such an interpretation could be especially sensible, if referred to a "new" asset, i.e., to an asset, which has been recently "created" or which has been recently introduced into the market, such as in the case of U.S. MBS in the first half of the 2000s.

In our analysis we derive (both necessary and sufficient) conditions, under which bubbles exist, in equilibrium, in any of the following four games:

- γ_1, in which intermediaries offer prices to sellers in period 0 and buyers offer prices to intermediaries in period 1,
- γ_2, in which sellers ask prices to intermediaries in period 0 and buyers offer prices to intermediaries in period 1,
- γ_3, in which intermediaries offer prices to sellers in period 0 and ask prices to buyers in period 1,
- γ_4, in which sellers ask prices to intermediaries in period 0 and intermediaries ask prices to buyers in period 1.

We define a bubble as the trade of a positive quantity of asset in any of the periods $t = 0$ and $t = 1$ at a price higher than highest among the beliefs of the agents in the model (sellers, intermediaries and buyers) about the asset cash-flow—or, under the assumption of common first-order beliefs (and second-order uncertainty) at a price higher than the belief of all the agents about the asset cash-flow. We consider all possible Perfect Bayesian Equilibria (PBE) of all the four games and, by using the words equilibrium and equilibria in the following sections of the paper, we refer to PBE and PBEs respectively.

Some notation

We introduce some notation, which will be used in our analysis. Let:

groups and differed among groups, uncertainty could be interpret as agents in each group being uncertain about discount factors or return on projects of agents in the other groups.

- s_v (intermediary i_v, buyer b_v) denote type v of any seller s (intermediary i, buyer b), i.e., any seller s, with $s = 1, \ldots, S$ (intermediary i, with $i = 1, \ldots, I$, buyer b, with $b = 1, \ldots, B$), after receiving signal $v_S = v$ ($v_I = v$, $v_B = v$), with $v \in \{L, M, H\}$;

- $p_{s,v}$ ($p_{i,v,t}$, $p_{b,v}$) be the price, which s_v asks in $t = 0$ (i_v asks (offers) in $t : t = 0, 1$, b_v offers in $t = 1$), with $s = 1, \ldots, S$ ($i = 1, \ldots, I$, $b = 1, \ldots, B$) and $v \in \{L, M, H\}$;

- $\bar{p}_{I,v,0}$ ($\bar{p}_{B,v}$) be the highest among the prices offered by type v, with $v \in \{L, M, H\}$, of all intermediaries in period 0 (buyers in period 1);

- i'_v, with $i'_v = 1, \ldots, I'_v$ and $1 \leq I'_v \leq I$ (b'_v with $b'_v = 1, \ldots, B'_v$ and $1 \leq B'_v \leq B$), be any intermediary i_v (buyer b_v), who offers $p_{i,v,0} = p_{i',v,0} = \bar{p}_{I,v,0}$ ($p_{b,v} = p_{b',v} = \bar{p}_{B,v}$), with and $v \in \{L, M, H\}$;

- $q_i \geq 0$ be the quantity of asset that any intermediary i buy from sellers in period 0 and therefore can sell to buyers in period 1; $Q_I = \sum_{i=1}^{I}$ be the quantity of asset that all intermediaries, buy from sellers in period 0 and therefore can sell to buyers in period 1;

- $hl = \pi^H H + (\pi^M + \pi^L)L$; $ml = (\pi^H + \pi^M)M + \pi^L L$; $hm = \pi^H H + (\pi^M + \pi^L)m$; and $hml = \pi^H H + \pi^M M + \pi^L L$.

1.3.2 Trade

In all the four games γ_1, γ_2, γ_3, and γ_4,

- since both intermediaries and buyers are risk-neutral and not wealth-constrained, if any of them demands asset, then he demands an infinite quantity of asset, and,

- since both sellers and intermediaries are risk-neutral, if any of them supplies asset, then he supplies all his endowment of asset,

and this is true for each intermediary, buyer and seller. Therefore:

- the highest price, which any buyer b_v (intermediary i_v) is willing to pay, in order to buy asset, is equal to v_B ($Ec_{i,v}$), with $v \in \{L, M, H\}$, and

- the lowest price, which any seller s_v (intermediary i_v) is willing to accept, in order to sell asset, is equal to v_S (v_I), with $v \in \{L, M, H\}$.

Clearly:

- both in γ_1 and in γ_3, if any seller is willing to sell at a price offered by more than one intermediary, the endowment of that seller is randomly distributed among the intermediaries, who offer that price;
- both in γ_1 and in γ_2, if any intermediary is willing to sell at a price offered by more than one buyer, the endowment of that intermediary is randomly distributed among the buyers, who offer that price;
- both in γ_3 and in γ_4, if more than one buyer is willing to buy at a price asked by one or more intermediary, the endowment of the intemediary (or intermediaries), who offers (or offer) such a price, is randomly distributed among those buyers; and
- both in γ_2 and in γ_4, if more than one intermediary is willing to buy at a price asked by one or more seller, the endowment of the seller(s), who offer (or offers) such a price, is randomly distributed among those intermediaries.

In the following sections we let

- $Ec_{s,v}$ be the cash-flow, which any seller s_v, with $v \in \{L, M, H\}$, expects to obtain, if he chooses to participate to the market in $t = 0$; it is equal to $p_{s,v}$ times the probability of selling the asset at $p_{s,v}$ plus v_S times the probability of not selling the asset at $p_{i,v,0}$, both in γ_2 and in γ_4;
- $Ec_{i,v}$ be the cash-flow, which any intermediary i_v, with $v \in \{L, M, H\}$, expects to obtain, if he chooses to participate to the market in $t = 1$; it is equal to $q_i p_{i,v,0}$ times the probability of selling the asset at $p_{i,v,0}$ plus $q_i v_i$ times the probability of not selling the asset at $p_{i,v,0}$, both in γ_3 and in γ_4.

1.4 Trade between intermediaries and buyers

1.4.1 Buyers offer prices to intermediaries

Consider the two games γ_1 and γ_2, in which buyers offer prices to intermediaries in period 1. Clearly any buyer b_v, who offers $p_{b,v} < \bar{p}_{B,v}$ buys a quantity of asset equal to zero and any buyer b'_v, who offers $p_{b',v} = \bar{p}_{B,v}$ buys a quantity of asset equal to:

- Q_I with probability $1/B'_v$, if $p_{b',v} = \bar{p}_{B,v} \geq H$,
- Q_I with probability $(\pi^M + \pi^L)/B'_v$, if $p_{b',v} = \bar{p}_{B,v} : M \leq \bar{p}_{B,v} < H$,

- Q_I with probability π^L/B'_v, if $p_{b',v} = \bar{p}_{B,v} : L \leq \bar{p}_{B,v} < M$, and
- 0 with probability 1, if $p_{b,v} = \bar{p}_{B,v} : \bar{p}_{B,v} < L$.

Assume that either γ_1, in which intermediaries offer prices to sellers in period 0, or γ_2, in which sellers ask prices to intermediaries in period 0, has at least one equilibrium—or that both games have at least one equilibrium. Then, in all such equilibria, buyers offer prices in period 1 as follows. The highest among the prices offered in $t = 1$ is equal to H, if buyers are of type H, and to M, if they are of type M, and at least any two buyers (not necessarily the same buyers if $v_B = H$ and if $v_B = M$) offer such highest prices. If buyers are of type L, each of them offers any price between 0 and L.

Lemma 1: Assume there exists at least one equilibrium in any of the games γ_1 and γ_2. Then in all such equilibria

(1) $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 1, \ldots, B'_v$, $2 \leq B'_v \leq B$, and $v = M, H$,

(2) $p_{b,L} \leq L$, $\forall b$.

Proof See Appendix.

Corollary 1: Assume there exists at least one equilibrium in any of the games γ_1 and γ_2. Then in all such equilibria, $\forall i$,

- $Ec_{i,H} = Hq_i$,
- $Ec_{i,M} = hmq_i$, and
- $Ec_{i,L} = hmlq_i$.

Proof Considering Lemma 1, it is clear that, if there exists at least one equilibrium in any of the games γ_1 and γ_2, then in all such equilibria

- i_H expects to sells q_i at a price equal to H with probability π^H and to sell 0—expecting a cash flow equal to Hq_i—with probability $\pi^M + \pi^L$. Therefore $Ec_{i,H} = Hq_i$;
- i_M expects to sell q_i at a price equal to H with probability π^H, at a price equal to M with probability π^M, and to sell 0—expecting a cash flow equal to Mq_i—with probability π^L. Therefore $Ec_{i,M} = hmq_i$; and
- i_L expects to sell q_i at a price equal to H with probability π^H, at a price equal to M with probability π^M, and, with probability π^L, either to sell q_i at a price equal to L or to sell 0—expecting a cash flow equal to Lq_i. Therefore $Ec_{i,L} = hmlq_i$.

This is true for each intermediary i.

Corollary 1 implies that, if there is any equilibrium in any of the games γ_1 and γ_2, then in all such equilibria type H (M, L) of each intermediary is willing to pay at most H (hm, hml) in order to buy asset from sellers in $t = 0$. This can be easily proved by considering that the price, which any intermediary i_v is willing to pay at most to sellers in $t = 0$, is equal to the expected period 1 cash-flow divided by q_i, i.e., to $Ec_{i,v}/q_i$, and that this is true for each intermediary i_v, $\forall v$.

1.4.2 *Intermediaries ask prices to buyers*

Consider the two games γ_3 and γ_4, in which intermediaries ask prices to buyers in period 1. Assume that either γ_3, in which intermediaries offer prices to sellers in period 0, or γ_4, in which sellers ask prices to intermediaries in period 0, has at least one equilibrium—or that both games have at least one equilibrium. Then, in all such equilibria, intermediaries ask prices in period 1 as follows. Either if intermediaries are of type H or if they are of type M, each of them asks a price equal to H. If intermediaries are of type L, each of them asks a price either equal to H or to M, depending on the (relative) values of hl and ml.

Lemma 2: Assume there exists at least one equilibrium in any of the games γ_3 and γ_4. Then in all such equilibria

(3) $p_{i,v,1} = H$, with $v = M, H$, $\forall i$,

(4) $p_{i,L,1}$:

  (a) $= H$, $\forall i$, if $hl > ml$,

  (b) $\in \{H, M\}$, $\forall i$, if $hl = ml$,

  (c) $= M$, $\forall i$, if $ml > hl$.

Proof See Appendix.

Corollary 2: Assume there exists at least one equilibrium in any of the games γ_3 and γ_4. Then in all such equilibria, $\forall i$,

- $Ec_{i,H} = Hq_i$,
- $Ec_{i,M} = hmq_i$, and
- $Ec_{i,L} =$

– hlq_i, if $hl > ml$,
– $hlq_i = mlq_i$, if $hl = ml$,
– mlq_i, if $ml > hl$.

Proof Considering Lemma 2, it is clear that, if there exists at least one equilibrium in any of the games γ_3 and γ_4, then in all such equilibria:

- i_H expects to sell q_i at $p_{i,H,1} = H$ with probability π^H and to sell 0—expecting a cash flow equal to Hq_i—with probability $\pi^M + \pi^L$. Therefore $Ec_{i,H} = Hq_i$.
- i_M expects to sell q_i at $p_{i,M,1} = H$ with probability π^H and to sell 0—expecting a cash flow equal to Mq_i—with probability $\pi^M + \pi^L$. Therefore $Ec_{i,M} = hmq_i$.
- expectations about $Ec_{i,L}$ are as follows:
  – If $hl > ml$, i_L expects to sell q_i at $p_{i,L,1} = H$ with probability π^H and to sell 0—expecting a cash flow equal to Lq_i—with probability $\pi^M + \pi^L$. Therefore $Ec_{i,L} = hlq_i$, if $hl > ml$.
  – If $hl = ml$ and $p_{i,L,1} = H$, i_L expects to sell q_i with probability π^H and to sell 0—expecting a cash flow equal to Lq_i—with probability π^L. If $hl = ml$ and $p_{i,L,1} = M$, i_L expects to sell q_i with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to Lq_i—with probability π^L. Therefore $Ec_{i,L} = hlq_i = mlq_i$, if $hl = ml$.
  – If $ml > hl$, i_L expects to sell q_i at $p_{i,L,1} = M$ with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to Lq_i—with probability π^L. Therefore $Ec_{i,L} = mlq_i$, if $ml > hl$.

This is true for each intermediary i.

Corollary 2 implies that, if there is any equilibrium in any of the games γ_3 and γ_4, then in all such equilibria

- type H (M) of each intermediary is willing to pay at most H (hm) and
- type L of each intermediary is willing to pay at most hl, if $hl > ml$, or $hl = ml$, if $hl = ml$, or ml, if $ml > hl$,

in order to buy asset from sellers in $t = 0$. This can be easily proved by considering that the price, which any intermediary i_v is willing to pay at most to sellers in $t = 0$, is equal to the expected period 1 cash-flow divided by q_i, i.e., to $Ec_{i,v}/q_i$, and that this is true for each intermediary i_v, $\forall v$.

1.5 Trade between sellers and intermediaries

1.5.1 Intermediaries offer prices to sellers

Buyers offer prices to intermediaries in $t = 1$

Consider game γ_1, in which intermediaries offer prices to sellers in period 0 and buyers offer prices to intermediaries in period 1. The game has many equilibria. In all of them, buyers offer prices in period 1 as explained in Lemma 1 and intermediaries offer prices in period 0 as follows. The highest among the prices offered in $t = 0$ is equal to H, if intermediaries are of type H, to hm, if they are of type M, and to hml, if they are of type L, and at least any two intermediaries (not necessarly the same intermediaries if $v_I = H$, if $v_I = M$, and if $v_I = L$) offer such highest prices.

Lemma 3: In γ_1 there exist many equilibria. In all of them, prices in period 1 are as explained in Lemma 1 and prices in period 0 are as follows:

(i) $p_{i'_H,0} = \bar{p}_{I,H,0} = H$, with $i'_H = 1, \ldots, I'_H$ and $2 \leq I'_H \leq I$,

(ii) $p_{i'_M,0} = \bar{p}_{I,M,0} = hm$, with $i'_M = 1, \ldots, I'_M$ and $2 \leq I'_M \leq I$,

(iii) $p_{i'_L,0} = \bar{p}_{I,L,0} = hml$, with $i'_L = 1, \ldots, I'_L$ and $2 \leq I'_L \leq I$.

Proof According to Lemma 1, if there exists at least one equilibrium in γ_1, then in all such equilibria

(1) $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 2, \ldots, B'_v$, $2 \leq B'_v \leq B$, and $v = M, H$,

(2) $p_{b,L} \leq L$, $\forall b$.

Let $\boldsymbol{v_I = H}$. According to Corollary 1, if there exists at least one equilibrium in γ_1, then in all such equilibria $Ec_{i,H} = Hq_i$, $\forall i$. Therefore, as explained in section 4.1, in all such equilibria, the highest price, which i_H is willing to pay, in order to buy asset from sellers in $t = 0$, is equal to H and this is true for each intermediary i. With a reasoning similar to the one in the proof of Lemma 1—in the case, in which $v_B = H$—it can be easily proved that, among the strategy profiles, in which both (1) and (2) are satisfied,

- any strategy profile, in which $\bar{p}_{I,H,0} > H$, is not part of an equilibrium, or, which is equivalent, any strategy profile, in which any intermediary i_H offers $p_{i,H,0} > H$, cannot be part of an equilibrium;
- any strategy profile, in which $\bar{p}_{I,H,0} < H$, cannot be part of an equilibrium;

- any strategy profile, in which $p_{i'_H,0} = \bar{p}_{I,H,0} = H$, $i'_H = 1, \ldots, I'_H$, $I'_H = 1$, cannot be part of an equilibrium;

- given any strategy profile, in which $p_{i'_H,0} = \bar{p}_{I,H,0} = H$, $i'_H = 1, \ldots, I'_H$, $2 \leq I'_H \leq I$, neither any intermediary i_H offering $p_{i,H,0} < \bar{p}_{I,H,0}$ is better-off, by deviating to any other price, nor any intermediary i'_H offering $p_{i'_H,0} = \bar{p}_{I,H,0} = H$ is better-off, by deviating to any other price, and this is true for each intermediary i.

Let $\boldsymbol{v_I = M}$**.** The reasoning is the same as in the case, in which $v_I = H$, after substituting M to H, when H denotes a type of an intermediary, and hm to H, when H denotes either a cash-flow or a price.

Let $\boldsymbol{v_I = L}$**.** The reasoning is the same as in the case, in which $v_I = H$, after substituting L to H, when H denotes a type of an intermediary, and hml to H, when H denotes either a cash-flow a price.

Therefore, if there exists at least one equilibrium, then, among the strategy profiles, in which both (1) and (2) are satisfied, only those, in which all the three conditions (i), (ii), and (iii) are satisfied, can be part of an equilibrium. Clearly any assessment, in which all the five conditions (1), (2), (i), (ii), and (iii) are satisfied and beliefs are as assumed in section 3.1, is an equilibrium. Therefore there exist many equilibria in γ_1. In all of them all the five conditions (1), (2), (i), (ii), and (iii) are satisfied and beliefs are as assumed in section 3.1.

Intermediaries ask prices to buyers in $\boldsymbol{t = 1}$

Consider game γ_3, in which intermediaries offer prices to sellers in period 0 and ask prices to buyers in period 1. The game has many equilibria. In all of them, intermediaries ask prices in period 1 as explained in Lemma 2 and offer prices in period 0 as follows. The highest among the prices offered in $t = 0$ is equal to H, if intermediaries are of type H, to hm, if they are of type M, and, if they are of type L, either equal to hl or to ml—or to $hl = ml$—depending on the (relative) values of hl and ml, and at least any two intermediaries (not necessary the same intermediaries if $v_I = H$, if $v_I = M$, and if $v_I = L$) offer such highest prices.

Lemma 4: In γ_3 there exist many equilibria. In all of them, prices in period 1 are as explained in Lemma 1 and prices in period 0 are as follows:

(iv) $p_{i'_H,0} = \bar{p}_{I,H,0} = H$, with $i'_H = 1, \ldots, I'_H$ and $2 \leq I'_H \leq I$,

(v) $p_{i'_M,0} = \bar{p}_{I,M,0} = hm$, with $i'_M = 1, \ldots, I'_M$ and $2 \leq I'_M \leq I$,

(vi) $p_{i'_L,0} = \bar{p}_{I,L,0} =:$

(a) hl, with $i'_L = 1, \ldots, I'_L$ and $2 \leq I'_L \leq I$, if $hl > ml$,

(b) $hl = ml$, with $i'_L = 1, \ldots, I'_L$ and $2 \leq I'_L \leq I$, if $hl = ml$,

(c) ml, with $i'_L = 1, \ldots, I'_L$ and $2 \leq I'_L \leq I$, if $ml > hl$,

Proof According to Lemma 2, if there exists at least one equilibrium in γ_3, then in all such equilibria

(3) $p_{i,v,1} = H$, with $v = M, H$, $\forall i$,

(4) $p_{i,L,1}$:

(a) $= H$, $\forall i$, if $hl > ml$,

(b) $\in \{H, M\}$, $\forall i$, if $hl = ml$,

(c) $= M$, $\forall i$, if $ml > hl$.

Let $\boldsymbol{v_I = H}$***.*** The reasoning is the same as in the proof of Lemma 3, in the case, in which $v_I = H$, after substituting "Corollary 2" to "Corollary 1".

Let $\boldsymbol{v_I = M}$***.*** The reasoning is the same as in the case, in which $v_I = H$, after substituting M to H, when H denotes a type of an intermediary, and hm to H, when H denotes either a cash-flow or a price.

Let $\boldsymbol{v_I = L}$***.*** The reasoning is the same as in the case, in which $v_I = H$,

- after substituting L to H, when H denotes a type of an intermediary, and hl to H, when H denotes either a cash-flow or a price, if $hl > ml$;
- after substituting L to H, when H denotes a type of an intermediary, and $hl = ml$ to H, when H denotes either a cash-flow or a price, if $hl = ml$; and
- after substituting L to H, when H denotes a type of an intermediary, and $ml = hl$ to H, when H denotes either a cash-flow or a price, if $ml > hl$.

Therefore, if there exists at least one equilibrium, then, among the strategy profiles, in which both (3) and (4) are satisfied, only those, in which (iv), (v), and (vi) are satisfied, can be part of an equilibrium. Clearly any assessment, in which all the five conditions (3), (4), (iv), (v), and (vi) are satisfied and beliefs are as assumed in section 3.1, is an equilibrium. Therefore there exist many equilibria in γ_3. In all of them all the four conditions (3), (4), (iv), (v), and (vi) are satisfied and beliefs are as assumed in section 3.1.

1.5.2 Sellers ask prices to intermediaries

Buyers offer prices to intermediaries in $\boldsymbol{t = 1}$

Consider game γ_2, in which sellers ask prices to intermediaries in period 0 and buyers offer prices to intermediaries in period 1. The game has many equilibria. In all of them, buyers offer prices in period 1 as explained in Lemma 1 and sellers ask prices in period 0 as follows. Either if sellers are of type H or if they are of type M, each of them asks a price equal to H. If sellers are of type L, each of them asks a price equal to hml.

Lemma 5: In γ_2 there exist many equilibria. In all of them, prices in period 1 are as explained in Lemma 1 and prices in period 0 are as follows:

(vii) $p_{s,v} = H$, with $v = M, H$, $\forall s$,

(viii) $p_{s,L} = hml$, $\forall s$.

Proof According to Lemma 1, if there exists at least one equilibrium in γ_2, then in all such equilibria

(1) $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 1, \ldots, B'_v$, $2 \leq B'_v \leq B$, and $v = M, H$,

(2) $p_{b,L} \leq L$, $\forall b$.

As explained in section 4.1, Corollary 1 implies that, if there is any equilibrium in γ_2, then in all equilibria type H (M, L) of each intermediary is willing to pay at most H (hm, hml) in order to buy asset from sellers in $t = 0$. Therefore any seller s expects to sell his endowment of asset, i.e., 1, with probability

- π^H, if $H \geq p_s > hm$,
- $\pi^H + \pi^M$, if $hm \geq p_s > hml$,
- 1, if $hml \geq p_s$,

and this is true for each seller s.

Let $\boldsymbol{v_S = H}$***.*** If $p_{s,H} = H$, any seller s_H expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to H—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,H} = H$. If $p_{s,H} < H$, he expects to sell 1 with probability equal to or greater than π^H and to sell 0—expecting a cash flow equal to H—with probability equal to or greater than $\pi^M + \pi^L$, and this implies $Ec_{s,H} < H$.

Let $\boldsymbol{v_S = M}$***.*** If $p_{s,M} = H$, any seller s_M expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to M—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,M} = hm$. If $H > p_{s,M} > hm$, he expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to M—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,M} < hm$. If $hm \geq p_{s,M} > Hml$, he expects to sell 1 with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to M—with probability π^L and this implies $Ec_{s,M} = (\pi^H + \pi^M)p_{s,M} + \pi^L M < hm$. If $Hml \geq p_{s,M}$, he expects to sell 1 at $p_{s,M}$ with probability 1, and this implies $Ec_{s,M} \leq Hml < hm$.

Let $\boldsymbol{v_S = L}$. If $p_{s,L} = H$, any seller s_L he expects to sell 1 at $p_{s,L}$ with probability π^H and to sell 0—expecting a cash flow equal to L—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,L} = hl$. If $H > p_{s,L} > hm$, he expects to sell 1 at $p_{s,L}$ with probability π^H and to sell 0—expecting a cash flow equal to L—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,M} < hl$. If $hm \geq p_{s,L} > Hml$, he expects to sell 1 at $p_{s,L}$ with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to L—with probability π^L, and this implies $Ec_{s,L} = (\pi^H + \pi^M)p_{s,L} + \pi^L M < hl$. If $Hml \geq p_{s,L}$, expects to sell 1 at $p_{s,L}$ with probability 1, and this implies $Ec_{s,L} \leq Hml < hl$.

Therefore both for type H and for type M of any seller s, asking a price equal to H strictly dominates all other actions in period 1 and this is true for each seller s. Moreover for type L of any seller s, asking a price equal to hml strictly dominates all other actions in period 1 and this is true for each seller s.

Therefore, if there exists at least one equilibrium, then, among the strategy profiles, in which both (1) and (2) are satisfied, only those, in which both (vii) and (viii) are satisfied, can be part of an equilibrium. Clearly any assessment, in which all the four conditions (1), (2), (vii), and (viii) are satisfied and beliefs are as assumed in section 3.1 is an equilibrium. Therefore there exist many equilibria in γ_2. In all of them all the four conditions (1), (2), (vii), and (viii) are satisfied and beliefs are as assumed in section 3.1.

Intermediaries ask prices to buyers in $\boldsymbol{t = 1}$

Consider game γ_4, in which sellers ask prices to intermediaries in period 0 and intermediaries ask prices to buyers in period 1. The game has many equilibria. In all of them, buyers offer prices in period 1 as explained in Lemma 2 and sellers ask prices in period 0 as follows. Either if sellers are of type H or if they are of type M, each of them asks a price equal to H. If sellers are of type L, each of them either is indifferent among asking a price equal to H, hm, and hl or is indifferent among asking a price equal to H and hl or asks a price equal to hm, depending on the (relative) values of hm and hl.

Lemma 6: In γ_4 there exist many equilibria. In all of them, prices in period 1 are as explained in Lemma 2 and prices in period 0 are as follows:

(ix) $p_{s,v} = H$, with $v = M, H$, $\forall s$,

(x) $p_{s,L}$:

(a) $\in \{H, hm, hl\}$, $\forall s$, if both $hl > ml$ and $hl = (\pi^H + \pi^M)hm + \pi^L L$,

(b) $\in \{H, hl\}$, $\forall s$, if both $hl > ml$ and $hl > (\pi^H + \pi^M)hm + \pi^L L$,

(c) $= hm$, $\forall s$, either if both $hl > ml$ and $hl < (\pi^H + \pi^M)hm + \pi^L L$ or if $ml \geq hl$.

Proof See Appendix.

1.6 Bubbles

Under the assumption of common first-order prior beliefs, equilibrium bubbles are possible in period 0, but not in period 1. If all agents are of type L, bubbles occur in period 0, under all trading rules, which we have considered so far. If all agents are of type M, period 0 bubbles occur, if and only if intermediaries offer prices to sellers in period 0. If all agents are of type H, no period 0 bubble occurs.

Proposition 1: Let $v_S = v_I = v_B = v$. There are no equilibrium bubbles in period 1, either in γ_1, γ_2, γ_3, or γ_4. If $v = L$, there is a bubble in period 0 in all equilibria in γ_1, γ_2, and γ_3. If both $hl > ml$ and $hl \geq (\pi^H + \pi^M)hm + \pi^L L$, there is a bubble in period 0 in all equilibria, in which $p_{s,L} = hl$, for at least one seller s, but not in the other equilibria, in γ_4. If $v = M$, there is a bubble in period 0 in all equilibria both in γ_1 and γ_3, but there are no equilibrium bubbles either in γ_2 or in γ_4. If $v = H$, there is a bubble in period 0 in all equilibria in all the four games.

Proof

Consider either γ_1, γ_2, γ_3 or γ_4

Clearly no buyer b_v would buy at any price higher than v, and this is true for each b and for each v. Therefore no buyer posts a price higher than v in any equilibrium, either in γ_1 or in γ_2, and none of them accepts to buy at any price higher than v in any equilibrium, either in γ_3 or in γ_4. Therefore there is no bubble in period 1 in any equilibrium, either in γ_1, γ_2, γ_3 or γ_4.

Assume $v = H$. According to Lemmas from 3 to 6, in any of the four games, all agents—either intermediaries or sellers—who post a price in period 0, post a price equal to H in all equilibria. Since $v = H$, there is no bubble in period 0 in any equilibrium.

Consider γ_1

Assume $v = L$. According to Lemma 3, in all equilibria $p_{i'_L,0} = \bar{p}_{I,L,0} = hml$, $i'_L = 2, \ldots, I'_L$, $I'_L \in \{2, \ldots, I\}$. Trade occurs at $\bar{p}_{I,L,0} = hml$. At such a price sellers sell all their endowment of asset to at least one intermediary i'_L (see both Corollary 1 and Lemma 3). Since $p_{i'_L,0} = hml > v = L$, there is a bubble in period 0 in all equilibria.

Assume $v = M$. According to Lemma 3, in all equilibria $p_{i'_M,0} = \bar{p}_{I,M,0} = hm$, $i'_M = 2, \ldots, I'_M$, $I'_M \in \{2, \ldots, I\}$. Trade occurs at $\bar{p}_{I,M,0} = hm$. At such a price sellers sell all their endowment of asset to at least one intermediary i'_M (see both Corollary 1 and Lemma 3). Since $p_{i'_M,0} = hm > v = M$, there is a bubble in period 0 in all equilibria.

Consider γ_2

Assume $v = L$. According to Lemma 5, in all equilibria $p_{s,L} = hml$, $\forall s$. At such a price sellers sell all their endowment S of asset to at least one intermediary i (see both Corollary 1 and Lemma 5). Since $p_{s,L} = hml > v = L$, $\forall s$, there is a bubble in period 0 in all equilibria.

Assume $v = M$. According to Lemma 5, in all equilibria $p_{s,M} = M$, $\forall s$. At such a price sellers sell all their endowment S of asset to at least one intermediary i (see both Corollary 1 and Lemma 5). Since $p_{s,M} = M = v = L$, $\forall s$, there is no bubble in period 0 in any equilibrium.

Consider γ_3

Assume $v = L$. According to Lemma 4, in any equilibrium either $p_{i,L,0} = hl$, $\forall i$, or $p_{i,L,0} = hl = ml$, $\forall i$, or $p_{i,L,0} = ml$, $\forall i$. Given the prices offered by intermediaries, sellers sell all their endowment of asset to at least one intermediary i (see both Corollary 2 and Lemma 4). Since $p_{i,L,0} > v = L$, $\forall i$, there is bubble in period 0 in all equilibria.

Assume $v = M$. According to Lemma 4, in all equilibria $p_{i,M,0} = hm$, $\forall i$. At such a price sellers sell all their endowment S of asset to at least one intermediary i (see both Corollary 2 and Lemma 4). Since $p_{i,M,0} = hm > v = L$, $\forall i$, there is a bubble in period 0 in all equilibria.

Consider γ_4

Assume $v = L$. According to Lemma 6, in any equilibrium either $p_{s,L} \in \{H, hm, hl\}$, $\forall s$, or $p_{s,L} \in \{H, hl\}$, $\forall s$, or $p_{s,L} = hm$, $\forall s$, depending on the (relative) values of hm and hl. Trade occurs only in those equilibria, in which $p_{s,L} = hl$, for at least one seller s. Sellers asking such a price sell all their endowment of asset to at least one intermediary i (see both Corollary 2 and Lemma 6). Since $p_{s,L} = hl > v = L$, there is a bubble in period 0 in all equilibria, in which $p_{s,L} = hl$, for at least one seller s, but not in the other equilibria. Since necessary conditions for $p_{s,L} = hl$, for at least one seller s in equilibrium, are $hl > ml$ and $hl \geq (\pi^H + \pi^M)hm + \pi^L L$ (see Lemma 6), such conditions are necessary also for any equilibrium bubble in period 0.

Assume $v = M$. According to Lemma 6, in all equilibria $p_{s,M} = M$, $\forall s$. At such a price sellers sell all their endowment of asset to at least one intermediary i_M (see both Corollary 2 and Lemma 6). Since $p_{s,M} = v = M$, $\forall s$, there is no bubble in period 0 in any equilibrium.

1.7 Conclusion

In our analysis we have shown that in a financial market, in which trade is intermediated, a bubble can emerge, under the assumptions that short-selling is not allowed and that the supply of asset is limited relative to the demand, because of initial second-order uncertainty, even if, differently from previous models on bubbles under rational expectations, agents have common first-order prior beliefs and there is no agency problem between agents.

We believe that our model could be modified, in order to describe how information—either about asset returns or about agents expectations about asset returns—is revealed (or not revealed) either by prices or by agents' trades—or by both—in markets, in which:

- information is asymmetric and
- not all agents are able to trade together at the same time, e.g., because of search cost in locating sellers or (potential) buyers of an asset.

Appendix

Proof of Lemma 1

Consider either γ_1 or γ_2. We denote as p' any price different from $p_{b,v}$.

Consider any buyer b_v, with $v \in \{L, M, H\}$

Let $p_{b,v} > v$. If $p_{b,v} < \bar{p}_{B,v}$, b_v buys 0 and makes 0. If $p_{b,v} = \bar{p}_{B,v}$, he expects to buy a positive quantity and to make a loss, unless both $p_{b,v} \geq H$ and type v of all other buyers offers less than $\bar{p}_{B,v}$. In such a case, he buys a positive quantity and makes a loss.

If, e.g., $p' = v$, he expects to buy a positive quantity and to make 0, unless both $v = H$ and type H of all other buyers offers less than H. In such a case, he buys a positive quantity and makes 0.

Therefore any buyer b_v offering $p_{b,v} > v$, with $p_{b,v} = \bar{p}_{B,v}$, is better-off by deviating to, e.g., $p' = v$. This implies that any strategy profile, in which $\bar{p}_{B,v} > v$, or, which is equivalent, any strategy profile, in which type v of any buyer b offers $p_{b,v} > v$, cannot be part of an equilibrium.

Consider any buyer b_L

Let $p_{b,L} \leq L$. If $p_{b,L} < L$, any buyer b_L buys 0 and makes 0. If $p_{b,L} = L$, consider the following cases:

1. if $p_{b,L} < \bar{p}_{B,L}$, he buys 0 and makes 0,
2. if $p_{b,L} = \bar{p}_{B,L}$, he expects to buy a positive quantity and to make 0.

If $p' < L$ ($p' = L$), the reasoning is the same as in the case, in which we consider $p_{b,L} < L$ ($p_{b,L} = L$). If $p' > L$, consider the following cases:

1. if $p' < \bar{p}_{B,L}$, he buys 0 and makes 0,
2. if $p' = \bar{p}_{B,L}$, he expects to buy a positive quantity and to make a loss, and
3. if $p' > \bar{p}_{B,L}$, he either buys a positive quantity and makes a loss, if $p' \geq H$, or expects to buy a positive quantity and to make a loss, otherwise.

Therefore type L of any buyer b offering $p_{b,L} \leq L$ is no better-off by deviating to any other price p' and receives the same utility from offering $p_{b,L} \leq L$ as any other price $p' \leq L$.

Let $v_B = M$

Assume $\bar{p}_{B,M} < M$. Consider any strategy profile, in which type M of at least one buyer, but not of all buyers, offers $p_{b,M} = \bar{p}_{B,M} < M$, or, more formally, any strategy profile, in which $p_{b',M} = \bar{p}_{B,M} < M$, $b'_M = 1, \ldots, B'_M$, and $1 \leq B'_M < B$. Consider any buyer b_M offering $p_{b,M} < \bar{p}_{B,M}$. He buys 0 and makes 0. If both $p' \geq \bar{p}_{B,M}$ and $L \leq p' < M$, he expects to buy $\pi^L Q_I$ and to make a profit. Therefore type M of any buyer b offering $p_{b,M} < \bar{p}_{B,M} < M$ is better-off by deviating to $p' \geq \bar{p}_{B,M}$, with $L \leq p' < M$. This implies that any strategy profile, in which type M of any buyer b offers $p_{b,M} < \bar{p}_{B,M} < M$, cannot be part of an equilibrium.

Consider any strategy profile, in which $p_{b,M} = \bar{p}_{B,M} < M$ for all buyers, or, more formally, any strategy profile, in which $p_{b',M} = \bar{p}_{B,M} < M$, $b'_M = 1, \ldots, B'_M$, and $B'_M = B$. For any buyer b_M consider the following cases:

1. If $\bar{p}_{B,M} < L$, he buys 0 and makes 0. If, e.g., $p' = L$, he expects to buy a positive quantity and to make a profit.

2. If $\bar{p}_{B,M} \geq L$, he expects to buy $\pi^L Q_I/B$ and to make a profit of $(\pi^L Q_I/B)(M - p_{b,M})$. If, e.g., $p' = \bar{p}_{B,M} + \varepsilon$, with ε positive and close to 0, he expects to buy $\pi^L Q_I/B$ and to make a profit of $(\pi^L Q_I)(M - (p_{b,M} + \varepsilon))$.

Therefore type M of any buyer b is better-off by deviating either to $p' = L$, if $\bar{p}_{B,M} < L$, or to $p' = \bar{p}_{B,M} + \varepsilon$, if $\bar{p}_{B,M} \geq L$. This implies that any strategy profile, in which $p_{b,M} = \bar{p}_{B,M} < M$ for all buyers, cannot be part of an equilibrium.

Assume $\boldsymbol{\bar{p}_{B,M} = M}$***.*** Consider any strategy profile, in which type M of at least one buyer, but not of all buyers, offers $p_{b,M} = \bar{p}_{B,M} = M$, or, more formally, in which $p_{b',M} = \bar{p}_{B,M} = M$, $b'_M = 1, \ldots, B'_M$, and $1 \leq B'_M < B$. Consider any buyer b_M offering $p_{b,M} < \bar{p}_{B,M}$. He buys 0 and makes 0. If $p' < L$, he buys 0 and makes 0. If $p' \geq L$, consider the following cases:

1. If $p' < \bar{p}_{B,M}$, he buys 0 and makes 0.

2. If $p' = \bar{p}_{B,M}$, he expects to buy a positive quantity and to make 0.

3. If $p' > \bar{p}_{B,M}$, he either buys a positive quantity and makes a loss, if $p' \geq H$, or expects to buy a positive quantity and to make a loss, otherwise.

Therefore type M of any buyer b offering $p_{b,M} < \bar{p}_{B,M} = M$ is no better-off by deviating to any other price p'.

Consider any strategy profile, in which type M of only one buyer offers $p_{b,M} = \bar{p}_{B,M} = M$, or, more formally, in which $p_{b',M} = \bar{p}_{B,M} = M$, $b'_M = 1, \ldots, B'_M$, and $B'_M = 1$. Consider buyer b'_M. He expects to buy $(\pi^M + \pi^L)Q_I$ and to make 0. If, e.g., $p' = \bar{p}_{B,M} - \varepsilon$, with ε positive and close to 0, he expects to buy $\pi^L Q_I$ and to make a profit of $\pi^L Q_I/\varepsilon$. Therefore buyer b'_M, or, which is equivalent, type M of buyer b offering $p_{b,M} = \bar{p}_{B,M} = M$ is better-off by deviating to $p' = \bar{p}_{B,M} - \varepsilon$. This implies that any strategy profile, in which only one buyer b'_M offers $p_{b'_M,1} = \bar{p}_{B,M} = M$, or, which is equivalent, type M of only one buyer b offers $p_{b,M} = \bar{p}_{B,M} = M$, cannot be part of an equilibrium.

Consider any strategy profile, in which type M of more than one buyer offers $p_{b',M} = \bar{p}_{B,M} = M$, or, more formally, in which $p_{b',M} = \bar{p}_{B,M} = M$, $b'_M = 1, \ldots, B'_M$ and $2 \leq B'_M \leq B$. Consider any buyer b'_M. He expects to buy $(\pi^M + \pi^L)Q_I/B'_M$ and to make 0. If $p' < \bar{p}_{B,M}$, he buys 0 and makes 0. If $p' > \bar{p}_{B,M}$, he either buys Q_I and makes a loss, if $p' \geq H$, or expects to buy $(\pi^M + \pi^L)Q_I$ and to make a loss, otherwise. Therefore any buyer b'_M, or, which is equivalent, type M of any buyer b offering $p_{b,M} = \bar{p}_{B,M} = M$, is no better-off by deviating to any other price p'.

Let $v_B = H$

The reasoning is similar to the one in the case, in which $v_B = M$. Therefore it can be easily proved that

1. any strategy profile, in which $p_{b,H} = \bar{p}_{B,H} < H$ for one or more, but not all, buyers, cannot be part of an equilibrium,
2. any strategy profile, in which $p_{b,H} = \bar{p}_{B,H} < H$ for all buyers, cannot be part of an equilibrium,
3. given any strategy profile, in which $p_{b,H} < \bar{p}_{B,H} = H$ for any buyer b, b_H is no better-off by deviating to any other price p',
4. any strategy profile, in which $p_{b,H} = \bar{p}_{B,H} = H$ for any and only one buyer, cannot be part of an equilibrium, and
5. given any strategy profile, in which $p_{b,H} = \bar{p}_{B,H} = H$ for any buyer b and type H of at least one other buyer offers $\bar{p}_{b,H}$, b_H is no better-off by deviating to any other price p'.

So far we have proved that:

1. any strategy profile, in which $\bar{p}_{B,v} > v$, with $v \in \{L, M, H\}$, cannot be part of an equilibrium;
2. given any strategy profile, in which $p_{b,L} \leq L$ for any buyer b, b_L is no better-off by deviating to any other price p' and receives the same utility from offering $p_{b,L} \leq L$ as any other price $p' \leq L$;
3. any strategy profile, in which $\bar{p}_{B,v} < v$, with $v \in \{L, M, H\}$, cannot be part of an equilibrium;
4. given any strategy profile, in which $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 1, \ldots, B'_v$, $1 \leq B'_v < B$, and $v = M, H$, type v of any buyer b offering $p_{b,v} < \bar{p}_{B,v}$ is no better-off by deviating to any other price p';
5. any strategy profile, in which $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 1, \ldots, B'_v$, $B'_v = 1$, and $v = M, H$, cannot be part of an equilibrium;
6. given any strategy profile, in which $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 1, \ldots, B'_v$, $2 \leq B'_v \leq B$, and $v = M, H$, type v of any buyer b' is no better-off by deviating to any other price p'.

Therefore, if there exists at least one equilibrium in any of the games γ_1 and γ_2, then each strategy profile, which is part of any of such equilibria, satisfies the following two conditions:

(1) $p_{b',v} = \bar{p}_{B,v} = v$, with $b'_v = 1, \ldots, B'_v$, $2 \le B'_v \le B$, and $v = M, H$,

(2) $p_{b,L} \le L$, $\forall b$.

Therefore, if there exists at least one equilibrium in any of the games γ_1 and γ_2, then in all equilibria both (1) and (2) are verified.

Proof of Lemma 2

Consider either γ_1 or γ_2.

Let $\boldsymbol{v_S = H}$***.*** If $p_{i,H,1} = H$, any intermediary i_H expects to sell q_i with probability π^H and to sell 0—expecting a cash flow equal to Hq_i—with probability $\pi^M + \pi^L$, and this implies $Ec_{i,H} = Hq_i$. If $p_{i,H,1} < H$, he expects to sell q_i with probability equal to or greater than π^H and to sell 0—expecting a cash flow equal to Hq_i—with probability equal to or greater than $\pi^M + \pi^L$, and this implies $Ec_{i,H} < Hq_i$.

Let $\boldsymbol{v_S = M}$***.*** If $p_{i,M,1} = H$, any intermediary i_M expects to sell q_i with probability π^H and to sell 0—expecting a cash flow equal to Mq_i—with probability $\pi^M + \pi^L$, and this implies $Ec_{i,M} = hmq_i$. If $H > p_{i,M,1} > M$, he expects to sell q_i with probability π^H and to sell 0—expecting a cash flow equal to Mq_i—with probability $\pi^M + \pi^L$, and this implies $Ec_{i,M} < hmq_i$. If $p_{i,M,1} = M$, he expects to sell q_i with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to Mq_i—with probability π^L, and this implies $Ec_{i,M} = Mq_i$. If $M > p_{i,M,1} > L$, he expects to sell q_i with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to Mq_i—with probability π^L, and this implies $Ec_{i,M} < Mq_i$. If $p_{i,M,1} \le L$, he expects to sell q_i with probability 1 and this implies $Ec_{i,M} < Mq_i$.

Let $\boldsymbol{v_S = L}$***.*** If $p_{i,L,1} = H$, any intermediary i_L expects to sell q_i with probability π^H and to sell 0—expecting a cash flow equal to Lq_i—with probability $\pi^M + \pi^L$, and this implies $Ec_{i,L} = hlq_i$. If $H > p_{i,L,1} > M$, he expects to sell q_i with probability π^H and to sell 0—expecting a cash flow equal to Lq_i—with probability $\pi^M + \pi^L$, and this implies $Ec_{i,L} < hlq_i$. If $p_{i,L,1} = M$, he expects to sell q_i with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to Lq_i—with probability π^L, and this implies $Ec_{i,L} = mlq_i$. If $M > p_{i,L,1} > L$, he expects to sell q_i with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to Lq_i—with probability π^L, and this implies $Ec_{i,L} < mlq_i$. If $p_{i,L,1} \le L$, he expects to sell q_i with probability 1 and this implies $Ec_{i,L} < mlq_i$.

Therefore, both for type H and for type M of any intermediary i, asking a price equal to H strictly dominates all other actions in period 1, both in γ_3 and in γ_4, and this is true for each intermediary i. This implies that, if there exists at leat one equilibrium in any of such games, then $p_{i,v,1} = H$, $\forall i$, with $v = M, H$, in all such equilibria.

Moreover, both with reference to γ_3 and to γ_4, the following statements are true:

1. If $hl > ml$, for type L of any intermediary i, asking $p_{i,L,1} = H$ strictly dominates all other actions in period 1 and this is true for each intermediary i. Therefore, if there exists at least one equilibrium, then $p_{i,L,1} = H$, $\forall i$, in each of such equilibria.

2. If $hl = ml$, type L of any intermediary i is indifferent between asking $p_{i,L,1} = H$ and $p_{i,L,1} = M$, such actions strictly dominate all the others in period 1, and this is true for each intermediary i. This implies that, if there exists at least one equilibrium in any of such games, then $p_{i,L,1} \in \{M, H\}$, $\forall i$, in each of such equilibria.

3. If $ml > hl$, for type L of any intermediary i, asking $p_{i,L,1} = M$ strictly dominates all other actions in period 1, and this is true for each intermediary i. Therefore, if there exists at least one equilibrium in any of such games, then $p_{i,L,1} = M$, $\forall i$, in each of such equilibria.

Proof of Lemma 6

According to Lemma 2, if there exists at least one equilibrium in γ_4, then in all such equilibria

(3) $p_{i,v,1} = H$, with $v = M, H$, $\forall i$,

(4) $p_{i,L,1}$:

  (a) $= H$, $\forall i$, if $hl > ml$,

  (b) $\in \{M, H\}$, $\forall i$, if $hl = ml$,

  (c) $= M$, $\forall i$, if $ml > hl$.

As explained in section 4.2, Corollary 2 implies that, if there is any equilibrium in γ_4, then in all equilibria

- type H (M) of each intermediary is willing to pay at most H (hm) and

- type L of each intermediary is willing to pay at most hl, if $hl > ml$, or $hl = ml$, if $hl = ml$, or ml, if $ml > hl$, in order to buy asset from sellers in $t = 0$. Therefore any seller s expects to sell his endowment of asset, i.e., 1, with probability

- π^H, if $H \geq p_s > hm$,
- $\pi^H + \pi^M$, either if both $hm \geq p_s > hl$ and $hl > ml$, or if both $hm \geq p_s > hl = ml$ and $hl = ml$, or if both $hm \geq p_s > ml$ and $ml > hl$,
- 1, either if both $hl \geq p_s$ and $hl > ml$, or if both $hl = ml \geq p_s$ and $hl = ml$, or if both $ml \geq p_s$ and $ml > hl$,

and this is true for each seller s.

Let $v_S = H$. The reasoning is the same as in Lemma 5, in the case, in which $v_I = H$.

Let $v_S = M$. If $p_{s,M} = H$, any seller s_M expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to M—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,M} = hm$. If $H > p_{s,M} > hm$, he expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to M—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,M} < hm$.

Assume $hl > ml$. If $hm \geq p_{s,M} > hl$, s_M expects to sell 1 with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to M—with probability π^L, and this implies $Ec_{s,M} = (\pi^H + \pi^M)p_{s,M} + \pi^L M < hm$. If $hl \geq p_{s,M}$, he expects to sell 1 with probability 1, and this implies $Ec_{s,M} \leq hl < hm$.

Assume $hl = ml$. The reasoning is the same as in the case above, in which $hl > ml$, after substituting $hl = ml$ to hl.

Assume $ml < hl$. The reasoning is the same as in the case above, in which $hl > ml$, after substituting ml to hl.

Let $v_S = L$. If $p_{s,L} = H$, any seller s_L expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to L—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,L} = hl$. If $H > p_{s,L} > hm$, he expects to sell 1 with probability π^H and to sell 0—expecting a cash flow equal to L—with probability $\pi^M + \pi^L$, and this implies $Ec_{s,L} < hl$. If $p_{s,L} = hm$, he expects to sell 1 with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to L—with probability π^L, and this implies $Ec_{s,L} = (\pi^H + \pi^M)hm + \pi^L M$.

Assume $hl > ml$. If $hm > p_{s,L} > hl$, s_L expects to sell 1 with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to M—with probability π^L, and this implies $Ec_{s,L} = (\pi^H + \pi^M)p_{s,L} + \pi^L M < hm$. If $p_{s,M} = hl$, he expects to sell 1 with probability 1, and this implies $Ec_{s,L} = hl$. If $hl > p_{s,L}$, he expects to sell 1 with probability 1, and this implies $Ec_{s,L} < hl$.

Assume $hl = ml$. If $hm > p_{s,L} > hl = ml$, s_L expects to sell 1 with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to M—with probability π^L, and this implies $Ec_{s,L} = (\pi^H + \pi^M)p_{s,L} + \pi^L M < hm$. If $p_{s,M} = hl = ml$, he expects to sell 1 with probability 1, and this implies $Ec_{s,L} = hl = ml$. If $hl = ml > p_{s,L}$, he expects to sell 1 with probability 1, and this implies $Ec_{s,L} < hl = ml$.

Assume $ml > hl$. If $hm > p_{s,L} > ml$, s_L expects to sell 1 with probability $\pi^H + \pi^M$ and to sell 0—expecting a cash flow equal to M—with probability π^L, and this implies $Ec_{s,L} = (\pi^H + \pi^M)p_{s,L} + \pi^L M < hm$. If $p_{s,M} = ml$, he expects to sell 1 at $p_{s,L}$ with probability 1, and this implies $Ec_{s,L} = ml < hm$. If $ml > p_{s,L}$, s_L expects to sell 1 with probability 1, and this implies $Ec_{s,L} < ml < hm$.

Therefore both for type H and for type M of any seller s, asking a price equal to H strictly dominates all other actions in period 1 and this is true for each seller s. Moreover the following statements are true:

1. If both $hl > ml$ and $hl = (\pi^H + \pi^M)hm + \pi^L L$, type L of any seller s is indifferent among asking $p_{s,L} = H$, $p_{s,L} = hm$, and $p_{s,L} = hl$, such actions strictly dominate all other actions in period 0 and this is true for each seller s. $in\{H, hm, hl\}$

2. If both $hl > ml$ and $hl > (\pi^H + \pi^M)hm + \pi^L L$, type L of any seller s is indifferent between asking $p_{s,L} = H$ and $p_{,L} = hl$, such actions strictly dominate all the others in period 0, and this is true for each seller s.

3. Iif either both $hl > ml$ and $hl < (\pi^H + \pi^M)hm + \pi^L L$ or if $ml \geq hl$, for type L of any seller s, asking $p_{s,L} = hm$ strictly dominates all other actions in period 1, and this is true for each seller s.

Therefore, if there exists at least one equilibrium, then, among the strategy profiles, in which both (3) and (4) are satisfied, only those, in which both (ix) and (x) are satisfied, can be part of an equilibrium. Clearly any assessment, in which all the four conditions (3), (4), (ix), and (x) are satisfied and beliefs are as assumed in section 3.1 is an equilibrium. Therefore there exist many equilibria in γ_4. In all of them all the four conditions (3), (4), (ix), and (x) are satisfied and beliefs are as assumed in section 3.1.

2. LIQUIDITY AND COSTLY INFORMATION ACQUISITION

2.1 Introduction

In the present work we examine how asymmetric information and liquidity (or wealth) can affect the acquisition of information in financial markets. We analyze a general setting, in which some agents, which we name sellers, can both "verify the quality" of and invest in an asset, where by "verify the quality" of the asset we mean to acquire information about the future value of, or cash-flow from, the asset. We show that, in such a setting, the presence of potential buyers—or, simply, buyers—who have no liquidity (or wealth) constraint and who cannot verify the quality of the asset, induces sellers not to acquire information about the future value of asset, before investing in it.

First we show that, if there were no potential buyers for the asset, sellers, before deciding if to invest in it or not, would verify the quality of the asset and would invest, if and only if the quality were high. In other words, given that the asset can deliver different possible cash-flows in the future, sellers would invest, if and only if they would obtain the information that the asset will deliver any of the highest possible cash-flows (and not any of the lowest). Then we show that, in presence of buyers, who are not wealth-constrained, sellers would not verify the quality of the asset and would invest in it, and, moreover, that buyers would buy the asset.

The difference in behavior of sellers, in absence and in presence of buyers, is possible, since sellers are risk-averse. If there are no buyers, sellers, being risk-averse, prefer spending some wealth to verify the cash-flow from the asset, before taking their investment decision, in order to avoid the risk of investing in an asset, which could deliver a low cash-flow. If there are buyers, sellers, given the price at which buyers are willing to buy the asset with certainty, prefer directly investing in the asset in order to sell it, without spending resources to verify its quality. We could imagine sellers to be originators of the asset, such as originators of US residential mortgages, who have been at the origin of the 2007 financial crisis, and buyers to be financial institutions in general, such as investment banks.

In section 2 we present assumptions common to the models, which we discuss in sections from 3 to 5. In section 6 we conclude.

2.1.1 Related literature

Grossman and Stiglitz (1980) and Verrecchia (1982) discuss models, in which agents decide whether to acquire information about assets before trading. In Grossman and Stiglitz agents can purchase a common signal about the return on a risky asset at a certain cost. In Verrecchia the signal about the return on a risky asset is private, i.e., each agent chooses the quality of the signal, which he purchases, by paying a higher cost in order to obtain a signal with a higher precision, i.e., with a lower variance of its error term. In both settings, differently from the present work, trade is modeled only as a non-strategic interaction. Moreover no investment decision is taken by agents, i.e., traders, after deciding whether to acquire information, simply trade their endowments of assets, and no distinction between agents, who can invest in a risky asset and agents, who can only buy the asset from "investors".

2.2 The models: common assumptions

2.2.1 All models

All the models, which we analyze, are characterized by the following common assumptions.

All characteristics of a model are common knowledge among the agents featured in that model, unless we specify otherwise.

There are $t = 1, \ldots, T$ dates, with $T \geq 3$. There are $s = 1, \ldots, S$ agents, which we denote as sellers. Each of them has an initial endowment of one unit of consumption good, or wealth, which can be used for consumption. Each seller has the option to verify at date 1 the "quality" of a risky asset, and to invest his wealth in it at date 2. For simplicity we assume that the minimum investment, which can be made in the asset, is exactly one unit of consumption good. Such an investment allows a seller to obtain one unit of asset. Therefore a seller's decision in period 2 will be simply between investing 1 unit of consumption good in the asset (I) and not investing it (NI), by keeping it "as it is", e.g., in a safe storage technology. A seller chooses $a_s^2 \in A^2 = \{a^2 | a^2 = I, NI\}$, $\forall s$, with A^2 being the set of possible period 2 actions for all sellers.

Either if sellers invest in the risky asset or not, consumption takes place in period T, when the asset delivers a return, i.e., cash-flow of consumption good (and the safe storage technology delivers a cash-flow equal to 1). Four possible states $\omega \in P^1 = \{1, 2, 3, 4\}$ can occur at date T, with $P^1 = P_s^1$, $\forall s$, where P^1 denotes sellers' initial partition of date T state space. In each state, the asset delivers a different return. Let v^ω be the cash-flow from the asset in state ω, with $v^1 > v^2 > v^3 > v^4$. A state ω occurs with probability $\pi^\omega \in (0, 1)$, with $\sum_{\omega=1}^{4} \pi^\omega = 1$.

At date 1 sellers can verify the quality (V) of the risky asset or not (NV), i.e., a seller chooses $a_s^1 \in A^1 = \{a^1 | a^1 = V, NV\}$, with A^1 being the set of possible period 1 actions $\forall s$.

Either if he verifies or if he does not verify, he obtains a signal m_s about the asset return. In the former case the signal is both costly and informative, while in the latter case it is both costless and uninformative. Let $m_s|a_s^1$ being the value, which the signal assumes, given that seller s chooses action $a_s^1 \in A^1$. If $a_s^1 = V$, a seller spends $c > 0$ units of his wealth and the signal assumes value $m_s|V = H$, if the asset will deliver a cash-flow equal to either v^1 or v^2 (i.e., if the state will be either 1 or 2), and value $m_s|V = L$, if the cash-flow will be either v^3 or v^4 (i.e., if the state will be either 3 or 4). This is true $\forall s$. We can write $m_s|V \in \{H, L\}$ with $\Pr(m_s|V = H) = \Pr(H|V) = \pi^1 + \pi^2$, $\Pr(\omega \in \{1, 2\}|(H|V)) = \Pr(\omega \in \{1, 2\}|H) = 1$, $\Pr(m_s|V = L) = \Pr(L|V) = \pi^3 + \pi^4$, and $\Pr(\omega \in \{3, 4\}|(L|V)) = \Pr(\omega \in \{3, 4\}|L) = 1$, $\forall s$. If $a_s^1 = NV$, the signal is both costless and uninformative, i.e., a seller spends no wealth and the signal assumes value HL, independently of the true date T state. We can write $m_s|NV = HL$, i.e., $\Pr(m_s|NV = HL) = \Pr(HL|NV) = 1$, and $\Pr(\omega|(HL|NV)) = \Pr(\omega|HL) = \pi^\omega$, $\forall \omega \in P^1$, $\forall s$.

Let

$$E_s v = \sum_{\omega=1}^{4} \pi^\omega v^\omega$$

denote the expected cash-flow (or expected value of the asset) according to the initial belief of seller s, i.e., to s' belief before receiving any signal, $\forall s$. Therefore we define the expected value of the asset, according to s' belief (updated by applying Bayes rule) after receiving $m_s|V = H$, as

$$E_s v|H = (v^1\pi^1 + v^2\pi^2)/(\pi^1 + \pi^2),$$

the expected value of the asset, after receiving $m_s|V = L$, as

$$E_s v|L = (v^1\pi^1 + v^2\pi^2)/(\pi^1 + \pi^2),$$

and the expected value of the asset, after receiving $m_s|NV = HL$, as

$$E_s v|HL = Ev,$$

$\forall s$. With a slight abuse of terminology, let

$$E_s v = HL, \quad E_s v|H = H, \quad \text{and } E_s v|L = L,$$

$\forall s$. Notice that $H > HL > L$ and let $HL > 1 > L$. We also let

$$H - c = \sum_{\omega=1}^{2} \pi^{\omega}(v^{\omega} - c)/(\pi^1 + \pi^2)$$

and

$$L - c = \sum_{\omega=3}^{4} \pi^{\omega}(v^{\omega} - c)/(\pi^3 + \pi^4).$$

We denote as $g^{E_s v|(m_s|a_s^1)}$ the gamble between the cash-flows delivered in the states, which seller s know can occur, after observing $m_s|a_s^1$. Moreover we denote as

- g^{HL} the gamble between the four possible cash-flows v^{ω}, $\omega = 1, 2, 3, 4$,
- g^{H} the gamble between v^1 and v^2, given $m_s|V = H$,
- g^{L} the gamble between v^3 and v^4, given $m_s|V = L$,
- g^{H-c} the gamble between $v^1 - c$ and $v^2 - c$, given $m_s|V = H$, and
- g^{L-c} the gamble between $v^3 - c$ and $v^4 - c$, given $m_s|V = L$.

Sellers are risk-averse and do not discount utility. We assume for simplicity that a seller s has a constant-absolute risk-aversion (CARA) utility function

$$u_s(w_s^T) = u_S(w_s^T) = 1 - \exp(-r w_s^T),$$

$\forall s$, where $r > 0$ is sellers' constant level of risk-aversion and w_s^T is the wealth, which s has and can use for consumption at date T.

We let

$$Eu_S|(a_s^1, (a_s^2|(m_s|a_s^1)))$$

be the utility, which a seller expects to obtain by choosing both action $a_s^1 \in A^1$, and, for a given value of the signal $m_s|a_s^1$, action $a_s^2 \in A^2$. For instance, we denote as

$$Eu_S|(V, (NI|(m_s|V))) = u_S|(V, NI) = u_S(1 - c),$$

with $m_s|V = H, L$, the utility, which a seller obtains, if he first verifies the quality of the asset and then decides not to invest, whatever the value of $m_s|V$, and as

$$Eu_S|(NV, (NI|(m_s|NV))) = u_S|(NV, NI) = u_S(1)$$

the utility, which he obtains, if he just "keeps his wealth safe", without verifying the quality of the asset (in such a case $m_s|NV = HL$).

We assume that

$$Eu_S(g^{H-c}) > Eu_S(g^{HL}) > u_S(1) > u_S(1-c) > Eu_S(g^L). \tag{2.1}$$

The assumption that $Eu_S(g^{HL}) > u_S(1)$ guarantees that, if a seller does not verify, he prefers investing to not investing, and the assumption that $u_S(1-c) > Eu_S(g^L)$ guarantees, that, if a seller verifies the quality of the asset and discovers it is low, he prefers not investing to investing.

We also assume that, if a seller is indifferent between verifying the quality of the asset and not verifying it, he chooses to verify it. Moreover, if he is indifferent between investing and not investing in the asset, he invests.

2.2.2 Models with buyers

The following features are common to the models, which we describe in sections from 3 to 5.

In such models we set $T = 5$ and allow sellers to sell the asset to buyers $b = 1, \ldots, B$, with $\mathcal{B} = \{1, \ldots, B\}$ being the set of all the buyers, in a market, which takes place at dates 3 and 4. If a seller s does not invest in the asset, he does nothing after period 2, except consuming in period 5 the unit of wealth, which in period 2 he decides not to invest. Therefore, if he does not invest, he obtains utility equal either to $u_S|(V, NI) = u_S(1-c)$ or to $u_S|(NV, NI) = u_S(1)$, depending on his choice of date 1 action a_s^1. The market takes place, if and only if at least one seller invests in the asset.

Buyers are not wealth-constrained. For simplicity we assume that each buyer has an infinite wealth. Their initial partition is the same as sellers', i.e., $P_b^1 = P^1$, $\forall b$. Buyers cannot observe the actions taken by sellers at date 1. Since they are rational, they can use all the information, which is available to them, to try to infer such actions, if such an inference is worthwhile. Buyers are risk-neutral and consume in period 5. A buyer b has a utility function $u_b(w_b^5) = w_b^5$, $\forall b$, where w_b^5 is the wealth, which b can use for consumption at date 5.

We denote as $ce(g) \in [0, \infty]$ the certainty equivalent of a gamble g, i.e., the amount of wealth such that $u_S(ce(g)) = u_S(g)$. Notice that, for each possible gamble g, $ce(g) < Eg$, where Eg denotes the expected value of gamble g, e.g., $ce(g^H) < H$, $ce(g^L) < L$ and $ce(g^{HL}) < HL$. Notice also that, since $u_S(g^{H-c}) > u_S(g^{HL}) > u_S(1)$, then

$$ce(g^{H-c}) > ce(g^{HL}) > 1.$$

We also assume that

$$ce(g^{H-c}) + c > HL. \tag{2.2}$$

We assume that if an agent is indifferent between trading and not trading, i.e., between selling and not selling, if he is a seller, and between buying and not buying, if he is a buyer, he trades, i.e., he sells, if he is a seller, and he buys, if he is a buyer.

In such models we analyze choices of players in perfect bayesian equilibria (PBE) in pure strategies.

2.3 Absence of buyers

We consider sellers' maximization problem under the assumption that, once sellers have invested in the asset, they cannot sell it at a later date, and consume the return delivered at $T = 3$.

A seller s decides as follows. At date 2, given his choice $a_s^1 = a_1$ of date 1 action, he chooses $a_s^2 = a_s^{*2}$ such that

$$Eu_S|(a^1, (a_s^{*2}|(m_s|a^1))) \geq Eu_S|(a^1, (a_s'^2|(m_s|a^1))),$$

with $a_s^{*2} \neq a_s'^2$, $a_s^{*2}, a_s'^2 \in A^2$. Let

$$Eu_S|V = \Pr(H|V)u_S|(V, (a_s^{*2}|(H|V))) + \Pr(L|V)u_S|(V, (a_s^{*2}|(L|V)))$$

and

$$Eu_S|NV = Eu_S|(NV, a_s^{*2}|(m_s|NV)).$$

At date 1 he chooses

$$a_s^1 = \begin{cases} V & \text{if } Eu_S|V \geq Eu_S|NV \\ NV & \text{otherwise.} \end{cases}$$

After he verifies, if both $m_s|V = H$ and he invests, his expected utility will be $Eu_S|(V, (I|(H|V)) = u_S(g^H - c)$, but, if both $m_s|V = L$ and he invests, his expected utility will be $Eu_S|(V, (I|(L|V)) = u_S(g^L - c)$. Moreover, if he does not invest after verifying, his expected utility will be $Eu_S|(V, (NI|(\{H, L\}|V))) = u_S(1 - c)$. Therefore he knows that, after he chooses V, he is better-off by investing, if $m_s|V = H$, but he is better-off by not investing, if $m_s|V = L$, i.e., that

$$\begin{aligned} Eu_S|V &= \Pr(H|V)u_S|(V, (I|(H|V))) + \Pr(L|V)u_S|(V, (NI|(L|V))) = \\ &= (\pi^1 + \pi^2)u_S(g^{H-c}) + (\pi^3 + \pi^4)u_S(1 - c), \end{aligned}$$

letting

$$Eu_S(g^{H-c,1-c}) = (\pi^1 + \pi^2)u_S(g^{H-c}) + (\pi^3 + \pi^4)u_S(1-c).$$

After he does not verify, his expected utility will be $u_S(g^{HL})$, if he invests, or $u(1)$, if he does not invest. Therefore he knows that, if he chooses NV, he is better-off by investing, i.e.,

$$Eu_S|NV = Eu_S|(NV, (I|(HL|NV))) = Eu_S(g^{HL}).$$

Proposition 1: In absence of potential buyers for the asset, each seller prefers to verify the quality of the asset, before choosing if to invest or not.

Proof It follows from $Eu_S|V > Eu_S|NV$.

Proposition 2: In absence of potential buyers for the asset, each seller invests in the asset, if and only if the quality of the asset is high.

Proof It follows from $Eu_S|(V, (I|(H|V)) > Eu_S|(V, (NI|(\{H, L\}|V))) >$
$> Eu_S|(V, (I|(L|V))$.

2.4 Buyers are not wealth-constrained and an auctioneer sets the price

We consider the case, in which sellers can sell the asset, in which they invested, to buyers, in a market, in which both sellers and buyers are price takers.

At date 3, sellers submit supply schedules and the buyers submit to him demand schedules to an auctioneer. Each seller (buyer) submits only one schedule. A supply (demand) schedule specifies the quantity of asset $q(p)_s$ $(d(p)_b)$, which a seller s (buyer b) is willing to sell (buy) at any possible price $p \in [0, \infty]$. Let

$$Q(p) = \sum_{s=1}^{S} q(p)_s$$

be the total supply, or, simply, supply, at price p and

$$D(p) = \sum_{b=1}^{B} d_b(p)$$

the total demand, or, simply, demand, at p. Let $p^* \in [0, \infty]$ be any price, at which $Q(p^*) = D(p^*)$. At date 4, if there is no price, at which supply equals demand, the auctioneer declares no price, and trade does not occur. If there is only one price, at which supply equals demand, the auctioneer declares that price, and if there is more than one price, at which supply equals demand, he declares randomly one of those prices. Trade occurs at the price p^* declared by the auctioneer, unless at such a price both supply and demand are equal to 0. In such a case no trade occurs. We define p^* an equilibrium price. Notice that the auctioneer does not receive any payoff. Notice also that both sellers and buyers can condition their choice of actions on the equilibrium prices.

2.4.1 Supply

Sellers do not act strategically at date 3, when they submit supply schedules. The supply schedule of a seller s simply depends on his choice of both period 1 action and, given the value of the signal, which he observes, period 2 action. The supply schedule of those sellers, who have invested in the asset is given by

$$q_s(p)|I = q(p)|I = \begin{cases} 0 & \text{if } p < Eu_S(g^{Ev|(m_s|a_s^1)}) \\ 1 & \text{otherwise,} \end{cases}$$

$\forall s$ such that $a_s^2 = I$, and the supply schedule of those, who have not invested in the asset, is given by

$$q_s(p)|NI = q_s(p)|NI = 0,$$

$\forall p$ and $\forall s$ such that $a_s^2 = NI$.

Notice that a seller can condition his choice of strategy $((a_s^1, (a_s^2|(m_s|a_s^1)))$ on the equilibrium price p^*, at which trade might occur. To see this, consider that at date 2 a seller, given his choice of date 1 action $a_s^1 = a_1$, chooses $a_s^2 = a_s^{*2}$ such that

$$Eu_S|(a^1, ((a_s^{*2}, q_s(p^*))|(m_s|a^1)) \geq Eu_S|(a^1, ((a_s'^{*2}, q_s(p^*))|(m_s|a^1)),$$

with $a_s^{*2} \neq a_s'^2$, $a_s^{*2}, a_s'^2 \in A^2$, and let

$$\begin{aligned} Eu_S|(V|p^*) = \Pr(H|V)u_S|(V, ((a_s^{*2}, q_s(p^*))|(H|V))) + \\ + \Pr(L|V)u_S|(V, ((a_s^{*2}, q_s(p^*))|(L|V))) \end{aligned}$$

and

$$Eu_S|(NV|p^*) = Eu_S|(NV, ((a_s^{*2}, q_s(p^*))|(m_s|NV))).$$

Now consider that

$$Eu_S|(V|p^*) = $$
$$= \begin{cases} Eu_S|(V, \{I|H, NI|L\}, (0|(H,L))) = & \\ = Eu_S(g^{H-c,1-c}) & \text{if } p^* - c < 1 - c \\ Eu_S|(V, (I|(H,L)), \{0|H, 1|L\}) = & \\ = Eu_S(g^{H-c,1-c}) = Eu_S(g^{H-c,p^*-c}) & \text{if } p^* - c = 1 - c \\ Eu_S|(V, I|(H,L)), \{0|H, 1|L\}) = & \\ = Eu_S(g^{H-c,p^*-c}) & \text{if } 1 - c < p^* - c < ce(g^{H-c}) \\ Eu_S|(V, (I|(H,L)), 1|(H,L)) = & \\ = u_S(p^* - c) = Eu_S(g^{H-c,p^*-c}) & \text{if } p^* - c = ce(g^{H-c}) \\ Eu_S|(V, (I|(H,L)), 1|(H,L)) = & \\ = u_S(p^* - c) & \text{if } ce(g^{H-c}) < p^* - c, \end{cases}$$

with $Eu_S(g^{H-c,p^*-c}) = (\pi^1+\pi^2)Eu_S(g^{H-c}) + (\pi^3+\pi^4)u_S(p^*-c)$, and consider also that

$$Eu_S|(NV|p^*) = $$
$$= \begin{cases} Eu_S|(NV, (I,0)|HL) = Eu_S(g^{HL}) & \text{if } p^* < ce(g^{HL}) \\ Eu_S|(NV, (I,1)|HL) = u_S(p^*) = Eu_S(g^{HL}) & \text{if } p^* = ce(g^{HL}) \\ Eu_S|(NV, (I,1)|HL) = u_S(p^*) & \text{if } p^* > ce(g^{HL}). \end{cases}$$

Clearly a seller s chooses

$$a_s^1 = \begin{cases} V & \text{if } Eu_S|(V|p^*) \geq Eu_S|(NV|p^*) \\ NV & \text{otherwise.} \end{cases}$$

In order to analyze sellers' choice in equilibrium, let $g^{H-c,p-c}$ be the gamble between the following two outcomes: the gamble g^{H-c}, with probability $\pi^1+\pi^2$, and an amount of wealth equal to $p-c$, with probability $\pi^3+\pi^4$. Consider also that there exists a unique price $\widehat{p}$, such that

$$Eu_S(g^{H-c,p-c}) \begin{cases} > u_S(p) & \text{if } p < \widehat{p} \\ = u_S(p) & \text{if } p = \widehat{p} \\ < u_S(p) & \text{if } \widehat{p} < p, \end{cases}$$

and assume that

$$ce(g^{HL}) < \widehat{p} < HL.$$

Compare now $Eu_S|(V|p^*)$ and $Eu_S|(NV|p^*)$ at any possible value of p^*, as follows:

$p^* < 1$ $\qquad Eu_S|(V|p^*) = Eu_S|(V, \{I|H, NI|L\}, 0|(H,L)) =$

$$
\begin{array}{ll}
 & = Eu_S(g^{H-c,1-c}) > \\
 & > Eu_S|(NV|p^*) = Eu_S|(NV,I,0) = Eu_S(g^{HL}) \\
p^* = 1 & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),\{0|H,1|L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) = Eu_S(g^{H-c,1-c}) > \\
 & > Eu_S|(NV|p^*) = Eu_S|(NV,I,0) = Eu_S(g^{HL}) \\
1 < p^* < ce(g^{HL}) & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),\{0|H,1|L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) > \\
 & > Eu_S|(NV|p^*) = Eu_S|(NV,I,0) = Eu_S(g^{HL}) \\
p^* = ce(g^{HL}) & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),\{0|H,1|L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) > \\
 & > Eu_S|(NV|p^*) = Eu_S|(NV,I,1) = Eu_S(g^{HL}) = \\
 & = u_S(p^*) \\
ce(g^{HL}) < p^* < \widehat{p} & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),\{0|H,1|L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) > \\
 & > Eu_S|(NV|p^*) = Eu_S|(NV,(I,1)|HL) = u_S(p^*) \\
p^* = \widehat{p} & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),\{0|H,1|L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) = \\
 & = Eu_S|(NV|p^*) = Eu_S|(NV,(I,1)|HL) = u_S(p^*) \\
\widehat{p} < p^* < ce(g^{H-c}) + c & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),\{0|H,1|L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) < \\
 & < Eu_S|(NV|p^*) = Eu_S|(NV,(I,1)|HL) = u_S(p^*) \\
p^* = ce(g^{H-c}) + c & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),1|\{H,L\}) = \\
 & = Eu_S(g^{H-c,p^*-c}) = u_S(p^* - c) < \\
 & < Eu_S|(NV|p^*) = Eu_S|(NV,(I,1)|HL) = u_S(p^*) \\
ce(g^{H-c}) + c < p^* & Eu_S|(V|p^*) = Eu_S|(V,(I|\{H,L\}),1|\{H,L\}) = \\
 & = u_S(p^* - c) < \\
 & < Eu_S|(NV|p^*) = Eu_S|(NV,(I,1)|HL) = u_S(p^*)
\end{array}
$$

From the comparison between $Eu_S|(V|p^*)$ and $Eu_S|(NV|p^*)$ at any value of p^*, it is easy to derive the supply of asset for a seller s at any value of p^*, i.e.,

$$
q_s(p^*) = q(p^*) = \begin{cases} 0 & \text{if } p^* < 1 \\ \{0|H,1|L\} & \text{if } 1 \leq p^* \leq \widehat{p} \\ 1 & \text{if } \widehat{p} < p^*, \end{cases}
$$

$\forall s$, with $Q(p^*) = Sq(p^*)$ being the supply in equilibrium.

2.4.2 Demand

Buyers do not act strategically at date 3, when they submit demand schedules. The supply schedule of a seller b simply depends on his belief $E_b v$ about the cash-flow from the asset supplied by sellers. Notice that, since all buyers share the same initial partition of date T state space, i.e., $P_b^1 = P^1$, $\forall b$, have access to the same information, and are rational, i.e., use all such information to construct and update (by applying Bayes rule) their beliefs, then they also share the same belief about the cash-flow from the asset supplied by sellers, i.e., $E_b v = E_B v$, $\forall b$. Therefore the demand schedule of a buyer b is given by

$$d_b(p) = d(p) = \begin{cases} 0 & \text{if } p > E_B v \\ 1 & \text{otherwise,} \end{cases}$$

$\forall b$, with $D(p) = Bd(p)$ being the demand at price p.

By comparing $Eu_S|(V|p^*)$ and $Eu_S|(NV|p^*)$, buyers can infer sellers' information about the asset supplied in equilibrium at any possible value of p^*. Let $E_B v(p^*)$ be buyers' belief about the asset supplied in equilibrium. The actual supply of asset in equilibrium depends on the signal m_s, which buyers cannot observe. However they can infer the following expected supply of asset at any possible value of p^*:

$$Eq_s(p^*) = Eq(p^*) = \begin{cases} 0 & \text{if } p^* < 1 \\ \pi^3 + \pi^4 & \text{if } 1 \leq p^* \leq \widehat{p} \\ 1 & \text{if } \widehat{p} < p^*, \end{cases}$$

$\forall s$, with $EQ(p^*) = SEq(p^*)$ being the expected supply in equilibrium. Clearly $E_B v(p^*)$ is not well-defined for those values of p^*, at which sellers do not supply asset, i.e., at which $EQ(p^*) = 0$. For simplicity, we assume buyers' belief at such values of p^* to be equal to buyers' belief given the initial partition, i.e., $E_B v(p^*) = HL$, with p^* such that $EQ(p^*) = 0$. Therefore we can write

$$E_B v(p^*) = \begin{cases} HL & \text{if } p^* < 1 \\ L & \text{if } 1 \leq p^* \leq \widehat{p} \\ HL & \text{if } \widehat{p} < p^*, \end{cases}$$

and we can denote the demand for asset by a buyer in equilibrium as

$$d_b(p^*) = d(p^*) = \begin{cases} \infty & \text{if } p^* < 1 \\ 0 & \text{if } 1 \leq p^* \leq \widehat{p} \\ [0, \infty] & \text{if } \widehat{p} < p^* \leq HL \\ 0 & \text{if } p^* > HL, \end{cases}$$

$\forall b$, with $D(p^*) = Bd(p^*)$ being the demand in equilibrium.

2.4.3 Equilibrium

Proposition 3: If sellers can trade with buyers in a market, in which both sellers and buyers are price takers, and if buyers are not wealth-constrained, then there exists an infinite number of pure-strategy PBE. In all of them no seller verifies the quality of the asset, each seller invests in the asset, and a quantity of asset equal to S is traded in the market.

Proof Notice that $Q(p^*) = D(p^*)$, if and only if $\widehat{p} < p^* \leq HL$. Notice also that, if $\widehat{p} < p^* \leq HL$,

1. $Eu_S|(V|p^*) = Eu_S|(V, (I|\{H, L\}), \{0(p^*)|H, 1(p^*)|L\}) =$
$= Eu_S(g^{H-c,p^*-c}) < Eu_S|(NV|p^*) = Eu_S|(NV, (I, 1(p^*))|HL) =$
$= u_S(p^*)$, i.e., a seller chooses not to verify the quality of the asset and to invest in the asset, and this is true for each seller, and

2. $Q(p^*) = D(p^*) = S$, i.e., a quantity of asset equal to S is traded in the market.

2.5 Buyers are not wealth-constrained and set the prices

We consider the case, in which sellers can sell the asset, in which they invested, to buyers, in a market, in which buyers set the prices.

At date 3 each buyer b offer a prices $p_b \in [0, \infty]$ for the asset. At date 4 sellers can either accept or refuse buyers' offers. Sellers can observe all the prices posted by buyers and, among such prices, each seller s chooses the one, at which he wants to sell his unit of asset. If he does not want to sell his unit of asset, he does not choose any price. The units of asset belonging to all the sellers, who want to trade at a given price, are randomly assigned to the buyers, who have posted that price. Clearly each buyer commits himself to buy any number of unit of asset, which are assigned to him, at the price, which he posts.

Let $d_b(p)$ be the quantity of asset, which a buyer b is willing, i.e., commit himself, to buy at price $p \in [0, \infty]$. We define $d_b(p)$ as the demand of a buyer b at price p. Clearly $d_b(p) = \infty$, if $p_b = p$, and $d_b(p) = 0$, otherwise. Let also $D(p)$ be the total demand at p, calculated as the sum of the demands at p of all the buyers, who post a price equal to p. Let $q_s(p)$ be the quantity of asset, which a seller s is willing to sell at price p, given that p is one of the prices posted by buyers, i.e., given that $p = p_b$ for at least one buyer $b \in \mathcal{B}$. We define $q_s(p)$ as the supply of a seller s at price p. Clearly $q_s(p) = 1$, if s chooses p, and $q_s(p) = 0$, otherwise. Let also $Q(p)$ be the total supply at p, calculated as the sum of the supplies of all the sellers, who choose p.

Let $\bar{p}$ be the highest among the prices posted by the buyers. Clearly seller s chooses to sell at $\bar{p}$, if $\bar{p} \geq Eu_S|(a_s^1, (a_s^2|m_s))$, he will refuse to sell at any of the prices posted, otherwise, and this is true for all the sellers, who invest in the asset. Since each buyer has an infinite wealth, $D(\bar{p})$ will also be infinite and higher that $Q(\bar{p})$, since sellers are collectively endowed with a finite quantity of asset. Therefore, in any candidate equilibrium, buyers posting a price lower that $\bar{p}$ do not buy any unit of asset, while those buyers posting a price equal to $\bar{p}$ are randomly rationed, i.e., assuming $b' = 1, \dots, B' \leq B$ buyers, with $b' \in \mathcal{B}$, offer $p_{b'} = \bar{p}$, each of them expects to buy $Q(\bar{p})/B'$ units of asset.

2.5.1 Equilibrium

We consider the following assessments in order to determine if any of them is a PBE in pure strategies. By assessment we mean a set, whose elements are a profile of strategies and a system of beliefs.

1 Assessments, in which (all) buyers believe that any seller s invests, if he verifies, whatever quality he discovers, and does not invest, otherwise.

In each of such assessments buyers believe that $\Pr((a_s^1 = V)|(a_s^2 = I)) = 1$, $\Pr((m_s|V = H)|(a_s^2 = I)) = \pi^1 + \pi^2$, and $\Pr((m_s|V = L)|(a_s^2 = I)) = \pi^3 + \pi^4$. Consider the choices of buyers, assuming that at least one seller invests.

Let $\bar{p} < ce(g^L)$. Then any buyer b' buys and makes 0. Any of them, by deviating to, e.g., $p = ce(g^L)$, expects to buy $Q(\bar{p})$ with probability $\pi^3 + \pi^4$ and to make a profit of $(L - ce(g^L)Q(\bar{p})/(\pi^3 + \pi^4)$. Therefore he prefers deviating to p. This is true $\forall b'$.

Let $ce(g^L) \leq \bar{p} \leq L$. Consider first the case, in which $B' > 1$. Any buyer b' expects to buy $Q(\bar{p})$ with probability $(\pi^3 + \pi^4)B'$ and to make a profit of $(L - \bar{p})Q(\bar{p})/(\pi^3 + \pi^4)B'$. By deviating to $p = \bar{p} + \varepsilon$, with ε positive and close to 0, he expects to buy Q with probability $\pi^3 + \pi^4$ and to make a profit of $(L - (\bar{p} + \varepsilon))Q(\bar{p})/(\pi^3 + \pi^4)$. Therefore he prefers deviating to p. This is true $\forall b'$. Consider now the case, in which $B' = 1$. Any of the buyers posting $p_b < \bar{p}$ buys and makes 0. He prefers deviating to $p = \bar{p} + \varepsilon$ to make an expected profit of $(L - (\bar{p} + \varepsilon))Q(\bar{p})/(\pi^3 + \pi^4)$. This is true for all buyers posting $p_b < \bar{p}$.

Let $\bar{p} = L$. Consider first the case in which $B' > 1$. Any buyer b' expects to make 0. By deviating to $p > L$, he expects a loss, if he buys any positive quantity of asset. By deviating to any price $p < L$, he buys and makes 0. This is true $\forall b'$. Any of the buyers posting $p_b < \bar{p}$ buys and makes 0. By deviating to $p > L$, he expects a loss, if he buys any positive quantity of asset. By deviating to any price $p \leq L$, he buys and makes 0. This is true for all buyers posting $p_b < \bar{p}$. Consider now the case in which $B' = 1$. Buyer b' expects to make 0. By deviating to $p = L - \varepsilon$, with ε positive and close to 0, he expects to buy $Q(\bar{p})$ with probability $\pi^3 + \pi^4$ and to make a profit of $\varepsilon Q(\bar{p})/(\pi^3 + \pi^4)$. Therefore he prefers deviating to p.

Let $\bar{p} > L$. Then any buyer b' buys a positive quantity of asset with a positive probability and expects to make a loss, if he buys. Therefore, if $B' > 1$, he is incentived to deviate to any price lower than $\bar{p}$, so that he buys a quantity of asset equal to 0 and makes 0, and, if $B' = 1$, he is incentived to deviate to any price lower than the second-highest price among those posted by buyers, so that he buys a quantity of asset equal to 0 and makes 0. This is true $\forall b'$.

Therefore the only strategy profiles, which could be part of an equilibrium, are those, in which at least two buyers post $p_b = \bar{p} = L$. Consider now such strategy profiles. Given that at least two buyers post $p_b = \bar{p} = L$, seller s prefers $a_s^1 = V$, and, after verifying, he prefers investing, if $m_S|V = H$, in order to obtain a payoff equal to $Eu_S(g^{H-c})$, and not investing, if $m_S|V = L$, in order to obtain a payoff equal to $u_S(1-c)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that any seller s invests, if verifies, and does not invest, if he does not verifies, cannot be an equilibrium.

2 *Assessments, in which (all) buyers believe that any seller s invests, if he verifies and—after he verifies—discovers that the asset has a low quality, and does not invest, otherwise.*

In each of such assessments buyers believe that $\Pr((a_s^1 = V)|(a_s^2 = I)) = 1$, $\Pr((m_s|V = L)|(a_s^2 = I)) = 1$, and $\Pr((m_s|V = H)|(a_s^2 = I)) = 0$. Consider the choices of buyers, assuming that at least one seller invests.

The reasoning is very similar to the one developed in paragraph 1. It can be proved that the only strategy profiles, which could be part of an equilibrium, are those, in which at least any two buyers post $p_b = \bar{p} = L$, and, if one considers such strategy profiles, it can be easily seen that, given that at least two buyers post $p_b = \bar{p} = L$, seller s chooses $a_1^s = V$, and, after verifying, he prefers investing, if $m_S|V = H$, in order to obtain a payoff equal to $Eu_S(g^{H-c})$, and not investing, if $m_S|V = L$, in order to obtain a payoff equal to $u_S(1-c)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that any seller s invests, if he verifies and—after he verifies—discovers that the asset has a low quality, and does not invest, otherwise, cannot be an equilibrium.

3 *Assessments, in which (all) buyers believe that any seller s invests, if he verifies and—after he verifies—discovers that the asset has a high quality, and does not invest, otherwise.*

In such an assessment buyers believe that $\Pr((a_s^1 = V)|(a_s^2 = I)) = 1$, $\Pr((m_s|V = H)|(a_s^2 = I)) = 1$, and $\Pr((m_s|V = L)|(a_s^2 = I)) = 0$. Consider the choices of buyers, assuming that at least one seller invests.

The reasoning is very similar to the one developed in paragraph 1. It can be proved that the only strategy profiles, which could be part of an equilibrium, are those, in which at least any two buyers post $p_b = \bar{p} = H$, and, if one considers such strategy profiles, it can be easily seen that, given that at least any two buyers post $p_b = \bar{p} = H$, seller s chooses $a_s^1 = NV$, and, after not verifying, he prefers investing, in order to obtain a payoff equal to $u_S(H)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that any seller s invests, if he verifies and—after he verifies—discovers that the asset has a high quality, and does not invest, otherwise, cannot be part of an equilibrium.

4 *Assessments, in which (all) buyers believe that any seller s*

- *invests, in the two following cases: if he verifies and—after he verifies—discovers that the asset has a high quality, and if he does not verify, and*
- *does not invest, otherwise.*

In any of such assessments buyers believe that $\Pr((a_s^1 = V)|(a_s^2 = I)) = \alpha$, $\Pr((a_s^1 = NV)|(a_s^2 = I)) = 1 - \alpha$, $\Pr((m_s = H)|(a_s^2 = I)) = \alpha$, and $\Pr((m_s = HL)|(a_s^2 = I)) = 1-\alpha$, with $\alpha \in (0, 1)$. Consider the choices of buyers, assuming that at least one seller invests.

The reasoning is very similar to the one developed in paragraph 1. It can be proved that the only strategy profiles, which could be part of an equilibrium, are those, in which at least any two buyers post $p_b = \bar{p} = HL$, and, if one considers such strategy profiles, it can be easily seen that, since at least any two buyers post $p_b = \bar{p} = HL$, seller s chooses $a_s^1 = NV$, and, after not verifying, he prefers investing, in order to obtain a payoff equal to $u_S(HL)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that any seller s

- invests, in the two following cases: if he verifies and—after he verifies—discovers that the asset has a high quality, and if he does not verify, and
- does not invest, otherwise,

cannot be an equilibrium.

5 *Assessments, in which (all) buyers believe that any seller s*

- *invests, in either one of the two following cases: if he verifies and—after he verifies—discovers that the asset has a low quality, and if he does not verify, and*

- *does not invest, otherwise.*

In any of such assessments buyers believe that $\Pr((a_s^1 = V)|(a_s^2 = I)) = \alpha$, $\Pr((a_s^1 = NV)|(a_s^2 = I)) = 1 - \alpha$, $\Pr((m_s = L)|(a_s^2 = I)) = \alpha$, and $\Pr((m_s = HL)|(a_s^2 = I)) = 1 - \alpha$, with $\alpha \in (0, 1)$. Consider the choices of buyers, assuming that at least one seller invests.

The reasoning is very similar to the one developed in paragraph 1. It can be proved that the only strategy profiles, which could be part of an equilibrium, are those, in which at least any two buyers post $p_b = \bar{p} = L$, and, if one considers such strategy profiles, it can be easily seen that, since at least any two buyers post $p_b = \bar{p} = L$, seller s chooses $a_s^1 = V$, and, after verifying, he prefers investing, if $m_S|V = H$, in order to obtain a payoff equal to $Eu_S(g^{H-c})$, and not investing, if $m_S|V = L$, in order to obtain a payoff equal to $u_S(1 - c)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that any seller s

- invests, in either one of the two following cases: if he verifies and—after he verifies—discovers that the asset has a low quality, and if he does not verify, and

- does not invest, otherwise,

cannot be an equilibrium.

6 Assessments, in which (all) buyers believe that any seller s invests, either if he verifies—whatever quality he discovers—or if he does not verify.

In any of such assessments buyers believe that $\Pr((a_s^1 = V)|(a_s^2 = I)) = \alpha$, $\Pr((a_s^1 = NV)|(a_s^2 = I)) = 1 - \alpha$, $\Pr((m_s = H)|(a_s^2 = I)) = (\pi^1 + \pi^2)\alpha$ $\Pr((m_s = L)|(a_s^2 = I)) = (\pi^1 + \pi^2)\alpha$, and $\Pr((m_s = HL)|(a_s^2 = I)) = 1 - \alpha$, with $\alpha \in (0, 1)$. Consider the choices of buyers, assuming that at least one seller invests.

The reasoning is very similar to the one developed in paragraph 1. It can be proved that the only strategy profiles, which could be part of an equilibrium, are those, in which at least any two buyers post $p_b = \bar{p} = L$, and, if one considers such strategy profiles, it can be easily seen that, given that at least two buyers post $p_b = \bar{p} = L$, seller s chooses $a_s^1 = V$, and, after verifying, he prefers investing, if $m_S|V = H$, in order to obtain a payoff equal to $Eu_S(g^{H-c})$, and not investing, if $m_S|V = L$, in order to obtain a payoff equal to $u_S(1-c)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that any seller s invests, either if he verifies—whatever quality he discovers—or if he does not verify, cannot be an equilibrium.

7 *Assessments, in which (all) buyers believe that each seller s invests, if he does not verifies, and does not invest, otherwise.*

In each of such assessments buyers believe that $\Pr((a_s^1 = NV)|(a_s^2 = I)) = 1$, $\Pr((m_s = H)|(a_s^2 = I)) = 0$, $\Pr((m_s = L)|(a_s^2 = I)) = 0$, and $\Pr((m_s = HL)|(a_s^2 = I)) = 1$. Consider the choices of buyers, assuming that at least one seller invests.

The reasoning is very similar to the one developed in paragraph 1. It can be proved that the only strategy profiles, which could be part of an equilibrium, are those, in which at least any two buyers post $p_b = \bar{p} = HL$. if one considers such strategy profiles, it can be easily seen that, given that at least any two buyers post $p_b = \bar{p} = HL$, seller s chooses $a_s^1 = NV$, and, after not verifying, he prefers investing, in order to obtain a payoff equal to $u_S(HL)$. This is true $\forall s$.

Therefore any assessment, in which (all) buyers believe that each seller s invests, if he does not verifies, and does not invest, otherwise, is an equilibrium.

Proposition 4: If sellers can trade with buyers in a market, in which buyers set the prices, and if buyers are not wealth constrained, then there exists an infinite number of pure-strategy PBE. In all of them no seller verifies the quality of the asset, each seller invests in the asset, and a quantity of asset equal to S is traded in the market.

Proof In sections from 5.1.1 to 5.1.7 we have proved that no assessment, in which (all) buyers have beliefs different from $\Pr((a_s^1 = NV)|(a_s^2 = I)) = 1$, $\Pr((m_s = H)|(a_s^2 = I)) = 0$, $\Pr((m_s = L)|(a_s^2 = I)) = 0$, and $\Pr((m_s = HL)|(a_s^2 = I)) = 1$ about sellers is an equilibrium. We have also proved that any assessment, in which (all) buyers believe that each seller s invests, if he does not verifies, and does not invest, otherwise, is an equilibrium.

Therefore only those assessments, in which (all) buyers believe that each seller s invests, if he does not verifies, and does not invest, otherwise, are (pure-strategy) PBE. In any of such assessments at least any two buyers post a price equal to $\bar{p} = HL$. Given this, each seller s prefers $a_s^1 = NV$, and, after not verifying, he prefers investing, in order to obtain a payoff equal to $u_S(HL)$, by accepting to sell his unit of asset at $\bar{p} = HL$. Therefore S units of asset are traded in the market.

2.6 Conclusion

We have analyzed models to show that the presence of agents, who are both uninformed (i.e., they don't have direct access to information) about an asset and wealthy (i.e., they are not liquidity-constrained), can disincentive the acquisition, or research, of information in financial markets, given that such acquisition is costly.

It could be interesting to extend the analysis by considering the decisions of those agents, who have direct access to information about the asset, at different levels of wealth, with a utility function, which is not CARA, to examine how their own wealth affects their choice. Another interesting extension of the analysis could consist in letting those agents face a continuous choice, i.e., in letting them choose to invest in the asset any quantity of wealth between 0 and their initial endowment. Moreover one could analyze in more detail how the cost of information acquisition relative to the size of wealth of those agents, who can acquire information, affects their decision.

3. INFORMATION, BELIEFS AND BUBBLES IN FINANCIAL MARKETS

3.1 Introduction

In the economic literature, the word bubble refers to the price of an asset being higher than the fundamental value of the asset. In the theoretical literature about bubbles, the notion of fundamental value differs across models according to assumptions about agents' preferences and beliefs. Nevertheless according to a general notion, which can apply to all models about bubbles, the fundamental value of an asset for an agent can be defined as the price which that agent is willing to pay for that asset, in order to hold it forever, i.e., until maturity, after buying it. Clearly, an agent is willing to pay more than the fundamental, if and only if he expects to resell the asset in the future at a higher price higher than the fundamental.

In the present survey, we discuss theoretical literature about bubbles by presenting models, in which agents are assumed not have the same information or not behave on the base of the same information or beliefs. Existing contributions are distinguished into three classes: the first one includes models with rational-expectations equilibria (REE) under asymmetric information, the second one comprises models, in which agents have different first-order prior beliefs, and the third one consists of models, in which some agents are denoted as "irrational" or "noise traders".

fundamental value of an asset (or, simply, fundamental) at a given point in time can be defined as the highest among the prices, which agents are willing to pay in order to buy the asset at that point in time and hold it forever. In the theoretical literature on bubbles, the fundamental equals different values according to different assumptions about agents' preferences and beliefs. [1]

[1] See Allen et al. (1993), Allen and Gorton (1993), Brunnermeier (2008 and 2001).

3.2 Bubbles in REE under asymmetric information

Bubbles can emerge in settings, in which at least some agents are uncertain about some features of the economy, such as asset returns, and different agents have different information, but share a common distribution of prior beliefs, about such features. This implies that all market participants have rational expectations, i.e., they have beliefs, which are on average correct, and update them by using all the available information. In such models the fundamental of an asset for a trader is defined as the price, which that trader would pay, if he were obliged to hold the asset forever, after buying it. Clearly, if all traders are risk-neutral and have the same information about an uncertain asset return, the fundamental is the same for all traders and is equal to the expected net present value of the asset return.

Tirole (1982) shows that, in a market for a risky asset with risk-neutral traders and $t = 1, \ldots, T$ periods,

- if it is common knowledge that the initial allocation is (ex-ante) efficient and
- if agents are not myopic, i.e, if, in each period t, they maximize expected discounted gains from t to T and do not base their behavior only on the comparison, in each period t, between current asset prices and the probability distribution of prices in $t + 1$,

bubbles are impossible in t in a REE. The reasoning in the proof is that, since the initial allocation is (ex-ante) efficient and all agents are rational, all agents know that there is no possibility to gain by buying an asset, whose price is higher than the fundamental, therefore no agent would be willing to buy an asset at such a price. Tirole suggests that bubbles are possible if one introduces at least one "non-rational" agent, i.e., a trader, whose demand or supply does not depend on market prices, or if one let agents have different (first-order) prior beliefs, which are not updated.

Allen, Morris and Postlewaite (1993, AMP) discuss necessary conditions either for "expected" or for "strong" bubbles are discussed. In their setting time is finite (and discrete), agents are either risk-averse or risk-neutral, and trade a risky asset for a divisible riskless asset (money). An expected bubble in period t is defined as the price of the (risky) asset being higher than every agents' valuation in t (i.e., expected utility) of the expected return of the asset, whereas by strong bubble it is meant the price being higher than any possible pay-off from the asset. The authors prove three necessary conditions for a bubble to emerge in period t a dynamic REE, in addition to the condition that the initial allocation has to be inefficient. The three conditions are the following ones

1. each agent is short-sale constrained with positive probability at some future time after t and the constraint is binding;

2. each agent has private information at t,

3. agents' trades are not common knowledge.

Condition (1) (and the condition that the initial allocation has to be inefficient) are necessary both for an expected and for a strong bubble (since any strong bubble is also an expected bubble). Condition (1) can be explained by the fact that, if an agent assigns positive probability to be prevented from selling the asset short in the future and also to all other agents being short-sale constrained in the future, he can decide to hold an asset, whose price is higher than the fundamental, since the constraint should prevent the price from falling. Such a condition also implies that there cannot be a bubble neither in period T nor in period $T-1$. Conditions (2) and (3) are necessary for a strong bubble. They make possible that, in equilibrium, each trader knows that, in t, the price of the asset is above any possible realization of the dividend, but does not know that the other traders also know this fact. If condition (2) did not hold, prices were fully revealing. Then it would be common knowledge that the price of the asset is higher that any possible relization of the dividend, therefore no agent would be willing to buy the asset at such a price. Moreover condition (3) prevents that, as proved by Geanakoplos (1994), "common knowledge of actions", i.e., of (net) trades in the case of AMP, "negates asymmetric information about events", i.e., about the fact the price is higher than any possible realization of the dividend. Therefore the third condition implies that there must be at least three traders for a bubble to emerge.

The authors show four possible ways to generate gains from trade for rational expectations, who update their beliefs, by making the initial allocation inefficient:

1. heterogeneous priors,

2. state-dependent utility functions,

3. random endowments and identical concave utility functions,

4. identical non-random endowments and different concave utility functions.

Bubbles can emerge in markets, in which there are principal-agent relationships among investors, i.e, in which some traders ("principals") cannot directly invest their money in assets and entrust other traders ("agents") with such a task. "Agents" expect to gain at the expense of the "principals", since the contracts between the latter and the former ones condition neither on the type nor on the type of investments and limits "agents"' liability, i.e., losses, in case the investments have a low return. The contract gives to "agents" an incentive to "overinvest" either in riskier assets or in assets with lower (expected) returns, therefore driving the prices of such assets above the fundamentals.

Allen and Gorton (1993) discuss a model with risk-neutral traders. In their setting portfolio managers trade on behalf of investors. Good managers are able to choose stock of undervalued firms, i.e., assets with positive (discounted) expected value. Bad managers are not. Investors cannot distinguish between good and bad portfolio managers. Therefore the optimal contract allocate to managers a share of the profits, which they make, and nothing, if they make losses, either if they are good or bad managers. In equilibrium bad managers buy assets with negative expected value at a bubble price. Allen and Gale (2000) traders are risk-neutral. Banks borrow money from investors to invest either in a riskless or in a risky asset. Investors lend their money with a "simple debt contract", which allows banks to avoid losses by defaulting on the loan, if the risky asset delivers a low cash-flow. Therefore banks invest in the risky asset more than they would, if they invested their own money, therefore bidding up the price of the asset.

3.3 Bubbles when agents' priors are heterogeneous

A strand of literature comprises models, in which bubbles emerge, under the assumption that agents have heterogeneous prior beliefs about expected dividends from an asset. We refer mainly to the models by Harrison and Kreps (1978) and Scheinkman and Xiong (2003). In such settings there exist two groups of agents and each group has different priors, with agents within each group sharing the same beliefs. Beliefs are (implicitly) assumed to be common knowledge among all agents, therefore agents in one group do not try to infer information of agents in the other group from prices.

Since agents are risk neutral, the fundamental value of the asset is defined as the expected net present value of the future stream of dividends, and, since the fundamental differs for each group of agents, a bubble is defined as the price of the asset being higher than the highest among the two fundamental.

Bubbles emerge in equilibrium, since today's buyers, who are the most optimistic agents, can resell the asset at a future date to today's sellers, who are the least optimistic agents, at a price higher than their valuation, i.e., higher than the valuation of today's buyers and, therefore, also of today's sellers. This is possible, since agents' first-order beliefs evolve over time according to stochastic processes, in such a way that today's sellers expects to become more optimistic than today's buyers.

Necessary conditions for bubbles are binding short-sale constraints and limited supply of the asset.

Harrison and Kreps (1978) do not provide an explicit explanation for differences in agents' beliefs, whereas in Scheinkman and Xiong (2003) disagreements among agents about the fundamental are generated by a "behavioral bias". In their models agents are "overconfindent", i.e., each group of agent receives a different signal about asset dividends and agents in each group overestimate the precision of the signal, which they receive.

3.4 Bubbles when some agents are "irrational"

Some models, such as DeLong et al. (1990) investigate how asset prices can depart from fundamentals, i.e., how they can be either or lower than fundamentals, assuming that the expectations of some agents about asset returns are not rational, i.e., on average not correct. Such agents are therefore defined as "irrational" or "noise" traders. Since "rational" agents are risk-averse and have a "short" horizon (i.e., they live for a limited number of periods), "noise trader risk" limits their willingness to arbitrage assets in such a way that asset prices are not fully driven down (or up) to fundamentals.

Noise trader risk is defined as the risk that noise traders' beliefs, which evolves stochastically over time, will be in the future even further from the fundamental than they are today. In equilibrium rational traders buy the asset, if noise traders are "pessimistic", i.e., they underestimante the fundamental. Since noise traders' beliefs are stochastic, rational traders know that in the future noise traders can become more even more pessimistic. When rational traders, given their short horizon, liquidate their position, they might resell the asset at a price lower than the price, at which they bought the asset. Therefore rational traders' risk aversion reduces their willingness to bet against noise traders and limits the amount of asset they are willing to buy. Moreover fear that optimistic noise traders can become even more optimistic in the future, therefore increasing the price of the asset, limits the amount of rational traders are willing to sell short to optimistic noise traders.

Clearly, noise trader risk is priced since noise traders' beliefs are correlated with each other (in Delong et al. (1990) all noise traders share the same beliefs), otherwise the "optimism" of some noise traders would cancel out the "pessimism" of the others.

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