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Essays on Trade and Cooperation

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Abstract

This dissertation is devoted to non-cooperative game theory. It consists of two surveys, one on strategic market games, another on non-cooperative coalition formation and two research papers. One research paper studies mixed strategies for strategic market games with wash sales, another offers an self-financed enforcement mechanism for non-cooperative coalition formation.

In Chapter 1 presents a survey on strategic market games. Chapter 2 presents the model of mixed strategies in strategic market game with wash sales.

In Chapter 3 presented a survey on coalition formation, including cooperative and non-cooperative approaches.

In Chapter 4 present a model of non-cooperative coalition formation with self-financed enforcement mechanism.

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To my parents ...

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Chapter 1

Strategic Market games: a survey

1.1 Introduction

The strategic market game (SMG) is the general equilibrium mechanism of strategic reallocation of resources. It was suggested by Shapley and Shubik and it lies in the fundamentals of contemporary monetary macroeconomics with default. The survey highlights features of SMG and some the most important current applications of SMG, especially for monetary macroeconomic analysis.

1.2 The mechanism of strategic market games

The strategic market game (SMG) is the general equilibrium mechanism of strategic reallocation of resources. It was suggested by Shapley and Shubik ([333]) and it lies in the fundamentals of monetary macro economics with default.

The applied side of this mechanism is that it allows investigations of strategic reallocation of resources, which is especially important for monetary policy and contemporary financial turbulence (Shubik [348], also in Goodhart, Sunirand and Tsomocos, [168]), Tsomocos ([383]).

To be more specific, the approach offered in the seminal paper of Shubik and Wilson ([359]) is the application of SMG, which transforms the traditional macroeconomic problem of fiat money holding for a finite time, into a standard microeconomic problem. Thus, one can think about SMG as reallocation mechanism for the general equilibrium theory with fiat money (Shubik, [348, p.10]).

The survey has the following structure. First,we present basic details for this trading mechanism(s) of SMG, some special properties of SMG, and some traditional applications. Then we explain macroeconomic applications and the results of some experiments.

There is extensive literature on SMG. Many papers study the same question from different sides. The only survey on strategic market games was published by Giraud ([157]) as an introduction to the specialized issue of the Journal of Mathematical Economics on SMG.

1.2.1 Types of goods, which can serve as a means of exchange

Multiplicity of types of goods is important for organization of markets and for dynamical trade. Shubik and Quint ([304]) distinguished between the following types of goods, which can serve as a means of exchange.

1. Perishable, which have a single period of life, when they are traded and consumed.

2. Storable consumable, for example, cans of beans, spices or salt. They can be consumed, "at the option of an individual" or be carried over in time.

3. Durable good which supplies a stream of services during its finite life time.

4. Fiat money, "a fictitious durable" with no consumption value, but with value "derived from its participation and usage in transactions" (Kiyotaki and Wrigh. [226])

The key difference between fiat money and any consumable good is that money is a stock variable and all other goods are flow variables (useful for a finite number of periods). [1]

So comes the traditional question of a demand for fiat money in a finite time - why should one hold money since for a finite time interval it ultimately has no value?

1.2.2 Characterization of an economy

An economy is characterized by the following features.

1. There is a set of *players (traders)*, I, with a general element i. Each trader is characterized by an initial endowment, e^i, $e^i \in R^m_+$, where m is the total number of goods in the economy, both consumable and money.

2. A *message (a signal)* is a statement of a trader, which includes his decisions of what, how and under which conditions to buy or to sell.

3. A market (a trading post) processes messages and produces trades, i.e. collects messages of traders, determines prices, performs settlements, payments and delivery procedures following some predefined rules.

1.2.3 Market organization

Shubik ([348]) described different organization of markets in economies with SMG. Let L be a number of consumable goods, which can be traded.

1. There is one trading post which collects all messages from traders. This trading post is responsible for all the operations with this good (like the Walrasian auctioneer).

[1] Shubik and Smith, ([355] supplied the detailed formal analysis of differences between flow and stock variables for a multi-period SMG trade.

2. There is a trading post for every pair of commodities. So there are $\frac{(L-1)L}{2}$ different markets.

3. There is at least one trading post for each good, where it is traded for fiat money. So there are L markets if there is only one trading post for every good.

The organization of markets matters as traders should have enough means of payments for operations at every market one wants to trade (Shubik, [341]) otherwise some of the markets may not open. This problem does not appear in a centralized trade of Walrasian auctioneer. In terms of trade with fiat money this means that if there is not enough money, then the market loses its liquidity, for example Shubik ([342, 344]). Transactions costs affect number of markets and lead to appearance of a good which serves as a means of payment (more in Shubik and Rogawski, [317]).

Capie, Tsomocos and Wood, ([68]) used SMG to analyze conditions under which fiat money dominates electronic barter and to study implications for monetary policy from changes in technologies of financial transactions. Shubik's experiments demonstrated that the organization of markets matters for an equilibrium transition path (see the section on experiments with SMG).

1.2.4 Variety of strategies in SMG

Strategic market games supply multiple options to construct trading strategies. The options differ by dimensions of individual strategy sets. For example, traders may use the sell-all strategy at the centralized trade with one trading post or trade in each good at an individual trading posts. Shubik and Quint ([304]) identified three basic trading mechanisms for single period strategic market game.

1. The sell-all model.

2. The buy-sell model (wash sales).

3. The double auction model.

Let L be a number of commodities, $m = L+1$, number of commodities and money.

Sell all model

The model assumes that a trader sells all the quantity of the good he has and pays with his money for what he buys. The dimension a typical strategy set is m, a vector of bids in money for every good. This model was studied for example by Shubik ([339]).

Wash sales

The model assumes that a trader may buy and sell a good simultaneously at the same trading post at any feasible quantity. The dimension of a typical strategy set is $2m$. Some properties of this trading mechanism are discussed further.

Motivations for using wash sells come strategic signaling reasons, different bookkeeping and tax-reduction interests. In some countries for some markets wash sales are illegal and prohibited.

Peck and Shell ([289, 291]) studied liquidity of the market with wash sales and proved the existence of multiple-trading equilibria for this game.

Ray ([314]) studied how individually achievable allocations depend on the total bid and the total offer made by other traders. He demonstrated that games with wash sales and games with buy or sell strategies are not individually decision equivalent (Ray, [315]) and have different sets of achievable allocations.

The double auction model

Traders can sell and buy goods by sending prices for buying/selling and by sending quantity limits for execution of transactions for these prices. The dimension of an individual strategy set is $4m$. These models were studied by Dubey, ([111]) and

Shubik and Dubey, ([127]). Extensive investigation of the limit order market was done by Mertens, ([263]).

1.2.5 The basic model

The presented variant of SMG assumes that there is a consumable good, which serves as commodity money.

A good is labeled as $l, l \in \{1, \ldots, L\}$, where L is the number of different types of consumable commodities. The good $L + 1$ is commodity money.

Each trader i from I has a preference relation, represented by a utility function $u^i \colon R^m_+ \mapsto R$, strictly concave and differentiable on R^m_+, $m = L + 1$. Before the game each trader receives an endowment of commodities and an endowment of commodity money, $e^i \in R^m_+, e^i \neq 0$, $\sum_{i \in I} e^i_l > 0$ for every $l = 1, L$. $\mathcal{E} = \{(R^m_+, u^i, e^i), i \in I\}$ is an economy.

Every commodity $i = 1, L$ is traded only at one post. Traders can forward their bids and offers for this good to this post. Trade takes place only for fiat money.

Commodity $L + 1$ is the commodity money, which is used as means of payment.

Every trader i has a strategy set which describes all his bids and offers for every commodity:

$$S^i = \{s^i = (b^i, q^i) \in R^L_+ \times R^L_+ \colon$$
$$0 \ \leq q^i_l \leq e^i_l, i = 1, L;$$
$$0 \leq \sum_{l=1}^{L} b^i_l \leq e^L_h, i = 1, L - 1, \} \quad (1.1)$$

with a general element $s^i = (s^i_l)_{l=1,L-1}$ where $s^i_l = (b^i_l, q^i_l)$ - is a pair, which consists of a bid, b^i_l, and an offer, q^i_l, for the good i from the trader i.

Let $S^{-i} = \times_{n \neq i} S^i$ be a set of strategies of all other players besides i with a general element s^{-i}, S^{-i} is a non-empty convex compact in $R_+^{(\#I-1)(2\times L)}$. The set of strategies $S = \times_{i\in I} S^i$ is a non-empty convex compact in $R_+^{(\#I)(2\times L))}$.

Pricing

Each good has a trading post where it is traded for commodity money.[2] All traders send their messages on the good to the trading post where it is traded.

An aggregated bid for the commodity l is $B_l = \sum_{n\in I} b_l^n$ and an aggregated offer is $Q_l = \sum_{n\in I} q_l^n$. Every trader i has an aggregated bid from his competitors $B_l^{-i} = \sum_{n\neq i} b_l^i$ for the commodity l and an aggregated offer from his competitors $Q_l^i = \sum_{n\neq i} q_l^i$ for the same commodity.

The pricing mechanism for the good $l = 1, L$ is defined as

$$p_l(b_l^i, q_l^i, B_l^{-i}, Q_l^{-i}) = \begin{cases} \frac{b_l^i + B_l^{-i}}{q_l^i + Q_l^{-i}}, & q_l^i + Q_l^{-i} \neq 0 \\ 0, & \text{else} \end{cases}$$

If there is a finite number of traders at the market then every trader has some market power to affect the final price. If there is a finite number of *types* of traders and every type contains a continuum of traders then the solution of the game is competitive.

1.2.6 Allocation rules

After-trade allocation for every trader $h \in H$ for every commodity $i \in \{1, \ldots, L-1\}$ follows the rule:

[2] Consumable goods can be traded also for other goods. This increases the number of markets, some of which may not open.

$$x_l^i = \begin{cases} e_l^i - q_l^i + \frac{b_l^i}{p_l}, & q_l^i > 0, p_l > 0, \\ & \sum_{l=1}^L p_l q_l^i \geq \sum_{l=1}^L b_l^i \\ e_l^i - q_l^i & q_l^i > 0, p_l > 0 \\ & \sum_{i=1}^L p_i q_l^i < \sum_{i=1}^L b_l^i \\ & \text{ the trader } i \text{ violates the budget constraint} \\ e_l^i & q^i = 0, b^i = 0 \end{cases}$$

Condition $\sum_{l=1}^L p_l q_l^i \geq \sum_{l=1}^L b_l^i$ means that the trader i can not violate his budget constraint. Otherwise he is punished by confiscation of all his offers.

Possible insolvency in money $\sum_{l=1}^L p_l q_l^i > \sum_{l=1}^L b_l^i$ plays a very important role in monetary applications and will be discussed further.

Allocation of the commodity money m for any trader $i \in I$ follows the rule:

$$x_i^m = \begin{cases} e_i^m - \sum_{l=1}^L b_l^i + \sum_{l=1}^L q_l^i p_l, & p_l > 0 \\ & \sum_{l=1}^L p_i q_l^i \geq \sum_{l=1}^L b_l^i \\ & \\ e_i^m - \sum_{l=1}^L b_l^i & p_l > 0 \\ & \sum_{l=1}^L p_i q_l^i < \sum_{l=1}^L b_l^i \\ & \text{the trader } i \text{ violates the budget constraint} \\ e_i^m & \sum_{l=1}^L b_l^i = 0, \sum_{l=1}^L q_l^i = 0 \end{cases}$$

1.2.7 Utility and maximization problem

Every trader i has a utility maximization problem:

$$\max_{(b_l^i, q_l^i)_{l=1}^L \in S^i} u^i(x_l^i(b_l^i, q_l^i, B_l^{-i}, Q_l^{-i})_{i=1}^L)$$
$$\text{subject to the pricing and allocation rules defined above.} \quad (1.2)$$

An equilibrium in pure strategies is a profile of strategies $(s^{i*})_{i\in I}$ such that

$$u^i(x^i(s^{i*}, s^{-i*})) \geq u^i(x^i(s^i, s^{-i*}))$$

for every $i \in I$ and any $s^i \neq s^{i*}$.

SMG is the game $(I, (S^i, u^i)_{i\in I})$ for strategic reallocation of resources within the general equilibrium framework. Thus strategic market games have properties of a resource reallocation mechanism and have properties of a non-cooperative game.

1.3 Existence of an equilibrium and efficiency

1.3.1 Existence

A trivial equilibrium, i.e. an equilibrium when all traders use trivial strategies or do not trade, is always an equilibrium in the game.

If a trader uses any trading strategy, then the utility function has discontinuity at origin, which complicates analysis of the game. In order to avoid this Dubey and Shubik ([129]) studied the existence of ϵ-Nash equilibrium. Their idea is to assume that there is an outside agency which supplies small $\epsilon > 0$ amounts of each good to each trading post, so that the price is defined as:

$$p_l(b_l^i, q_l^i, B_l^{-i}, Q_l^{-i}) = \frac{b_l^i + B_l^{-i} + \epsilon}{q_l^i + Q_l^{-i} + \epsilon}$$

Dubey and Shubik ([129]) proved that for any small and positive ϵ the price is positive and bounded from above. They proved that for this condition the game has an internal Nash equilibrium.

SMG with wash sales have multiple trading equilibria (Hubert and Shubik, ([208]). Bretton and Gobillard ([63]) studied stability of a set of equilibria when there are small transaction costs. They demonstrated that most of these equilibria are non-robust to the existence of transaction costs.

Peck, Shell and Spear [291] showed that if there is one trading equilibrium for a finite number of traders then there are other trading equilibria.

1.3.2 Efficiency

Dubey ([110]) demonstrated that usually SMG with a finite number of players are inefficient. Asymptotic efficiency properties of SMG were studied by Dubey, Mas-Colell and Shubik, ([121]). Their formal analysis was based on the following assumptions: convexity of strategies sets, anonymity of traders, continuity of outcomes with respect to strategies, aggregation of relevant messages and nondegeneracy (traders can affect market outcome through their strategies).

They proved that for a small number of traders a result of SMG is inefficient, but with an asymptotic convergence to the efficient Walrasian outcome when the number of traders increases infinitely.

Koutsougeras ([233]) studied degree of competition in SMG with a finite number of traders. He showed that "the proportion of individuals whose strategic behavior differs substantially from price taking, converges to zero" asymptotically, "regardless of the distribution of characteristics" of the players. Koutsogeras and Ziros ([236]) demonstrated that for an atomless economy there is a three way equivalence- between market game mechanisms, competitive equilibria and a core.

Dubey and Geanakoplos ([113]) constructed the Walrasian equilibrium as an asymptotic case of noncooperative games using a variant of SMG.

Amir and Bloch ([8]) reproduced the convergence of SMG with wash sales to competitive equilibria from supermodular optimization/games, using asymptotic replication of an economy.

1.3.3 Trivial strategy

The case when trivial equilibrium is the only equilibrium in SMG was studied by Cordella and Gabszewicz ([86]), Busetto and Codogniato ([64]). If the trivial equilibrium is the only equilibrium in the game then the initial allocation of endowments is already Pareto efficient and the game does not have trading equilibria.

Studying dynamical trade in SMG is complicated by possible equilibria multiplicity, if one uses wash sales.

1.3.4 Time and retrading

Ghosal and Morelli ([155]) studied dynamical retrading for SMG. Their game had sell-all strategies and demonstrated Pareto improvement in allocations and asymptotic competitiveness. This result matches replication of a one-period game with sell-all strategies, which asymptotically converges to competitive outcome in one period game.

1.4 Special properties of SMG with wash sales

Strategic market games with wash sales have some special features, which make the analysis more complex.

1.4.1 Identity strategy

If wash sales are allowed then we can ask a question - which strategies do not change allocation for a player i (presented in for example, Dubey and Shubik, [131]). These strategies must satisfy the following conditions:

$$e_l^i = e_l^i - q_l^i + b_l^i(q_l^i + Q_l^{-i})/(B_l^i + b_l^{-i}), \text{ for some consumable good } l = 1, L$$

and

$$e_m^i = e_m^i - \sum_{l=1,L} b_l^i + \sum_{l=1,L} q_l^i(b_l^i + B_l^{-i})/(Q_l^i + Q_l^{-i}), \text{ for money}$$

From here follows

$$Q_l^{-i} b_l^i = B_l^{-i} q_l^i$$

This pair of strategies $(b_l^i, q_l^i)_{l=1,L+1}$ maps initial endowment $e^i = (e_1^i, \ldots, e_L^i, e_m^i)$ of the player i into itself.

1.4.2 Concave line of individually achievable allocations

The set of individually achievable allocations serves as a budget constraint in the game. It is the strictly concave curve (Dubey and Shubik, [131]), but converges to a straight line with an asymptotic increase in the number of traders.

Let $G(q_1^i, \ldots, q_L^i) = (e_m^i + \frac{(e_l^i - q_l^i)B_l^{-i}}{Q_l^{-i} + e_l^i - q_l^i})_{l=1,L}$ be a point on this line. One can show that $\dfrac{\partial G}{\partial q_l^i} > 0$ and $\dfrac{\partial^2 G}{\partial (q_l^i)^2} < 0$.

1.4.3 Arbitrary choice of some components of an equilibrium strategy for wash sale trade

In order to show this we substitute the pricing and the allocations rules into the utility function. For a game with two goods the player i has utility maximization problem:

$$\max_{s^i \in S^i} u^i(e_1^i - q_1^i + b_1^i \frac{q_1^i + q_1^j}{b_1^i + b_1^j}, e_2^i - b_1^i + q_1^i \frac{b_1^i + b_1^j}{q_1^i + q_1^j})$$

The first order condition for the strategy q_1^i is

$$u_1^i * (-1 + \frac{b_1^i}{b_1^i + b_1^j}) + u_2^i(\frac{b_1^i + b_1^j}{q_1^i + q_1^j} - q_1^i \frac{b_1^i + b_1^j}{(q_1^i + q_1^j)^2}) = 0$$

and for the strategy b_1^i is

$$u_1^i * (\frac{q_1^i + q_1^j}{b_1^i + b_1^j} - b_1^i \frac{q_1^i + q_1^j}{(b_1^i + b_1^j)^2}) + u_2^i * (-1 + \frac{q_1^i}{q_1^i + q_1^j}) = 0$$

, where

$u_1^i = \frac{\partial}{\partial q_1^i} u^i(e_1^i - q_1^i + b_1^i \frac{q_1^i + q_1^j}{b_1^i + b_1^j}, e_2^i - b_1^i + q_1^i \frac{b_1^i + b_1^j}{q_1^i + q_1^j})$ and etc.

From here we obtain that the trader i has only one equation to determine two strategic variables b_1^i and b_1^j:

$$\frac{u_1^i(\zeta_1^i, \zeta_2^i)}{u_2^i(\zeta_1^i, \zeta_2^i)} = -\frac{(\frac{b_1^i + b_1^j}{q_1^i + q_1^j} - q_1^i \frac{b_1^i + b_1^j}{(q_1^i + q_1^j)^2})}{(-1 + \frac{b_1^i}{b_1^i + b_1^j})} = -\frac{-1 + \frac{q_1^i}{q_1^i + q_1^j}}{\frac{q_1^i + q_1^j}{b_1^i + b_1^j} - b_1^i \frac{q_1^i + q_1^j}{(b_1^i + b_1^j)^2}} \quad (1.3)$$

This means that if there is a wash sale then traders have a free choice of some components of their equilibrium strategies. This property of SMG with wash sales leads to multiplicity of trading equilibria.

1.4.4 Multiple individual strategies, which support the same price

Strategic market games with wash sales have the special property: a trader may have different strategies which generate the same market price. These strategies do not affect his utility, but affect marginal costs of other traders.

Shapley and Shubik, [333, p.964, footnote 30]) wrote: "... if at equilibrium Trader i is sending both goods and cash to the same trading post and if the price there is p, then he might consider decrease both q^i and b^i in the ratio 1 to p. This would not change his final outcome (...) or the price, but it would change the marginal cost of good to the other traders and so destroy the equilibrium".

We will show this formally. Consider for simplicity the game of two traders. Let price p can be formed in two different ways:

$$p = \frac{\bar{b}_1^i + b_1^j}{\bar{q}_1^i + q_1^j} = \frac{b_1^i + b_1^j}{q_1^i + q_1^j}$$

where $(\bar{b}^i, \bar{q}^i)$ and (b^i, q^i) are two different strategies of the trader i. Then $\bar{b}_1^i q + b_1^j q = \bar{q}_1^i b + q_1^j b$, where $b = b_1^i + b_1^j$, $q = q_1^i + q_1^j$

Let $\bar{b}_1^i = b_1^i + \Delta b^i$ and $\bar{q}_1^i = q_1^i + \Delta q^i$, then

$$\Delta b^i q + b_1^i q + b_1^j q = \Delta q^i b + q_1^i b + q_1^j b$$

$$\Delta b^i - \Delta q^i b = -q b_1^i q - b_1^j q + q_1^i b + q_1^j b = 0$$

Thus

$$\Delta b^i q = \Delta q^i b$$

and

$$\frac{\Delta b^i}{\Delta q^i} = \frac{b}{q} = p$$

which means if the player i changes his strategies in the proportion of 1 to p, then the market price does not change.

Now we will demonstrate that this change in strategies does not change allocation for the player i. Let x_1^i be an allocation in the good 1 for the trader i:

$$\begin{aligned} x_1^i(q_1^i + \Delta q^i, b_1^i + \Delta b^i, q_1^j, b_1^j) \\ = a_1^i - (q_1^i + \Delta q^i) + (b_1^i + \Delta b^i)\frac{q_1^i + q_1^j + \Delta q^i}{b_1^i + b_1^j + \Delta b^i} \\ = a_1^i - (q_1^i + \frac{\Delta b^i}{p}) + (b_1^i + \Delta b^i)\frac{1}{p} \\ = a_1^i - (q_1^i) + \frac{b_1^i}{p} = x_1^i(q_1^i, b_1^i, q_1^j, b_1^j) \end{aligned} \quad (1.4)$$

The same can be shown for the allocation of commodity money. Thus if i changes his strategy from (q_1^i, b_1^i) to $(q_1^i + \Delta q^i, b_1^i + \Delta b^i)$ he does not change his allocations and consequently his final payoff.

But from another side this changes the ratio of marginal utilities for the player j:

$$\frac{u_1^j(q_1^j, b_1^j, q_1^i, b_1^i)}{u_2^j(q_1^j, b_1^j, q_1^i, b_1^i)} = \frac{\frac{b_1^i + b_1^j}{q_1^i + q_1^j} - q_1^j \frac{b_1^i + b_1^j}{(q_1^i + q_1^j)^2}}{-1 + \frac{b_1^j}{b_1^j + b_1^i}} \neq$$

$$\frac{\frac{b_1^i + \Delta b^i + b_1^j}{q_1^i + \Delta q^i + q_1^j} - q_1^j \frac{b_1^i + \Delta b^i + b_1^j}{(q_1^i + \Delta q^i + q_1^j)^2}}{-1 + \frac{b_1^j}{b_1^j + \Delta b^i + b_1^i}} = \frac{u_1^j(q_1^j, b_1^j, q_1^i + \Delta q^i, b_1^i + \Delta b^i)}{u_2^j(q_1^j, b_1^j, q_1^i + \Delta q^i, b_1^i + +\Delta b^i)} \quad (1.5)$$

where $u_1^j(q_1^j, b_1^j, q_1^i, b_1^i) = \frac{\partial w^j(\zeta_1^j)}{\partial \zeta_1^j}$ and $u_2^j(q_1^j, b_1^j, q_1^i, b_1^i) = \frac{\partial w^j(\zeta_2^j)}{\partial \zeta_2^j}$

1.5 Production in strategic market games

There is very little literature which investigates production and related issues within the framework of SMG.

Production in SMG was introduced by Dubey and Shubik ([131]). Strategic behavior in their model covers both factor and final goods markets. Dubey and Shubik, ([129]) introduced analysis of actions of stockholders and managers into an economy with production.

1.6 Strategic market games and incomplete markets without money

Giraud and Weyers, ([162]) demonstrated the existence of a full subgame-perfect equilibria for finite-horizon economies with incomplete markets without default. In their finite horizon model with strategic investors, the price of a security may be different from its fundamental value even if asset markets are complete, and regardless of the (finite) number of agents.

Brangewitz, ([62]) extended the model of Giraud and Weyers for the cases with possibility of default with collaterals.

1.7 Transaction costs

Rogatsky and Shubik ([317]) studied the effects of transaction costs on trading activity. They demonstrate that the introduction of fiat money, which reduces transaction costs, also reduces the number of market from $m(m-1)/2$ to $m-1$, where m is the total number of goods in the economy, including commodity money. Shubik and Yao ([345]) studied how transaction costs affect demand for money.

1.8 Information and uncertainty in SMG

Shubik [362] presented the basic framework for SMG when traders have asymmetric information.

Dubey and Shubik, ([124]) studied a closed economy with SMG reallocation mechanism, where traders have exogenous uncertainty. Results converge asymptoticaly to those of the Arrow-Debreu competitive equilibrium .

Peck and Shell, ([288]) showed that the Arrow securities game and the contingent commodity games have different Nash equilibria. The two games differ as the market power of a trader depends on the way markets are organized. The only common equilibria between the games are those which involve no transfer of income across states (also Weyers, [389]). Peck and Shell, ([288]) concluded that imperfectly competitive economies are sensitive to details of market structures, which are insignificant for competitive economies.

Goenka ([166]) examined the effect of leakage of information in SMG through prices. He demonstrated three results: (a) if information is free, then information revelation is faster; (b) if information is not free, then there may be no acquisition of information; (c) information leakage leads to a decrease in value of information but does not affect the incentive for informed traders to sell the information.

Gottardi and Serrano ([170]) studied a strategic model of dynamic trading with asymmetric information between buyers and sellers. The structures of the sets of buyers and sellers in their paper are different - a continuum of buyers and a finite number of sellers, who have private information. Information revelation in the trade significantly depends on the possibility for a seller to exploit his information, a presence of clients, the structure of the sellers' information, and the intensity of competition allowed by the existing trading rules.

Minelli and Meier ([271]) proved the existence of an equilibrium in strategic market game for a large, anonymous markets in which both buyers and sellers may have private information.

Polemarchakis and Raj ([300]) studied correlated equilibria in strategic market games played in overlapping generation framework. They shown that it corresponds to sunspot equilibria in the associated, competitive economy.

Dubey, Geanakoplos and Shubik ([116]) used a two period strategic market game to criticize the approach of Rational Expectations Equilibrium to asymmetric information in general equilibrium, as it does not leave place for private information to come to the market.

1.9 Double auction

This variant of SMG can be considered as the two-sided Bertrand-Edgeworth model. The model was presented by Dubey and Shubik, ([127]).

In this section of the survey we follow the notation of Dubey and Shubik, ([127]). An economy E consists of traders, $N = \{1, \ldots, n\}$, with a trader indexed by i. Every trader i is characterized by an initial endowment $a^i \in R^k_+$ and a utility function, $u^i \colon R^k_+ \mapsto R$, where k is the number of goods. Utility function is assumed to be continuous, non-decreasing and strictly increasing in at least one variable. We skip the properties of allocation of the endowments, which serve to guarantee existence of trade.

Let for any price $p \in R^k_{++}$ the budget set of i is:

$$B^i(p) = \{x \in R^k_+ \colon px \leq pa^i\}$$

and let

$$\tilde{B}^i(p) = \{x \in B^i(p) \colon u^i(x) = \max_{y \in B^i(p)} u^i(y)\}$$

Competitive equilibrium of the economy is a list $(p; x^1, \ldots, x^n)$ of prices and allocations such that each player i maximizes his utility on the budget set, $x^i \in \tilde{B}^i(p)$ for every player $i \in$ and there is the balance of all goods in the economy $\sum_{i \in N} x^i = \sum_{i \in N} a^i$.

A strategy for the player $i \in N$ is a list

$$s^i = (p^i, q^i, \tilde{p}^i, \tilde{q}^i)$$

where $p^i \in R^k_{++}$, $q^i \in R^k_{++}$, $\tilde{p}^i \in R^k_{++}$, $\tilde{q}^i \in R^k_{++}$, $\tilde{q}^i_j \leq a^i_j$, for every good $j = 1, \ldots, k$. Elements of the strategy have the following interpretation: "if the price of commodity j is p^i_j or less, then i is willing to buy this good up to the quantity q^i_l; if the price of the good j is $\tilde{p}^i_l$ or more then the trader i is willing to sell this good up to the quantity $\tilde{q}^i_j$".

So a trader makes limit orders at both sides of the market. The dimension of the strategy sets is the important difference with the wash sales model. Another difference is that for a double auction with finite number of traders Nash equilibrium coincides with competitive equilibrium, like in Bertrand competition.

Dubey, ([132]) showed that strategic (Nash) equilibria of the double auction mechanism yield competitive (Walras) allocations, even if there is a bilateral monopoly. Dubey and Sahi, ([122]), studied mechanism of mapping signals into trade, which satisfy certain axioms and showed that there are only a finite number of such mechanisms, which satisfy these axioms. They also indicated that there are some open problems regarding a convexity property of these mechanisms. Some other properties of double-auctions were studied by Weyers ([390]). Giraud and Tsomocos ([161]) applied double-auction mechanism for studying dynamical monetary processes without tatonement.

1.9.1 Asset trading

Koutsougeras and Papadopoulos, ([235]) studied saving behavior within SMG framework. They demonstrated that in equilibrium with a finite number of traders there is a positive spread between the cost of a portfolio and the portfolio's returns.

Hens et al. ([198]) constructed two-fund separated strategic market game where traders use sell-all strategies.

Giraux and Stahn, ([160]) studied a two period financial economy and addressed the question of the existence of an equilibrium. They showed the existence of nice equilibria, i.e. situations in which prices for both assets and commodities are strictly positive.

1.10 Multiple post trading

Amir et al, ([9]) studied an economy with pair-wise trade in each good. They demonstrated that equilibrium prices may not satisfy parities. A similar argument was supplied by Sorin, ([369]) for studying SMG with multiple fiat money.

Investigation of equilibrium deviations from price parities was continued by Koutsougeras, ([232, 234]). He demonstrated that if a good can be traded at multiple trading posts, then equilibrium prices for the same good at different trading posts can be different in equilibrium. This leaves a place for free arbitrage for a trader who comes to the market. The important outcome is that the Law of One Price fails even when traders do not have liquidity constraints.

Let k_1, k_2 be two different trading posts for the same good l. B^i_{l,k_1} - is an aggregate bid of all other traders besides i for the good l trading at the trading post k_1. In the same way is defined B^i_{l,k_2}. Q^i_{l,k_1} is an aggregate offer of all other traders besides i for the good l trading at the trading post k_1. Q^i_{l,k_2} is designed in the same way.

Equilibrium price in every pair of trading posts k_1, k_2 of a commodity l, satisfies the following (no-arbitrage) conditions;

$$(p_{l,k_1})^2 = \frac{B^i_{l,k_1}}{Q^i_{l,k_1}} \frac{Q^i_{l,k_2}}{B^i_{l,k_2}} (p_{l,k_2})^2$$

when there are no liquidity constraints. The proof follows from the first order conditions of utility maximization problem.

If some traders have binding liquidity constraints then

$$(p_{l,k_1})^2 \leq \frac{B^i_{l,k_1}}{Q^i_{l,k_1}} \frac{Q^i_{l,k_2}}{B^i_{l,k_2}} (p_{l,k_2})^2$$

Gobillard ([165]), Bloch and Ferret, ([52]) demonstrated that the existence of wash sales is the necessary condition for the existence of this effect.

Application of this result to other markets was done by Koutsougeras and Papadopoulos, ([285]), Papadopouls, ([284]). They demonstrated that empirical deviations from interest rate parities, purchasing power parities and international Fisher equations can be explained in the same way.

1.11 Money and SMG

Shubik, ([352]) listed four properties of money: a numeraire, a means of exchange, a store of value and a source of liquidity. Further, he claimed that these properties are most naturally formalized with strategic market games.

Joint introduction of strategic market games and punishment for default has solved the traditional dichotomy between consumable goods (flow variable) and fiat money (stock variable). This erased the historical separation between microeconomics and macroeconomics.

1.11.1 Enough money

Enough money in the economy means that given prices the quantity of money (a means of payment) is equal to the value of all goods in the economy, nominated in fiat money. In strategic market games prices are endogenous, thus we can not exclude cases, when some traders may have not enough money to pay. If traders can not pay *before* a trade, they need credit, while if they do not enough have money *after* the trade to honor obligations, they have default. Strategic behavior makes players consider these problems before the trade starts.

Conditions for enough money split into three cases (Shubik ([346]): (1) Well distributed enough money - no trader has a liquidity constraint to trade. (2) Not well distributed enough money - there are traders who need liquidity and want to borrow, and there are traders who do not have the liquidity constraint and can give credits. (3) Not enough money in the whole economy.

Money borrowing/lending mechanism, based on strategic market game with incentive compatibility for participants is presented below.

1.11.2 Endogenous demand for money

The setup of the model is cash-in-advance model, but the existence of lending requires explicitly formulated incentive of a borrower to pay his credit back. Shubik and Wilson, ([359]) described the relation of money holding and default as follows: " If an individual ends up with a positive amount of money after having paid the bank, this has no positive value to him. If on the other hand he is unable to honor his debts in full, a penalty is leveled against him."

In their model trade takes place only with money, which traders do not have as endowments and they take credits from the Central Bank. The credit market is organized as a strategic market game - traders supply their bids, and the Central Bank supplies money. The resulting interest rate is endogenous.

The important part of the model is the paired appearance of two financial instruments - fiat money, issued by the Central Bank, and promises of traders to pay back their debts. Traders issue their promises in exchange for money.

Let v^1, v^2 be bids for credits from two traders.[3] The Central Bank is a strategic dummy in the model, which supplies fixed quantity of money M. The interest rate is determined in the model following the rule of strategic market games:

$$(1+r) = \frac{v^1 + v^2}{M}$$

If a trader bids for credit v^i then he will receive $\frac{v^i}{1+r}$ units of money in exchange for a promissory note IOU (I Owe You) size v^i, which is measured in fiat money. v^i is the promise of i to pay back to the bank.

If there is no punishment for not paying credit back, then demand for money will be infinite. If punishment is infinite (capital punishment, for example), then there will be no demand for money. Thus the size of the punishment affects individual trading strategies, including the demand for money. A trader has a utility maximization problem, where punishment enters as dis-utility[4]:

$$u^i((b^i, q^i), v^i) + \lambda^i \max\{0, q^i \frac{b^i + b^j}{q^i + q^j} - b^i + \frac{v^i}{(1+r)} - v^i\}$$

where λ^i is the punishment for not paying credit.

The most important result of the model is that introduction of punishment for credits implies the endogenous demand for money, which is based on individual decision making. In other words the paper of Shubik and Wilson constructed fundamentals of general equilibrium microeconomic theory with fiat money for a finite time.

[3] Or aggregated types of trades

[4] There are other ways to introduce punishment

Shubik and Wilson, ([359]) concentrated on a type symmetric Nash equilibrium. This means that there is a finite number of types of players (two types in their model) and there is continuous number of players of each type. In this case equilibrium in the model is competitive, which facilitates analysis.

Shubik and Wilson, ([359]) did not supply a proof of existence of an equilibrium, they supply numerical examples. They conclude that the optimal punishment must be equal to the marginal utility to usage of money in trade.

Using this approach Shubik and Tsomocos, ([358]) constructed a playable game where a government is able to extract seigniorage from the agents in an economy, who take credits in fiat money. The government attempts to reduce the interest rate subject to its requirement to replace worn out fiat money. Minimization of interest rates leads to minimization of the *effective* money supply. The strategic variable interest rate determines revenues. "The existence of an equilibrium requires that we believe that the government can announce *in advance* the correct interest rate and how it is going to spend revenues it has not yet received".

Shubik and Tsomocos, ([357]) studied existence of a mutual bank, where a continuum of traders can get credits. For a multiple period trade there is the important problem - how does default in the past affects access to credit resources in future? This problem was studied by Bhattacharaya et al. ([44]) using strategic market games.

The introduction of punishment for default in SMG has opened the new class of general equilibrium models with fiat money in finite time.

1.12 Strategic market games and money

1.12.1 Variety of models with money

As we can see from the model of Shubik and Wilson, "money is the substitute for trust" to use other's money. Shubik, ([353]) presented a taxonomy of models with fiat money and endogenous demand for money. There are 12 basic models, which differ by sources of money, uncertainty levels and time of trade. Each model has one feature from the following three lists.

1. Fiat money
2. Outside credit
3. Inside credit

1. No exogenous uncertainty
2. Exogenous uncertainty

1. Finite horizon
2. Infinite horizon

Outside credit means that there is an outside bank which is ready to borrow and to lend. Inside credit means that there is an initial issue of fiat and inside money market for borrowing and lending where the endogenous rate of interest is formed. Introduction of money into general equilibrium can be done following the mechanism of Shubik and Wilson.

So there are plenty of different variants of strategic market games, which are to be investigated.

Dubey and Geanakoplos, ([120]) proposed a way to combine inside money (loans) and outside money (money in wealth) into the general equilibrium framework. In their model money also has value in one period general equilibrium model.

The credit market operates in the style of Shubik and Wilson. Dubey and Geanakoplos, ([120]) showed existence of monetary equilibrium for a type symmetric model with continuum of traders of each type.

In brief their model consists of the following elements (we follow the notation of Dubey and Geanakoplos, [120]).

There is a set of players $H = \{1, \dots, h\}$ and there is a set of goods, $L = \{1, \dots, l\}$. A player $\alpha \in H$ has initial endowment $e^\alpha \in R^L_+$ and utility function $u^\alpha \colon R^L_+ \mapsto R$, concave and continuously differentiable. Restrictions, $e^\alpha = (e^\alpha_1, \dots, e^\alpha_L) \neq 0$, $\sum_{\alpha \in H} e^\alpha > 0$ are necessary conditions for trade in all L goods.

Fiat money in the economy serves as a medium of transactions. Let $M > 0$ be the supply of money by the Central Bank and let $m^\alpha \geq 0$ be the private endowment of money of the trader α. The trader can use his endowment to pay back his credit after the trade, as after the trade he does not need money. Thus after the trade the Central Bank accumulates all the money in the economy: M and $\sum_{\alpha \in H} m^\alpha$.

The model has a competitive banking sector. It gives the credit and imposes penalties for borrowers in cases of their default. There is no assumption that a stock of money balances government taxes.

The game has the following order of events.

1. Each player α borrows $c^\alpha \in R_+$ units of money from the bank. His debt is $\mu^\alpha = (1+\theta)c^\alpha$, where the interest rate is formed as $1 + \theta = \frac{M}{\sum_{\alpha \in H} c^\alpha}$.

2. All players trade with commodities using money for purchases. The set of strategies available for trade is

$$\left\{ (b^\alpha, q^\alpha) \in R^L_+ \times R^L_+ \colon : \forall j \in L q^\alpha_j \leq e^\alpha_j \sum_{j \in L} b^i_j \leq c^\alpha + m^\alpha \equiv \frac{c^\alpha}{1+\theta} + m^\alpha \right\}$$

. The final holding of fiat money is

$$\bar{c}^\alpha = c^\alpha + m^\alpha - \sum_{j \in L} b_j^\alpha + \sum_{j \in L} p_j q_j^\alpha$$

, which consists of his credit, initial endowment of money and monetary result of his net trade.

3. The player α chooses to repay $r^\alpha \leq \bar{c}^\alpha$ on his loan. His outstanding debt at the bank is $d^\alpha = d^\alpha(m^\alpha, r^\alpha) = \mu^\alpha - r^\alpha$.

The choice set of the player i is

$$\Sigma^\alpha(\theta, p) = \Big\{(\mu^\alpha, b^\alpha, q^\alpha, r^\alpha) \in R_+ \times R_+^L \times R_+^L \times R_+ \colon q_j^\alpha \leq e_j^\alpha, \forall j \in L$$
$$\sum_{j \in L} b_j^\alpha \leq \frac{\mu^\alpha}{1+\theta} + m^\alpha; r^\alpha \leq \frac{\mu^\alpha}{1+\theta} + m^\alpha - \sum_{j \in L} b_j^\alpha + \sum_{j \in L} p_j q_j^\alpha \Big\} \quad (1.6)$$

A trader can not pay back more than his net balance in fiat money after the trade.

The outcome functions, x_j^α and d_j^α, are continuous functions from $\Sigma^\alpha(\theta, p)$ into R_{++}.

The motivation to pay debt operates in the same way as in the model of Shubik and Wilson. If there is no punishment, then $r^\alpha = 0$ and there is infinite demand for credit. Prices will be driven to infinity and the value of money will be zero. If the punishment is very high then there is no demand for money, but the economy may have not enough money for trade.

Utility of the trader α is described as: $U^\alpha(x^\alpha, d^\alpha) = u^\alpha(x^\alpha) - \lambda^\alpha(\theta, p, \omega)\max\{0, d^\alpha\}$, where ω is other relevant macro-variables.

Monetary equilibrium is a triple (θ, p, y), $y = (y^1, \ldots, y^\alpha, \ldots, y^L)$ and $y^\alpha \in R_+^L$ for $\alpha \in H$ such that for every player $\alpha \in H$ there is: $(\mu^\alpha, b^\alpha, q^\alpha, r^\alpha)_{\alpha \in H}$ such that

1. •

$$(\mu^\alpha, b^\alpha, q^\alpha, r^\alpha) \in Arg \max_{(\mu^\alpha, b^\alpha, q^\alpha, r^\alpha) \in \Sigma^{alpha}} U^\alpha(x^\alpha(b,q,p), d^\alpha(\mu, r), \theta, p, \omega)$$

   • $y^\alpha = x^\alpha(b^\alpha, q^\alpha, p)$

2. $\omega = \eta(\theta, p, y)$

3. $\sum_{\alpha \in H} y^\alpha = \sum_{\alpha \in H} e^\alpha$

4. $\sum_{\alpha \in H} \frac{\mu^\alpha}{1+\theta} = M$

Every player maximizes his utility U^α, correctly anticipating macro-variable ω, with commodity markets clearing $\sum_{\alpha \in H} y^\alpha = \sum_{\alpha \in H} e^\alpha$ and the money market clearing $\sum_{\alpha \in H} \frac{\mu^\alpha}{1+\theta} = M$.

The definition of the monetary equilibrium is constructed in such a way that if it exists, then money has positive value.

If initial allocation is already Pareto-efficient, then there is a trivial equilibrium, money has no value and monetary equilibrium does not exist.

If initial endowment of fiat money in the economy $\sum_{\alpha \in H} m^\alpha > 0$ is positive, then in any monetary equilibrium there is a positive interest rate and the set of monetary equilibria is determinate. No trader will hold money after the trade. After the trade all the money $M + \sum_{\alpha \in H} m^\alpha$ will go back to the bank.

As $\frac{M}{\sum_{\alpha \in H} m^\alpha} \mapsto 0$ then the interest rate converges to 0 and the monetary equilibrium commodity allocations converge to the Arrow-Debreu allocations of the underlying economy without money.

The model has different variants of punishments (real and nominal), and these variants are suitably adjustable.

The model reacts differently to changes in private endowments of money $(m^\alpha)_{\alpha \in H}$ and to changes in borrowed money M. If there is an increase in M (holding $(m^\alpha)_{\alpha \in H}$

constant), then the interest rate is lower, while an increase in $(m^{\alpha}_{\alpha \in H})$ (holding M constant) raises the interest rate. But in both cases there is inflation and changes in allocations.

Multiple different extensions of this model exist. Dubey and Geanakoplos, ([114]) showed conditions when monetary equilibrium can exist and money has positive value, even a general equilibrium with incomplete markets may not exist. Their model has four special features in comparison to the standard uncertainty framework: "missing assets, in the sense that some imaginable contracts are not available for trade; missing market links, in the sense that not all pairs of instruments in the economy trade directly against each other; inside and outside fiat money; and a banking sector, through which agents can borrow and lend money."

The paper of Tsomocos et al., ([75]) has multiple features important for applied macroeconomic analysis: incomplete markets with money and default, non-bank private sector, banks, a central bank, a government and a regulator. The model has positive default in equilibrium. The model allows to "characterize contagion and financial fragility as an equilibrium phenomenon, based on individual rationality".

Another important development of the model is the paper of Goodhart, Sunirand and Tsomocos ([168]) which studied rationality of bank behavior, possible contagious default, approaches to design prudential regulation and approaches to limit incentives for excessive risk-taking by banks. Another applied side of this model is that it supplies micro-foundations for financial fragility mechanisms, and "highlights the trade-off between financial stability and economic efficiency".

Tsomocos et al. ([44]) studied multi-period inter-bank credit market. The authors demonstrated how banks become inter-connected through promissory notes, what results in dynamical spillover effects - the financial results of one bank affect profits and default rates of another sequentially.

Tsomocos ([384] investigated nominal indeterminacy in a monetary overlapping generation model of of the international economy. He demonstrated that the combined effect of the monetary sector together with the market and agent heterogeneity remove real and nominal indeterminacy. The important partial result is that existence of outside money removes the nominal indeterminacy. The resulting "monetary policy becomes non-neutral since monetary changes affect nominal variables which in turn determine different real allocations".

Karatzas, Shubik and Sudderth, ([223]) applied SMG for studying fiscal and monetary control and government decisions.

The application of SMG for studying monetary economics within general equilibrium framework is an actively growing field of research. Geanakoplos and Tsomocos ([153]) studied applications for international finance, while Peiris and Tsomocos, [293] studied international monetary equilibrium with default.

1.13 Experiments with SMG

There is some literature on experiments with strategic market games. Duffy, Matros and Tezemtelides, ([133]), experimentally investigated convergence of SMG to competitive outcome. They report that as the number of participants increases, the Nash equilibrium they achieve approximates the associated Walrasian equilibrium of the underlying economy.

Huber, Shubik and Sunder, ([207]), made experiments with endogenous demand for money. Players issued their own IOU promises. Settlement was done by a costless efficient clearinghouse. The results suggest that "if the information system and clearing are so good as to preclude moral hazard and any form of information asymmetry, then the economy operates efficiently at any price level without government money".

Similar experiments were done by Anger et al ([11]) who asked the following research question: "Is personal currency issued by participants sufficient for an economy to operate efficiently, with no outside or government money?" The results demonstrate that "if agents have the option of not delivering on their promises, a high enough penalty for non-delivery is necessary to ensure an efficient market; a lower penalty leads to inefficient, even collapsing, markets due to moral hazard."

Huber, Shubik and Sunder, ([210]), investigated the applicability of penalties for equilibrium selection in SMG. They report experimental evidence on the effectiveness of penalties for conversion to a desired equilibrium.

The results of the last three papers are very close to the predictions of a Prisoners' dilemma in terms of monetary economics. This prediction can be formulated by the statement of Shubik "money is a substitution for trust".

In another paper Huber, Shubik and Sunder, ([209]), reported the results of experiments, where they compared predictions of the theory of SMG with the results of different experimental strategic market games (sell-all, buy-sell, and double auction). Their data reveal different paths of convergence, and different levels of allocative efficiency in the three settings. These results suggest that institutional details matter to understand differences between the investigated games.

1.14 Conclusion

This small survey has highlighted some of the most important features of SMG and the most important current applications of SMG for macroeconomic analysis.

Chapter 2

Mixed strategies in strategic market games with wash sales

2.1 Introduction

Shapley ([336]) and Shaley and Shubik [333]) and Shubik ([340]) introduced strategic market games (SMG) and studied strategic behavior within closed economic models. Dubey and Shubik ([125]) established the existence of ϵ-Nash equilibrium in pure strategies.

The possibility of wash sales in this class of games, i.e. a trader can simultaneously buy and offer for sale a good at the same trading post has serious consequences. Wash sales on the one hand increase market activity, may conceal information and serve signaling purposes, but on the other hand, produce multiplicity of pure strategy Nash equilibria. Furthermore, wash sales increase the thickness of the market (Peck and Shell, ([289])). Finally, there is the possibility of lopsided markets which jeopardize the existence of equilibrium altogether.

Shapley and Shubik ([333, p.964]) emphasized that the existence of wash sales is "one major disadvantage" of SMG. Consequently, traders do not have enough first order conditions to identify uniquely their best response strategies and, hence, they need to choose arbitrarily some of the actions. Therefore, the resulting best responses correspondences results into multiplicity of pure strategy equilibria. Shubik ([348, p.163]) mentioned that "the strategy set would be arbitrarily points' in a closed convex set. This, in turn implies the case in which all other traders would have "conjectures on the part of the players, as to what a player will do" Aumann and Brandenburger ([19]). Thus, there is no reason, in principle, to exclude mixed strategies.

In this note we introduce mixed strategies in SMG that both remove the singularity in SMG caused by no-trade and produce unique Nash equilibria. Finally, we construct the unique equilibrium in SMG and provide a numerical example.

2.2 The model

Without loss of generality consider a two person SMG, (Dubey and Shubik ([125]). Traders are indexed by $i = 1, 2$ (two types of traders who differ only in endowments). There is one perfectly divisible good in the economy, and there is commodity money.

An initial allocation of trader i is a vector $e^i = (e^i_1, e^i_2) \in R^2_+$, where e^i_1 is the amount of the good available to trader i and e^i_2 represents the commodity money held by i. Utility of the trader i is represented by a real-valued function:

$$u^i \colon R^2_+ \mapsto R_+.$$

.

The utility functions are concave, smooth and non-decreasing.

The set of strategies of the trader i is

$$S_i = \{s_i := (q_i, b_i) \colon q_i \in R^1_+, b_i \in R^1_+, 0 \le q_i \le e^i_1;\ 0 \le b_i \le e^i_2\},$$

where q_i is the quantity of commodity trader i offers for sale, b_i is the bid i for the good.

The strategies set of both players is a product $S = S^1 \times S^2$ which is a compact and convex set with a general element $s \in S$.

The price of the good is determined by strategies of both players that yields $p(s) = \dfrac{b_1 + b_2}{q_1 + q_2}$ where $s = (q_1, b_1, q_2, b_2)$. If $q_1 + q_2 = 0$, then by definition $p = 0$. The final allocation for i in the consumable good is

$$\zeta_i(s) = \begin{cases} e_1^i + b_i/p(s) - q_i, & p(s) > 0 \\ e_1^i - q_i, & \text{otherwise} \end{cases}$$

In the commodity money

$$m_i(s) = e_2^i - b_i + q_i p(s)$$

The payoff to i is $u^i(\zeta_i(s), m_i(s))$, where $s = (s_1, s_2)$, $s_i = (q_i, b_i)$, $i = 1, 2$.

The strategic market game is defined by $\Gamma = ((i = 1, 2), (S_i, u^i))$, where $i = 1, 2$ are the players, S_i, $i = 1, 2$ are their sets of the strategies, and u^i, $i = 1, 2$ are their utility functions.

The trader i solves the problem of utility maximization:

$$s_i^* = argmax_{s_i \in S_i} u^i(\zeta_i(s_i, s_{-i}), m_i(s_i, s_{-i}))$$

subject to the pricing and allocation rules stated above, where $-i = \begin{cases} 1, & i = 2 \\ 2, & i = 1 \end{cases}$.

A Nash equilibrium of the game is a strategy profile $s^* = (s_1^*, s_2^*) \in S$ such that

$$u^i(\zeta_i(s^*), m_i(s^*)) \geq u^i(\zeta_i(s_i, s_{-i}^*), m_i(s_i, s_{-i}^*))$$

Indeterminacy in construction of strategies follows the following consideration.

The first order condition of i in q_i is

$$u_1^i(\zeta_i, m_i) \cdot \Big(-1 + \frac{b_i}{b_1 + b_2} \Big) + u_2^i(\zeta_i, m_i) \cdot \Big(\frac{b_1 + b_2}{q_1 + q_2} - q_i \frac{b_1 + b_2}{(q_1 + q_2)^2} \Big) = 0$$

and in b_i is

$$u_1^i(\zeta_i, m_i) \cdot \Big(\frac{q_1 + q_2}{b_1 + b_2} - b_i \frac{q_1 + q_2}{(b_1 + b_2)^2} \Big) + u^i \cdot (\zeta_i, m_i) * \Big(-1 + \frac{q_i}{q_1 + q_2} \Big) = 0,$$

where $u_j^i(x_1, x_2) \equiv \frac{\partial u^i(x_1,x_2)}{\partial x_j}$, i.e. a partial derivative of the utility function on the argument $j, j = 1, 2$.

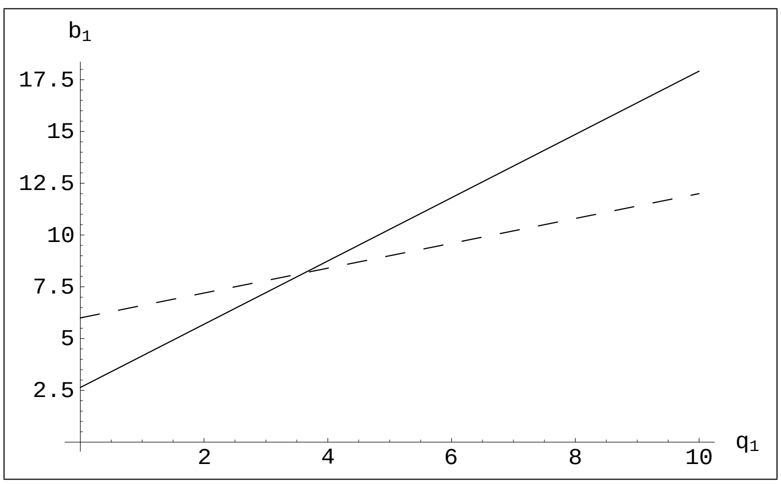

Figure 2.1: Best response correspondence of player 1 for the two sets of strategies of the player 2: solid line:$(b_2, q_2) = (5, 5)$; dashed line $(b_2, q_2) = (6, 20)$

The first equation can be transformed into the second by multiplication of the factor $-p = -\frac{q_1+q_2}{b_1+b_2}$ and the second into the first one by multiplication of the factor $-1/p = -\frac{b_1+b_2}{q_1+q_2}$. Thus the trader i has only one equation to determine two strategic variables b_i and q_i:

$$\frac{u_1^i(\zeta_i, m_i)}{u_2^i(\zeta_i, m_i)} = -\frac{\frac{b_1+b_2}{q_1+q_2} - q_i \frac{b_1+b_2}{(q_1+q_2)^2}}{-1 + \frac{b_i}{b_1+b_2}} = -\frac{-1 + \frac{q_i}{q_1+q_2}}{\frac{q_i+q_2}{b_1+b_2} - b_i \frac{q_1+q_2}{(b_1+b_2)^2}}.$$

We will take the consumable good 1 to define probability distribution over it for each player.

Based upon the previous argument, we can specify the bids and allow only the offers for sale to be indeterminate. Consequently, we introduce a probability distribution over the quantities offered for sale.

Formally, let $\Phi_i(q_i)$ be a set of all probability measures over a non-empty, convex, and compact $[0, e_1^i]$ with a general element $\pi_i(q_i)$, $i = 1, 2$ such that:

$$\pi_i(q_i) \in [0, 1], : \int_{q_i \in [0, e_1^i]} \pi_i(q_i) dq_i = 1$$

Let $\Phi = \Phi_1(q_1) \times \Phi_2(q_2)$ be a probability measure for all players with a general element $\pi = (\pi_1, \pi_2) \in [0, 1]^2$.

Expected payoff of the trader i from the trade is

$$g_i(\pi_i(q_i), \pi_{-i}(q_{-i})) = \int_{(q_1,q_2) \in [0,e_1^1] \times [0,e_1^2]} \pi_i(q_i) \pi_{-i}(q_{-i}) u^i(\zeta_i(s), m_i(s)) dq_i dq_{-i}$$

given the pricing and allocation rules for every profile of strategies. $s = (s_i, s_{-i})$, $s_i = (q_i, b_i)$, $s_{-i} = (q_{-i}, b_{-i})$

Mixed strategy Nash equilibrium $(\pi_i^*)_{i \in I}$ is a profile of mixed strategies such that for $\forall i \in I$ there exists

$$g_i(\pi_i^*, \pi_{-i}^*) \geq g_i(\pi_i, \pi_{-i}^*)$$

for every $\pi_i(s_i) \neq \pi_i^*(s_i)$.

2.3 Solution for mixed strategies equilibrium

Payoff maximization problem of the trader $i = 1, 2$ when s_{-i} uses his optimal strategy $\pi^*_{-i}(q_{-i})$ requires

$$\max g_i(\pi_i, \pi^*_{-i}) = \int_{(q_1,q_2)\in[0,e_1^1]\times[0,e_1^2]} \pi(s) u^i(\zeta_i(s), m_i(s))ds$$

$$\text{subject to } \int_{q_i\in[0,e_1^i]} \pi_i(q_i)dq_i = 1,$$

where payoff in pure strategies for the player $i = 1, 2$ is

$$u^i(\zeta_i(s), m_i(s)) \equiv u^i\Big(\zeta_i\Big(q_i, b_i(q_1, q_2), q_{-i}, b_{-i}(q_1, q_2)\Big), m_i\Big(q_i, b_i(q_1, q_2), q_{-i}, b_{-i}(q_1, q_2)\Big)\Big)$$

is the payoff function with the substituted pricing and allocations, e_1^i and e_2^i - are commodity endowments of the player i.

For simplicity of the notation we will write

$$\Pi_i(s_i, s_{-i}) := u^i\Big(\zeta_i\Big(q_i, b_i(q_1, q_2), q_{-i}, b_{-i}(q_1, q_2)\Big), m_i\Big(q_i, b_i(q_1, q_2), q_{-i}, b_{-i}(q_1, q_2)\Big)\Big)$$

for a payoff function of the player i when the game Γ has the strategy profile $s = (s_i, s_{-i})$.

In order to construct mixed strategies, we need the Euler-Lagrange equations for both players. The equation for the player $i, i = 1, 2$ is:

$$\mathcal{L}_i = \int_{(q_1,q_2)\in[0,e_1^1]\times[0,e_1^2]} \pi_i(q_i)\pi^*_{-i}(q_{-i})\Pi_i(s_i, s_{-i})dq_{-i}dq_i - \lambda_i \int_{q_i\in[0,e_1^i]} \pi_i(q_i)dq_i,$$

where $\pi^*_{-i}(q_{-i})$ is the optimal mixed strategy of $-i$ corresponding to q_{-i}, λ_i is the Lagrangian multiplier of i.

Mixed strategies are the two functions $\pi_i^* : [0, e_1^i] \mapsto [0, 1]$ satisfying the normalization condition

$$\int_{q_i \in [0, e_1^i]} \pi_i^*(q_i) dq_i = 1, \ \ i = 1, 2.$$

The Euler-Lagrange has the following interpretation. It describes the combination of all possible pure strategies equilibria, each appearing with some probability. For each equilibrium the probability of it's realization depends on probability weights of both players. Own probability of the player has the normalization condition. Probability of another a player takes as given.

In geometrical terms this is a smooth surface, where each point is a pure strategies equilibrium. The whole surface is the envelope defined over the set of best responses of both players. A trivial strategy has the zero measure and is effectively excluded.

In order to solve Euler-Lagrangian equations we need to calculate it's variation:

$$\delta \mathcal{L}_i = \int_{(q_1, q_2) \in [0, e_1^1] \times [0, e_1^2]} \delta \pi_i(q_i) \pi^*_{-i}(q_{-i}) \Pi_i(s_i, s_{-i}) dq_{-i} dq_i - \lambda_i \int_{q_i \in [0, e_1^i]} \delta \pi_i(q_i) dq_i = 0$$

$$\delta \mathcal{L}_i = \int_{q_i \in [0, e_1^i]} dq_i \delta \pi_i(q_i) \Big(\int_{(q_{-i}) \in [0, e_1^{-i}]} \pi^*_{-i}(q_{-i}) \Pi_i(s_i, s_{-i}) dq_{-i} - \lambda_i \Big) = 0$$

Using the main theorem of variational calculus (Gupta, [177]) we can derive the conditions for the mixed strategies. Mixed strategies can be calculated from the pair of equations $i = 1, 2$:

$$\lambda_i = \int_{q_{-i} \in [0, e_1^{-i}]} \pi_{-i}(q_{-i}) \Pi_i(s_i, s_{-i}) dq_{-i}, i = 1, 2.$$

,

It is very hard to find a solution of this system of equations in an analytical form. Thus we used numerical methods to calculate the mixed strategies distributions for both players.

2.3.1 Numerical calculations

Let payoffs of the player i is defined as

$$\Pi_i(s_i, s_{-i}) =$$
$$\log\left(e_1^i + b_i \frac{q_1 + q_2}{b_1(q_1, q_2) + b_2(q_1, q_2)} - q_i\right) + \log\left(e_2^i - b_i(q_1, q_2) + q_i \frac{b_1(q_1, q_2) + b_2(q_2, q_1)}{q_1 + q_2}\right) \tag{2.1}$$

Let also $e^1 = (10, 30)$ and $e^2 = (30, 10)$ be endowments of the two players. We used the constant $\epsilon_M = 2,22045 \times 10^{-16}$ as an approximation of the origin, the computer dependent variable of the system of Mathematica.

On the first stage of the calculations we have produced a grid of pure strategies equilibrium, where each point appeared from numerical optimization of a system of two non-linear equations. The system of equations appeared as a system of the two first order conditions of the players. On the second stage the numerical grid was numerically was numerically approximated with polynoms of the 5-th order. Equilibrium probability distributions are presented in the Figure 2.2. The complete listing of the program is presented in the Appendix to this Chapter.

We can observe from the graphs of probability distribution functions $\pi_i, i = 1, 2$ that probabilities have fluctuations in the 3-4rd digit to the right from a decimal point.

This in turn means that for both players the probability density functions $\pi_i, i = 1, 2$ are slowly changing. This property follows from the envelope theorem since we

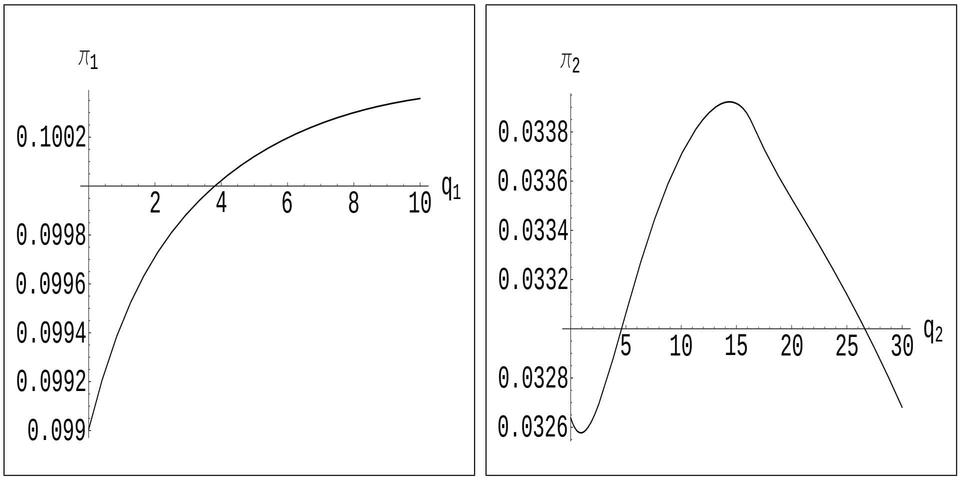

Figure 2.2: Mixed strategies for strategic market duopoly with wash sales, endowments are $e^1 = (10, 30)$ and $e^2 = (30, 10)$

	Expected utility (EU)	Var (EU)
Player 1	5.973	0.0166
Player 2	5.837	0.0057

Table 2.1: Characteristics of expected utility from using for mixed strategies in SMG with wash sales, endowments $(10, 30)$ and $(35, 10)$

study variations of final payoffs as we vary probability of quantity offered. That in effect constitute second order effect arising from the cross agent correlations that are small.

Since the density functions are smooth and slowly changing, the zero strategy has a zero measure within the obtained mixed strategy and therefore is effectively excluded. This is consistent with our approach based on the equilibrium concept of Shubik and Dubey (1978).

Finally payoff functions are presented in the Table 2.1 .

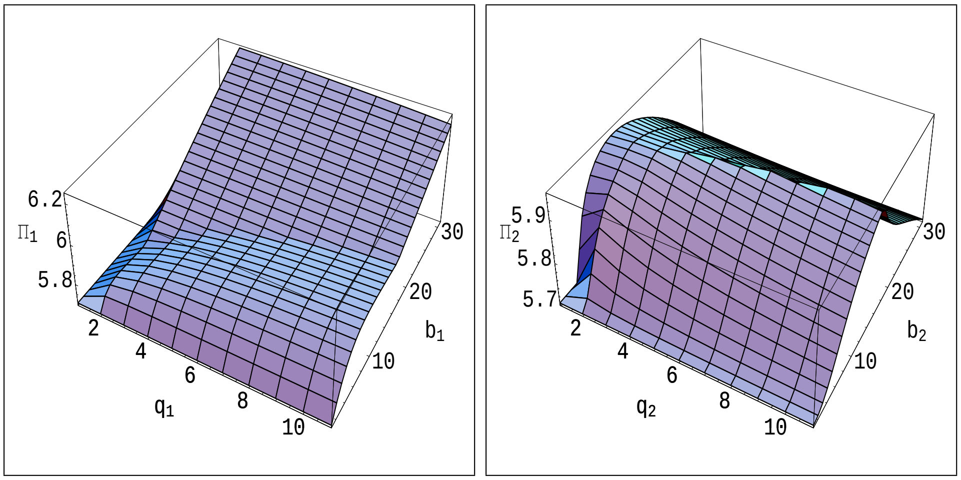

Figure 2.3: Indirect utilities for for the players with wash sales, endowments $(10, 30)$ and $(30, 10)$

2.4 Conclusion

The paper studies strategic market games with wash sales. This class of games posses best response correspondences that in turn generate non-uniqueness of pure strategy equilibria. We introduce a mixed strategy equilibrium that resolves the aforementioned indeterminacy, therefore, results into a unique equilibrium. Finally we provide an example that illustrates our equilibrium concept.

2.5 Appendix. The program.

This section contains the complete listing of the program, used for the calculations. Testing messages we not excluded from the program. The total time of the calculations about 30 minutes. **(* Loading packages *)**

(* CALCULATION OF MIXED STRATEGIES OVER*A*POSITIVE SET OF STRATEGIES*)

```
<< NumericalMath`PolynomialFit`
<< Graphics`Graphics`
```

```
(*================Cleanmemory============*)
ClearAll[approxsoln, values, errorplot, errorplot, ai, aj, probi,
normcoefi]

(*==================
Utility  functions with substituted non−
trivial  strategies and  prices==================*)
utilI[bi_, qi_?NumericQ, bj_, qj_?NumericQ]:=
If[qi + qj!=0,
N[Log[10 + bi(qi + qj)/(bi + bj) − qi]+
Log[30 − bi + qi(bi + bj)/(qi + qj)]],
N[Log[10.] + Log[30.]]]

utilJ[bj_, qj_?NumericQ, bi_, qi_?NumericQ]:=
If[qj + qi!=0,
N[Log[30 + bj(qi + qj)/(bi + bj) − qj]+
Log[10 − bj + qj(bi + bj)/(qi + qj)]],
Log[10.] + Log[30.]]
(* testing calculations of utilities*)
(*utilI[4, 0, 3, 0]
utilI[4, 0, 3, 5]
utilI[0, 6, 3, 0]
utilJ[4, 0, 3, 0]
utilJ[4, 0, 3, 5]
utilJ[0, 4, 3, 5]*)
```

```
(*============ = Define FOC=================== = *)
focI[bi_, qi_?NumericQ, bj_, qj_?NumericQ]:=
D[utilI[bi, qi, bj, qj], bi]
focJ[bj_, qj_?NumericQ, bi_, qi_?NumericQ]:=
D[utilJ[bj, qj, bi, qi], bj]
Print["–testing—"]
(*focI[bi, 0, 3, 0]
FindRoot[focI[bi, 0, 3, 5] == 0, {bi, 3, $MachineEpsilon, 30}]
FindRoot[focI[bi, 0, 4, 6] == 0, {bi, 3, $MachineEpsilon, 30}]

utilI[bi/.Solve [focI[bi, 0, 3, 5] == 0, bi][[1]], 0, 3, 5]
utilI[bi/.Solve[focI[bi, 4, 3, 7] == 0, bi][[1]], 4, 3, 7]
utilI[bi/.FindRoot[focI[bi, 0, 3, 0] == 0, {bi, 5, $MachineEpsilon, 30}][[1]],
0, 3, 5]
utilI[bi/.FindRoot[focI[bi, 0, 4, 6] == 0, {bi, 10, $MachineEpsilon, 30}][[1]],
4, 3, 7]*)
(*testing FOC——— – *)
(*
focJ[bj, 0, bi, 4]
focJ[bj, 0, bi, 5]
focJ[bj, 0, bi, 6]
Exit[] *)


(* ========= Price ==============*)
price[qi_, bi_, qj_, bj_]:=If[qi + qj==0, 0, (bi + bj)/(qi + qj)]
```

```
(*=========Allocations for --i—===============*)
xi1[p_, qi_?NumericQ, bi_?NumericQ]:=Which[
qi + bi == 0, 10,
p==0, 10 - qi,
p > 0, 10 - qi + bi/p]

xi2[p_, qi_?NumericQ, bi_?NumericQ]:=Which[
qi + bi == 0, 30,
p==0, 30 - bi,
p > 0, 30 - bi + qi * p]

(*=========Allocations of --j—===============*)
xj1[p_, qj_?NumericQ, bj_?NumericQ]:=Which[
qj + bj == 0, 30,
p==0, 30 - qj,
p > 0, 30 - qj + bj/p]
xj2[p_, qj_?NumericQ, bj_?NumericQ]:=Which[
qj + bj == 0, 10,
p==0, 10 - bj,
p > 0, 10 - bj + qj * p]
(* —testing prices ——— - *)
(*
price[3, 4, 5, 6]
price[0, 4, 0, 6]
price[3, 0, 5, 0]
xi1[price[3, 0, 5, 0], 3, 0]
xj1[price[0, 4, 0, 6], 0, 4] *)
```

```
(* ———Calculations of utilities from
 allocations———— – *)
 utilityI[qi_?NumericQ, bi_?NumericQ, qj_?NumericQ, bj_?NumericQ]:=If[
qi == 0, N[Log[300]],
N[Log[xi1[price[qi, bi, qj, bj], qi, bi]]+
Log[xi2[price[qi, bi, qj, bj], qi, bi]]]
]

utilityJ[qj_?NumericQ, bj_?NumericQ, qi_?NumericQ, bi_?NumericQ]:=
If[
qj == 0, N[Log[300]],
Log[xj1[price[qi, bi, qj, bj], qj, bj]]+
Log[xj2[price[qi, bi, qj, bj], qj, bj]]]

(*——– – Payoffs  from the game,
 given strategies which  can be chosen arbitrarily—– – — – *)
 payoffIU[qi_?NumericQ, qj_?NumericQ]:=
If[qi == 0, N[Log[300]],
utilityI[
 qi,
bi/.FindRoot[{focI[bi, qi, bj, qj] == 0, focJ[bj, qj, bi, qi] == 0},
{bi, 1, $MachineEpsilon, 30}, {bj, 1, $MachineEpsilon, 10}
][[1]],
qj,
bj/.FindRoot[{focI[bi, qi, bj, qj] == 0, focJ[bj, qj, bi, qi] == 0},
{bi, 1, $MachineEpsilon, 30}, {bj, 1, $MachineEpsilon, 10}
```

```
][[2]]
]]
payoffJU[qj_?NumericQ, qi_?NumericQ]:=
If[qj == 0, Log[300],
utilityJ[
 qj, bj/.FindRoot[
{focJ[bj, qj, bi, qi] == 0, focI[bi, qi, bj, qj] == 0},
{bj, 1, $MachineEpsilon, 10}, {bi, 1, $MachineEpsilon, 30}
][[1]],
qi, bi/.FindRoot[
{focJ[bj, qj, bi, qi] == 0, focI[bi, qi, bj, qj] == 0},
{bj, 1, $MachineEpsilon, 10}, {bi, 1, $MachineEpsilon, 30}
][[2]]
]]
(*—— – Testingpayoffs– – *)
(*Print["–"]
payoffIU[3, 4]
payoffIU[0, 4]
payoffIU[10, 30]
payoffJU[30, 9]*)


(*——— –  Calculation probabilitiesfori– – *)
(*– –  ai –  the integral to the right partof lambda in the equation,
on  the steadypath  for the playeri——*)
ai = Table[
NIntegrate[payoffIU[qi, qj], {qj, $MachineEpsilon, 30}],
```

```
{qi, $MachineEpsilon, 10}]

(*– values of indirect utility for i onthe  steady path—*)
aii = Table[
payoffIU[qi, qj], {qj, 0, 30},
{qi, 0, 10}];
ListPlot3D[aii,
AxesLabel → {StyleForm[\!\(q\_1\)], StyleForm[\!\(b\_1\)],
StyleForm[\!\(Π\_1\)]}];

heighti5[qi_]:=Interpolation[ai, InterpolationOrder → 5][qi];
N[%]

(*heighti5[qi]/normi5 – isaprobabilityforthestrategyqj*)
Plot[heighti5[qi], {qi, $MachineEpsilon, 10}, PlotLabel->" for i"];
(* Normalization coefficient for i *)
normi5 = Integrate[heighti5[qi], {qi, $MachineEpsilon, 10}]
N[%]
Plot[heighti5[qi]/normi5, {qi, $MachineEpsilon, 10},
AxesLabel → {StyleForm[\!\(q\_1\)], StyleForm[\!\(π\_1\)]}]

 (*——— – Calculations  for j——— – *)
(*– –  aj –  the integral to the right part of lambda in the equation,
 on the steady path for the player i——*)
aj = Table[
NIntegrate[payoffJU[qj, qi], {qi, 0, 10}],
{qj, $MachineEpsilon, 30}]
```

```
ListPlot[aj, PlotJoined → True, PlotLabel->"FOC- line for j"];

(*–valuesofindirectutilityforjonthesteadypath—*)
ajj = Table[
payoffJU[qj, qi], {qj, 0, 30},
{qi, 0, 10}];
ListPlot3D[ajj,
AxesLabel → {StyleForm[\!\(q\_2\)], StyleForm[\!\(b\_2\)],
StyleForm[\!\(Π\_2\)]}];

(* Normalization coefficient for j *)
heightj5[qj_]:=Interpolation[aj, InterpolationOrder → 5][qj];
Plot[heightj5[qj], {qj, $MachineEpsilon, 30}];
normj5 = NIntegrate[heightj5[qj], {qj, $MachineEpsilon, 30}]
N[%]
(*heightj5[qj]/normj5 − isaprobabilityforthestrategyqj*)
Plot[heightj5[qj]/normj5, {qj, $MachineEpsilon, 30},
AxesLabel → {StyleForm[\!\(q\_2\)], StyleForm[\!\(π\_2\)]}];

Exit[]
Plot3D[1/(normi5 ∗ normj5) ∗ N[heighti5[qi]] ∗ N[heightj5[qj]],
{qj, $MachineEpsilon, 30},
{qi, $MachineEpsilon, 10},
PlotLabel->"Joint probability density \n for the players i and j"]
(*—— − Expected utilities  − forI−−
 CHANGE  LEFT  BOUND  TO$MachineEpsilon–*)
(*Calculation of values ,
```

```
which in sum will  egiveexpected utility for i*)
euI = Table[N[payoffIU[qi, qj]]/(normi5 * normj5) * N[heighti5[qi]]*
N[heightj5[qj]], {qj, 0, 30},
{qi, 0, 10}];
Plus@@%
Plus@@%
euInumber = Plus@@%
(*Function Plus open the list and calculates sum of elements inside ,
but the table euI is alist of lists so need to apply twice *)

(*Calculation of values , which in sum willgivevariancefori*)
Table[((N[payoffIU[qi, qj]] − euInumber)^2/(normi5 * normj5))*
heighti5[qi] * heightj5[qj], {qi, $MachineEpsilon, 10},
{qj, $MachineEpsilon, 30}];
Plus@@%
Plus@@%
Plus@@%
vareuInumber = Plus@@%
Print["Expected utility for i is ", euInumber, "; sigma is ",
Sqrt[vareuInumber]]


(*—— – Expected utilities  –for J—– *)
(*Calculation of values ,
which in sum will  give expected utility for j*)
euJ = Table[N[payoffJU[qj, qi]]/(normi5 * normj5) * N[heighti5[qi]]*
N[heightj5[qj]], {qj, $MachineEpsilon, 30},
```

```
{qi, $MachineEpsilon, 10}];
Plus@@%
Plus@@%
euJnumber = Plus@@%
(*Calculation of values , which in sum will give variance for j*)
Table[((N[payoffJU[qj, qi]] − euJnumber)^2/(normi5 ∗ normj5))∗
heighti5[qi] ∗ heightj5[qj], {qi, $MachineEpsilon, 10},
{qj, $MachineEpsilon, 30}];
Plus@@%
Plus@@%
Plus@@%
vareuJnumber = Plus@@%
Print["Expected utility for j is ", euJnumber, "; sigma is ",
Sqrt[vareuJnumber]]

(* Plot best response correspondence for the player 1 *)
Plot[
{bi/.FindRoot[focI[bi, qi, bjcalc, qjcalc] == 0,
{bi, 1, $MachineEpsilon, 30}],
bi/.FindRoot[focI[bi, qi, 6, 20] == 0, {bi, 1, $MachineEpsilon, 30}]},
{qi, $MachineEpsilon, 10} ,
AxesLabel → {StyleForm[\!\(q\_1\)], StyleForm[\!\(b\_1\)]},
PlotStyle → {GrayLevel[0], Dashing[{.03}]}]
```

Chapter 3

Coalition formation: a survey

3.1 Introduction

The problem of allocation of strategically acting players into groups is one of the most important for social sciences. There are two views on this problem - non-cooperative games (Nash, [276]) and cooperative gamed (von Neumann and Morgenstern ([387]) and Nash, [277]) theories.

Brandenburger, ([57]) claims that they "differ in how they formalize independence among the players ... One might suggest that there is no place for cooperation in the former and no place for conflict, competition etc in the latter. In fact, neither is the case. One part of the non-cooperative theory (the theory of repeated games) studies the possibility of cooperation in ongoing relationship. And the cooperative theory embodies not just cooperation among players, but also competition in a particularly strong, unfettered form."

Greenberg, ([172, p.1311]) wrote that "it is doubtful that the process of coalition formation can be separated from the disbursement of payoffs". Thus there are two inter-connected problems for decision making in conflict situations:

1. "Which coalitions are likely to form?

2. How will the members of a coalition apportion their joint payoff?"

(Anatol Rapoport, [305, p.286])

Bloch, ([49]), poses the same questions in terms of endogenous coalition formation:

1. Which coalitions will be formed in equilibrium?

2. How will the coalitional wealth be divided among coalition members?

3. How does the presence of other coalitions affect the incentives to cooperate?

In a non-cooperative game players choose strategies. Execution of strategies of all the players leads to a coalition structure - "a coalition structure in an n-person game is a partition of the set of players" (Aumann and Dreze, [27])".

Carraro et al, ([70]), wrote that the third question of Bloch, ([49]), "dealing with the competition between coalitions, is simply ignored in traditional cooperative coalition theory (as well as in cooperative bargaining theory), where competition among players is not really taken in account...The [existing] non-cooperative approach is based on *partition function*, that assigns an individual payoff to each player for each possible coalition structure. This is a generalization of characteristic function games that allows for considerations of spillovers.. partition function form carriers more information than a game in a characteristic form ".

Non-cooperative construction of a coalition structure must prescribe every player all outcomes, including those when a coalition he chooses is not formed. An adequate non-cooperative theory of coalition formation must enable players to anticipate what happens not only inside equilibrium coalition structure(s), but also outside. Outside means - in those coalition partitions which are not formed. It is important to know as as off-equilibrium coalitions affect individual opportunity costs. Thus it is important to formulate conditions when these coalition partitions can not be formed due to non-cooperative strategic behavior of the players.

The interesting theory must also have a mechanism of individual motivations for players to support stability of coalitions, based on individual strategies with a possibly credible enforcement mechanism.

This survey presents the most influential approaches to formation of coalitions, based on non-cooperative and cooperative views of coalition formation.

3.2 The grand coalition

There are two basic coalitions which deserve special comments - trivial coalition and the grand coalition.

A trivial coalition is a coalition when a player stays alone. These coalitions are not so interesting for analysis, however it's existence must be considered for non-cooperative construction of coalitions. The mere existence of this option makes us to consider this strategy as any other strategy as it affects opportunity costs for every player.

Trivial coalitions may appear for different reasons - an intentional choice, a bad luck in formation of a desired coalition or opportunistic behavior of players in the coalition, etc.

If a player strategically chooses the trivial coalition then his payoff does not depend on the strategies of others. But there are other cases when the player can obtain the trivial coalition and his final payoff depends on the strategies of others. The trivial strategy can be an equilibrium result, if there are negative and monotonic externalities in the game.

A grand coalition is a set of all players in the game. These coalitions were investigated by Aumann and Dreze,([27]), for games with transferable utilities (TU). Aumann and Dreze,([27]), motivated the formation of such a coalition in the game by the implicit existence of positive externalities.

Informally, this means that the greater is the number of players in the coalition, the greater is the individual gain in this coalition. So the grand coalition is formed.

We can use a grand coalition to introduce the two criteria for stability of coalitions (Aumann, [20], also at Aumann and Peleg, [29]) - α and β effectiveness.

A finite game Γ in normal form (referring to the notation and work of Aumann, [20]) consists of a finite set of players N, a finite set P^i of pure strategies for every player i in N and the payoff function, $F\colon \times_{i \in N} P^i \mapsto R^N$, where F^i, the i-th coordinate of F^i is the payoff of i.

For a set of players $S\colon S \subset N$ we will write $P^S = \times_{i \in S} P^i$. A probability measure on P^S will be called c-strategy S-vector (c -for correlated). The set of correlated strategies of players from a subset S will be denoted as C^S. If $c^N \in C^N$. then $F(c^N)$ is expected payoff if the c-strategy N-vector c^N is played.

A coalition S is said to be α-effective of the payoff vector x^S of the coalition x if there is $c^S \in C^S$ such that for each $c^{N \setminus S} \in C^{N \setminus S}$ there is $F^S(c^S \times c^{N \setminus S}) \geq x^S$. This means that the coalition S can guarantee itself a payoff independent of strategies of all other players.

A coalition S is said to be β-effective of the payoff vector x if for each $c^{N \setminus S} \in C^{N \setminus S}$ there is $c^S \in C^S$ such that there is $F^S(c^S \times c^{N \setminus S}) \geq x^S$. This means that the coalition S can do that each players in S will always receive x^i.

Currarini and Marini, ([89, p.6]), noted that all players outside the selected coalition S, i.e. the players from the set $N \setminus S$, "stick together", which means that there is no coalition structure beyond the given coalition S. This remark is very important. It says that players outside S actually cannot make strategic actions. Thus from one side all players in N are equal in making a choice, but from another those who are beyond S can not do it *de facto*.

It is interesting to note that the value of a coalition (total gain of the players in the coalition) is described in terms of strategies in the paper from 1960-s, when there was not so sharp divergence between cooperative and non-cooperative game theories, as exists today.

3.2.1 Reasons for non-stability of the grand coalition

There are many reasons, why the grand coalitions can disintegrate for internal reasons: increasing coordination /communication costs, restrictive normative considerations, private interests, adverse selection, moral hazards, negative spill-overs and existence or appearance of outside options for the players in the coalition.

These reasons change (disturb) opportunity costs for player after the coalition is formed, which effects its stability. Non-cooperative theory of coalition formation must include these details.

Aumann and Dreze,([27, p.233]) supply the following example:

> "Acting together may change the nature of the game. For example, if two independent farmers were to merge their activities and share the proceeds, both of them might work with less care and energy; the resulting output might be less that under independent operations, in spite of a possible more efficient division of labor".

The reasons why the grand coalition cannot exist, is the source of ideas for construction of non-cooperative coalition formation. It is important to know as any the same reasons affect stability of any coalition, not only the grand coalition.

The problem of the existence of a grand coalition appears in many traditional economic problems, for example a supply of a public good with a lack of complete information ("a free-rider problem"). Guesnerie and Oddou,([176]), proposed to solve this problem by introduction of an outside agents (a government), which imposes proportional taxes. D'Aspremont et al.([100]) studied the size of a stable cartel. The result was that at the market with a finite number of firms, only one cartel is possible and it coincides with the grand coalition. Other applications of the grand coalition approach cover international trade cooperation (Baldwin and Kiander, ([30]), Kohler, ([228]) etc. For more recent example, we can take current financial crisis in the Eurozone, in which the stability of the grand coalition is corrupted by the opportunistic behavior of the few members.

Medlin ([261]) reported results of his experiments that even in a three-player game super-additivity is not enough to construct the grand coalition.

If one wants to study coalitions different from the grand, a tool is needed that is different from the characteristic cooperative theory. Carraro et al ([70, p.27]) wrote that "cooperative coalition theory basically coincides with the standard cooperative bargaining theory (Nash, [277])" for N players and it cannot really help in understanding the forces which drive the formation of (partial) coalitions".

3.3 Core

The recent textbook of Peleg and Sudholter, ([296]) covers the most important contemporary advantages in cooperative game theory. They remark that no solution of a cooperative game can be implemented by a bargaining model as a non-cooperative game in extensive form (Peleg, [295]).

A coalitional game (N, v) with a transferable utility consists of a finite[1] set of players N and a real number $v(S)$, which assigns a value to every subset of N, with the assumption $v(\{\emptyset\}) = 0$. $v(S)$ is the total gain of all players in the coalition S. The game (N, v) has other names - a game in coalition form, a game in characteristic form, coalitions game, or transferable utility (TU)-game.

Cooperative games with TU-utility have a particular property - after the players are exogenously collected together and they know their total payoff, they still have the problem of how to divide it. The first solution was offered by Nash, ([277]) as

[1] Studying a core with infinite number of players was initiated by Aumann, ([21])

a simultaneous bargaining game, and the important development as the sequential bargaining game was done by Rubinstein, ([321]), Binmore, Rubinstein and Wolinsky, ([46]). There are many surveys on the topic, for example, van Dame, ([386]) and also Guth and Tietz,([178]).

A super-additive game (N, v) has the property for the value function v

$$\forall S_1, S_2 \subset N, S_1 \cap S_2 = \{\emptyset\} \colon v(S_1) + v(S_2) \leq v(S_1 \cup S_2)$$

In most cases when an equilibrium coalition is the grand coalition the game has the property of the super-additive game.

In political games a constant-sum game often appears. It has the property of the value function

$$v(S) + v(N \backslash S) = v(N), \forall S \in N$$

, for example, Schofield, ([327]).

3.3.1 Solutions to the game

Let X be a set of feasible allocations for a game (N, v) defined as

$$X^*(N, v) = \{x \in R^N \colon x(N) \leq v(N)\}$$

$X^*(N, v)$ is a vector of possible final payoff distribution for the game (N, v). If $X^*(N, v) = \{x \in R^N \colon x(N) = v(N)\}$ then $X^*(N, v)$ is the Pareto efficient frontier.

The central concept in cooperative game theory is a core (introduced into game theory by Gillie, [156]).

$\mathcal{C}(N, v)$ is a core for the game (N, v) and is defined as

$$\mathcal{C}(N, v) = \{x \in X^*(N, v) \colon x(S) \geq v(S), \forall S \subset N\}$$

An allocation $x \in R^N$ is in the core $\mathcal{C}(N, v)$ iff there is no coalition, which can improve upon x. A core is defined in terms of values, but it is desirable to have a criteria for stability of a coalition in terms of strategies.

Strong Nash equilibrium, as a criteria for stability of coalitions, was introduced by Aumann, ([25]). This concept combines properties of α and β stability. There is no general criteria for existence of the Strong Nash equilibrium (Bernheim, Peleg, Whinston [43]). It can be applied for any coalition partition, but the problem is that at the moment there is no an adequate non-cooperative theory for construction coalition partitions with multiple cores from individual strategies.

Following α and β efficiency (Ichiishi, [213], also at Currarini and Marini, [89, p.6]) define α-characteristic function of a coalition S as:

$$v_\alpha(S) = \max_{y_S} \min_{y_{N\setminus S}} \sum_{i \in S} u^i(y_S, y_{N\setminus S})$$

and define β-characterization of a coalition S

$$v_\beta(S) = \min_{y_{N\setminus S}} \max_{y_S} \sum_{i \in S} u^i(y_S, y_{N\setminus S})$$

The key difference between α and β views is where to find players to join. α-view concentrates on members within the same coalition, while β-view looks for agents outside the coalition.

An allocation y_s is α (or respectively β) core iff it is feasible, $\sum_{i \in S} y_s^i = v(S)$ and there is no other coalition $S_1 \subset S$ (or $S_2 \in (N\setminus S)$) such that

$$v_\alpha(S_1) > \max_{y_S} \min_{y_{N\setminus S}} \sum_{i \in S} u^i(y_S, y_{N\setminus S})$$

or

$$v_\beta(S_2) > \max_{y_S} \min_{y_{N\setminus S}} \sum_{i \in S} u^i(y_S, y_{N\setminus S})$$

For any coalition S there is no assumption about an internal structure for the set $N \backslash S$, thus the natural way to investigate cores is to study cases when a core coincides with the grand coalition.

The concept of a core says little about how players come to the core. It operates more as an *ex post* criterion. To overcome this deficiency a non-cooperative game theory of bargaining and negotiations was developed.

In many cases the core may not exist, but payoffs of the players can closely approach the core payoffs. For such cases an ϵ-core is used (for example, at Wooders, [394]). An allocation x is in the ϵ core of the coalitional game (N, v) iff

$$\sum_{i \in N} x_i(N) = v(N)$$

and

$$\sum_{i \in S} x_i(S) \geq v(S) - \epsilon(\#S), \forall S \subset N$$

If a coalition is in the ϵ-core then no other coalition S will be able to guarantee its members more than they get from the ϵ-core.

The concept of a core is implicitly based on some pre-play agreements of players, which are not specifies explicitly.

There is a common knowledge of payoffs before the game starts. A core describes the allocations that do not need external enforcement. This is done through the definition - an increase in the size (number of participates) of the coalition cannot decrease payoffs of its members. However it becomes a problem to define a coalition partition with more than one core. And it is impossible to know the order in which players enter the core.

Peled and Sudholter, ([296]) describe cores for games when a coalition partition is not limited by the grand coalition.

Let $\mathcal{R}$ be a coalition partition for the game (N, v), then $(N, v, \mathcal{R})$ is a game with coalition partition. Then

$$X^*(N, v, \mathcal{R}) = \{x \in R^N : x(R) \leq v(R), \forall R \in \mathcal{R}\}$$

is the set of feasible payoff allocations vectors for $(N, v, \mathcal{R})$.

The core of a game with coalition partition $(N, v, \mathcal{R})$ has the core $\mathcal{C}(N, v, \mathcal{R})$ which is defined as

$$(N, v, \mathcal{R}) = \{x \in X^*(N, v, \mathcal{R}) : x(S) \geq v(S), \forall S \subset N\}$$

Construction of a core can be axiomatized, for example, Peleg, ([294]), Haimanko,([182, 183]), Lee and Volij, ([246]).

Banerjee et al, ([32]) studied cores for coalition formation games in which every player's payoff depends only on the membership in a coalition. They show that even for additively separable games the existence of a core allocation is problematic. McKelvey and Schofield ([260]) studies structural instability of a core. Zhao, ([400]), presented some conditions for existence of some partial cases of a core.

Problems with the existence of a core generated enormous literature to purify this equilibrium concept. Schmeidle ([325]) refined the core by introduction of a nucleolus, Mascheler, Peleg and Shapley ([257, 258]) studied a kernel and a bargaining set of a game.

3.3.2 Equilibrium concepts for coalition stability in terms of non-cooperative game theory

Strong Nash equilibrium is the development of the Nash equilibrium for co-existence of stability of multiple coalitions. It deals with immunity of a coalition to unilateral deviations of any group of it's members, Aumann, ([25]). In contrast the Nash equilibrium assumes possibility of a deviation of only one player.

Let N be a coalition and K is any subset of N. A profile of strategies $(s^* \equiv s^{i*})_{I \in N}$ is a strong Nash equilibrium profile if for every player $i \in N$ there is

$$u^i(s^*) \geq u^i(s_j, s^*_{N \setminus j})$$

where j is any sub-coalition from $N, j \subset N$ and s_j is a strategy profile of players from the coalition j.

In many cases strong Nash equilibrium seems to be too strong and it is hard to study coalitions with strong Nash equilibrium aside from the grand coalition.

Morreno and Wooders ([**?**]) and Raj ([313]) introduced strong correlated equilibrium for coalition formation. It has features of the correlated equilibrium and of the strong Nash equilibria concepts of the non-cooperative game theory.

Bernheim, Peleg and Winston, ([43]) purified the concept of a core and introduced a coalition-proof equilibrium, "an agreement is coalition-proof iff it is Pareto efficient within the class of self-enforcing agreements." It operates like an α version of the strong Nash equilibrium if to think in terms of α and β efficiencies.

Hedonic games is a wide class of cooperative games where players have preferences over coalitions they can participate in and cannot make payments to each other. This class includes many matching games (Bogomolnaia, Jackson, [55]).

Iehle, ([216]) studied hedonic games and provided the necessary and sufficient condition for core-partition existence in a hedonic game.

Bolch and Diamantoudi, ([51]), studied a bargaining procedure of coalition formation in the class of hedonic games, where preferences of the players depend only on the coalition they belong to. They provided an example of the nonexistence of a pure strategy stationary perfect equilibrium, and also provided the necessary and sufficient conditions for existence. "If the core of the hedonic game and its restrictions always consist of a single point, we show that the bargaining game admits a unique stationary perfect equilibrium, resulting in the immediate formation of the core coalition structure."

Cooperative game theory leaves very little space for individual decision making. It is not clear what players could do in order to form coalitions. From another side the issue of individual input into a value of a coalition is very important. A combinatorial approach for studying it was suggested by Shapley, ([335]). The Shapley value is defined

$$\phi_i(N, v) = \frac{1}{\#N!} \sum_{R \in \mathcal{R}} \Delta_i(S_i(R))$$

for each $i \in N$ such that $\mathcal{R}$ is the set of all $\#N!$ orderings of the players from N, $S_i(R)$ is the set of players preceding i in the ordering R and $\Delta_i(S_i(R)) = v(S \cup \{i\}) - v(S)$.

Shapley value is interpreted as an expected marginal contribution over all orders of player i to the set of players who proceed him.

3.4 Non-cooperative rules-based approach to coalition formation.

A rule-based approach to coalition formation is the substitute for the absence of a full-scale non-cooperative theory for construction of coalition partitions. The primary goal of these theories is to construct such a set of rules, that the resulting coalition partition is stable.

The primary difficulty of the approach is the absence of a well-defined individual set of strategies and payoffs over strategies of all players which let players form coalition within the standard framework of games in normal form.

3.4.1 Two stage game for coalition formation

Carraro et al, ([70]), describe the contemporary approach to non-cooperative coalition formation as a two stage game.

1. Stage 1. Players make non-cooperative decisions on joining a coalition using some rules of coalition formation.

2. Stage 2. Players implement their strategies

The view that coalition members act cooperatively to each other and non-cooperatively to non-members was developed by Zhao,([399]), in the concept of a hybrid equilibrium, also Ichiishi, ([214]).

Currarini and Marini, ([89]), describe the process a bit differently: "cooperation is modeled as a two stage process:

1. "The first stage players form coalitions" or "given rule of coalition formation maps players' announcements of coalitions into a well-defined coalition structure"

2. "At the second stage formed coalitions interact in well defined strategic settings"..."coalition structures ... turn into equilibrium strategies chosen by players at the second stage".

Existing theories of non-cooperative coalition formation (Hart and Kurz, [194, 195], Ray and Vohra, [311], Yi, [397], Bloch, [398], etc) do not contain an adequate components to construct best responses in the sense of the standard non-cooperative game theory.

Thus, existing models have implicit assumptions to support stability of coalitions, what usually is implicit coordination.

The deficiency of the two stage game for coalition formation is that there is the lack of description of all possible cases which may happen with players during coalition formation. For example, the desired coalition may not form or may disintegrate by internal reasons.

Anticipation of these cases is the standard for non-cooperative game theory. Any event which happens off-equilibrium cases matters for the existence an equilibrium: it has the effect on opportunity gains of every player.

3.4.2 Rules for coalition formation

The literature studies different rules for coalition formation.

1. Open membership rule, assumes that members of the coalition do not object if new members join the coalition. Originally it was utilized by D'Aspremont et al.,([100]), for construction of grand coalitions. Members of coalitions do not object against new-comers, which implicitly assumes positive externalities. It was implemented in the δ-rule (Hart and Kurz, [195]).

2. Coalition unanimity rule, implemented in the γ-rule (Hart and Kurz, [195]) assumes that a coalition is formed only if all members of the coalition agree for the list of members of the coalition. This approach was initially introduced by von Neumann and Morgenstern ([387]). A partial case of this rule is *exclusive* membership rule.

Carraro et al, ([70]), wrote that the difference between the rules is what happens after a coalition is formed, i.e. the second stage of the game. Implicitly this means that some coalitions may not be formed, some players may want to leave the coalition immediately etc.

3.4.3 The approach of Hart and Kurz,([195])

In the seminal paper Hart and Kurz, ([195]), have addressed the problem of why coalitions form. Their approach studies both rules (open and unanimous) with two modifications of the same model.

In order to explain coalition formation rules we need to explain the notation of Hart and Kurz, ([195]). A finite set of players is N, a strategy of a player i is a coalition S_i, he wants to participate, $S_i \subset N$. Let

$$U(\sigma) = \{S_i(\sigma) \in i \in N\}$$

be a utility of the player i if he chooses the coalition σ. Given the choice of a coalition the formation rule has the following construction: $S_i(\sigma)$:

$$S_i(\sigma) = \begin{cases} S_i, S_i = S_j, \forall j \in S_i \\ \{i\} \end{cases}$$

A coalition is constructed only from the unanimous agreement of all the players in the coalition.

The model leaves the question open what happens with players, who have not received the desired coalition. This deficiency is improved by the modification of the model, entitled the δ-rule. It assumes that a player can join the largest set of players he is willing to be associated with (Hart and Kurz, [195]). It assumes a different rule for payoff allocation;

$$U(\sigma) = \{S \subset N \colon i, j \in S \text{ iff } S_i = S_j\}$$

Chander and Tulkens, ([77]), have introduced the concept of γ-efficiency, which is based on the behavior of non-members of a coalition. It is based on interaction between a deviating coalition and the outside players. The limiting part of their definition is that outside players may be members of other coalitions, which may not be singletons.

Open membership (Yi and Shin (2000))

Open membership means that players can join other coalitions freely. Let

$$j_{i_1} \subset \ldots \subset j_{i_m} \in J_i$$

be a sequence of coalitions for a player i, m - is the length of the chain. If $j_{i_m} = j_g$, j_g - the grand coalition, then the open membership is open for all players. $j_{i_m} \neq j_g$ then the maximum coalition for i is different from the grand coalition.

Let

$$s^i(j_{i1}), \ldots, s^i(j_{im})$$

be the sequence of strategies for the player i, $s^i(j_{i1}) \in S^i(j_{i1})$ and etc.

Open membership assumes that if someone joins a coalition j, this does not deteriorate the payoffs of other players in this coalition j unless the coalition exceeds some size m. Let j_{i_m} be a coalition size m,$i \in j_{i_m}$:

$$u^i(s^i(j_{i1}), s^{-i}) = u^i(s^i(j_{i2}), s^{-i}) = \ldots = u^i(s^i(j_{im}), s^{-i})$$

or for the case of externalities with open membership

$$u^i(s^i(j_{i1}), s^{-i}) \leq u^i(s^i(j_{i2}), s^{-i}) \leq \ldots \leq u^i(s^i(j_{im}), s^{-i})$$

Open membership means that the player i does not mind an increase of a size of a coalition from j_{i_1} upto j_{i_m}.

The upper bound for open membership takes place if negative externalities appear at some size of a coalition . Some possible externalities were described in the section on the grand coalition.

Let k be a size of the coalition j_0, $k \in \{1, \#N\}$, $j_0 \in J$. j_{0i} is *a coalition* for player i of the size k, $\#N$ is the number of players in the game. Let for every player i there is

$$u^i(s^{i*}(j_{0i}), s^{-i}) > u^i(s^i(j), s^{-i})$$

, $\forall s^i(j) \in S^i$ given $s^{-i}(j)$. So every player i will choose to play only in different coalitions of the size k. Any player i may have more than one such coalitions so players need either to coordinate actions or to use mixed strategies.

If $k = 1$ or $k = \#N$ then this construction reproduces the results of Carraro and Siniscalco ([71]).

3.4.4 Problems with simultaneous games of coalition formation

Bloch ([49]) wrote about three problems with simultaneous games of coalition formation.

1. Multiplicity of Nash equilibria

2. Necessity to have equilibrium selection mechanism

3. Individual deviations cannot be counted by subsequent moves

The last option is considered by Bloch ([49]) as the most serious problem for non-cooperative coalition formation - members of a coalition cannot react to opportunistic action(s) of member(s) of a coalition, if he (they) deviate(s) from the coalition. This is impossible to implement in γ and δ rules.

Bloch,([49]), offered another solution, he introduced time and constructed *sequential games of coalition formation* (the development of Aumann and Myerson, [28]). The approach of far-sighted players was applied by Bloch, [48], to study a procedure when coalitions are formed in a sequence: once a coalition is formed, the game is played among the remaining players. The game is played by a sequential proposals: a player makes an offer to other players to join a coalition with him. If the prospective member rejects the proposal, than he makes a counter-offer for members of the coalition with him. If all members accept the coalition is formed.

After players agree to enter the coalition they are subject to an exogenous enforcement to stay there and not to leave it. The game has stationary perfect equilibria, which may differ from the coalition structures generated by γ and δ rules.

3.5 Partition functions and coalition partition

There is another approach to coalition formation. Instead of construction of strategies one can impose exogenous partition on the set of players. This approach was first offered by Thrall and Lucas, ([381]) and recently developed by Yi, ([397]).

In the voluminous literature on coalition formation there are two main directions for studying coalition partitions (Ray, [309]):

- Coalitions as units based on blocking, inclusion/exclusion, etc: Chwe ([82]), Haeringer ([179]), Thrall and Lucas ([381]), Ray ([308]), etc..
- Individuals as units where actions of players are based on a game in extensive form, for example Bloch ([48,50]), Chatterjee, Dutta and others ([79]), Okada ([280–282]), Seidman and Winter ([328]), Rubinstein ([321]) etc.

There is a larger literature on partition games, with exogenous partitions or exogenous partition functions, for example Deng ([104]), Kalai ([222]), Yi ([395,396]).

The approach of the literature is to take an exogenous partition of a set of players or a partition function (an allocation of payoffs over a *subset* of players) and then investigate properties of the resulting coalitions. There is also very big bargaining literature on endogenous coalition formation, for example Ray ([311]), Bennet ([40]), Chatterjee, Dutta, et al. ([311]).

3.5.1 Coalition formation with partition function

In the presentation of coalition formation with partition function we will use the notation of Yi, ([397]). A coalition structure $C = (B_1, \ldots, B_m)$ is a partition of the player set

$$P = (P_1, \ldots, P_N)\colon B_i \cap B_j = \{\emptyset\}, \forall i, j, and \cup_{i=1,m} B_i = P$$

A partition function of a player n_i is $\pi^i \colon C \mapsto R$, $\pi(n_i, C)$ the per member payoff of a member of the size-n_i coalition in the coalition structure $C = (n_1, \ldots, n_m)$. To avoid the problem of side-payments inside a coalition equal sharing inside the coalition is assumed.

This assumption does not solve the problem of stability of a resulting coalition, so one needs an operation, that reallocate players between coalitions - concentration. $\mathcal{C} = (n_1, \ldots, n_m)$ is a concentration of $\mathcal{C}' = (n_1, \ldots, n'_{m'}), m' \geq m$ iff there are exists a sequence of coalition structures

$$\mathcal{C}^1 = (n_1^1, \ldots, {n'}_{m'}^1), \ldots, \mathcal{C}^R = (n_1^R, \ldots, {n'}_{m'}^R)$$

such that

- $\mathcal{C} = \mathcal{C}^1$, and $\mathcal{C}' = \mathcal{C}^R$

- $C^{r-1} = C^r \backslash \{n^r_{i(r)}, n^r_{j(r)}\} \cup \{n^r_{i(r)} + 1, n^r_{j(r)} - 1\}$, $n^r_{i(r)} > n^r_{j(r)}$ for some $i(r)$, $j(r) = 1, \ldots, m(r)$ and for all $r = 2, R$.

In words this means that a final partition can be obtained by a finite sequence of concentrations.

Non-degenerative coalition structure $C = \{n_1, \ldots, n_m\}$ is stable iff there does not exist another coalition structure which can concentrate it.

By positive externalities Yi means cases when "formation, expansion or mergers of coalitions create positive external effects on non-members" of a coalition. Yi demonstrates how partition function can be applied for construction of coalitions with increasing sizes of these coalition with positive externalities.

A minor question in the analysis is the following - where from players get information, including which partition function to choose and concentration to follow. If this is the game with complete information, then players will agree only on partition function which supports only the final stable coalition partition. So concentration process becomes virtual and elusive.

The idea of equal division of shares inside coalitions supplies the benefit of the model - there is no inter-coalition transfer of wealth, however the same goal can be reached by the assumption that the game has non-transferable utility. This leaves an open question for analysis if concentrating coalition have different payoffs for members, how would a new payoff allocation in the new partition be justified.

To demonstrate applicability of the approach, Yi ([397]), constructs different partition functions. His examples cover cartels, research coalitions with spillovers, public goods, free-trade areas, All the examples are based on construction of payoffs, which depend on a number of players in a coalition, but not on what players do to join a coalition and do to support the stability of the coalition.

The implementation of different restriction on payoffs between different coalitions allows to construct concentrations (Yi [397]) or games with externalities (Yi and Shin [398], Bloch,([50]) and other authors.

3.5.2 Hybrid equilibrium

Zhao, ([399]), introduced hybrid equilibrium. It takes an exogenous coalition partition. He assumed that within a coalition, players are cooperators, and between coalitions they are competitors. Empirical justification for this approach comes from normative or historical facts of coalition formation, like different regional organizations or competition between different regions of the same country.

3.6 Sequential (non-cooperative) coalition formation.

Coalition formation can be considered dynamic. Traders' desires to improve their payoffs are the main reason for the appearance of a multiple-period coalition formation.

A sequential solution was developed in papers on multilateral bargaining, for example, Bennette [40], the survey of Serrano [329] presents the landscape of the problem. Other titles include Moldovanu and Winter, ([272]), Seidmann and Winter, ([328]) and Perry and Reny, ([297]), where dynamics come from alternating an bargaining model.

The recent paper of Chander and Wooders,([78]), studies sequential coalition formation and existence of sub-game perfect equilibrium in this process. Their approach is based on the assumption: "the idea that a coalition becomes a single player; given a game in extensive form with player set N, when a coalition S forms, a new game is created in which the players in S becomes one single player".

The model of Chander and Wooders,([78]), substitutes a non-cooperative game in an extensive form of coalition formation by a game of representative players from each coalition. This leaves aside the question of coordination of players inside coalitions.

3.7 Applied research coalition formation

3.7.1 Industrial organization

Cooperative game theory has multiple applications in industrial organization literature. One of the first papers on the topic was D'Aspremont et al., ([100]), who studied the existence of stable cartels. Their result is that the only stable cartel at the market is the grand coalition. Very close research but from another perspective was done by Donsimoni et al., ([107]), who have used linear demand and marginal cost function. Both papers show that under some additional conditions for cost efficiency the grand coalition is the stable cartel.

The important question was studied by Rotschield, ([320]). Using trigger strategies he has showed that the "stability of the cartel may depend crucially upon the relative efficiencies of the firm". This is the application of the very old economic concept, relative advantages of Ricardo, for studying coalition stability. The same question with the same conclusion but in a multi-period framework was studied by Escrihuela, ([141]).

Rotschield, ([319]), studied impact of changes in tariffs and quotas on the performance of an international "dominant-group" cartel.

Lambertini ([244]), studied the sensitivity of commitment to collusion to demands with different elasticities. When there is a small number of firms they prefer to compete in Cournout, otherwise, firms prefer to compete in Bertrand.

Rauscher, ([307]), investigated periodic commodity price shocks using a cartelized supply side. "It is shown how the interactions of sluggish demand and the inherent instability of the cartel create cyclical behavior and price shocks".

Caparros et al., ([67]), studied stability of coalitions with heterogeneous agents. Benchekroun, ([38], also [37]) studied dynamic stability of oligopolistic cartels.

Holtzm ([205]) and Ichiishi , ([214]), studied cooperative construction of a firm. The important outcome from their papers is that players can behave cooperatively inside a coalition and competitively between coalitions (see also hybrid equilibrium of Zhaom, [399]).

Other applications cover deal with multiple markets of fishing as different coalitions Szidarovszky et al., ([377]), multiplicity of financial markets were studied by Pirrong, ([298]), monetary unions were studied by Kohler, ([227]), etc.

3.7.2 Coalition formation in environmental agreements

Climate and environment protection is an old area of application for analysis of externalities. Recent results in cooperative game theory were used for analysis and comparison of national strategies in formation and supporting international agreements.

Ecchia and Mariotti, ([136]) discussed the role of international binding institutional agreements achieving effective International Environmental Agreements. They claimed that the game theoretic model of coalitional bargaining is the right tool for analysis.

Finus, et al. ([146]) studied formation and stability of coalitions to form international environmental agreements. They showed that stable coalitions can come only if there significant benefits or appropriate transfer scheme are introduced to motivate countries to stay in coalitions.

In another paper, Finus and Rundshagen, ([147]), studied endogenous coalition formation in global pollution control. They wrote that there are two types of free-riding in respect to International Environmental Agreement. The first type of free-riding is not to enter the agreement and have benefits from efforts of members of the Agreement. The second type of free-riding is to enter the agreement, but to violate the rules. Their classification comes along the standard α and β stability rules. This observation is very important as it captures general features of the coalition formation.

Tol, ([382]), studied a numerical model of coalition formation of nine world regions in the respect of their policies for greenhouse gas emission reduction. Asheim, ([16]) studied the interaction between regional and global cooperation in climate control.

Chapter 4

A mechanism for non-cooperative coalition formation in finite games

4.1 Introduction

A coalition is a subset from a set of players. There are few basic questions for social sciences related to existence of coalitions:

1. What holds members of the coalition together?

2. How did they come to the coalition and what they do to stay together?

3. What can an outside central planner do to hold them together?

Thus the concept of a coalition has two meanings - a group of players and a stable group of agents. For the discussion in this paper we will use the term "pre-coalition" meaning any group of players which decided to meet. The term coalition we will use only for pre-coalition where all the players in the pre-coalition do actions to stay together (we will also use the term a stable coalition).

Existing literature does not discriminate this detail. When there is no loss in generalization we will use the traditional term coalition.

There are two approaches to coalition construction. The first one is cooperative (Neumann- Morgenstern [387]), when players have a pre-play communication about the moves of each other and make non-binding claims. The game is described as a set of players, a partition of this set and a vector of payoffs of the players for each partition.

The development of this approach is the work of Nash ([277]), where he constructed cooperative bargaining game. A coalition of two players appears in his approach as a group which competes for a limited resource. The mechanism of the competition is a simultaneous bargaining with the implicit participation of an umpire, which excluded irrelevant bids.

Another approach to construction of coalitions is based on non-cooperative actions of players (Nash [276]). Non-cooperative construction is based on strategies and must prescribe for every player every possible detail. The game is described as a set of players, each player has a strategy set and a set of payoffs defined on a product of strategies of all players.

The most interesting problem is construction of (stable) coalitions from non-cooperative behavior. In other words the problem is how to construct a non-cooperative game when players make strategic decisions to form pre-coalitions and (stable) coalitions.

By stability of a coalition we mean immunity of this group of players against not only individual but also against any group deviations for the members of this coalition.

Different branches of game theory have different names for this kind of stability. In cooperative game theory it is described by the concept of a core. In non-cooperative game theory it is described by the concept of a strong Nash equilibrium.

It is known from cooperative game theory that a core may not exist in many cases, especially when it is different from the grand coalition (for example, Peleg and Sudhölter, [296]). from another side the strong Nash equilibrium is generally considered as too strong (Bernheim, Peleg, Whinston, [43]) and conditions of it's existence are still not clear, especially for cases with more than one coalition in the equilibrium.

The most important problems with construction of stable coalitions start when a set of players can be partitioned into many coalitions.

Cooperative view on coalition formation assumes that stability can be supported by pre-play agreements or by exogenous enforcement. Agreements are assumed to be credible without enough arguments. At the same time professional literature does not present a criteria of the existence for the strong Nash equilibrium.

Non-cooperative approach (for example, γ, δ-rules of Hart and Kurz,([194,195]) , does not supply enough tools to support stability against possible deviations, (Bloch, [49]).[1]

This paper studies the problem of non-cooperative construction of coalition(s) as stable coalitions, coalitions when players do actions to stay together. It is clear that not every coalition can be supported only by individual motivations of players. Thus we may assume the existence of a central planner - a virtual agent, equally distant from every player, who decides which coalitions to form and how. He imposes enforcement and makes players participate in the coalitions.

The set of coalitions for players is chosen by the central planner. If a coalition is implementable with enforcement we will call this coalition as an induced coalition. Induced means that the enforcement distorts payoffs of all players in this coalition in such a way that it is not rational to deviate in any way, individually or in a group.

Thus we apply the approach of Huber, Shubik and Sunder ([210]). They suggest that establishing " a precise specification of penalties " is a way to reach " any one of the available ... equilibrium".

[1] Partition approach, Yi, [397] takes partition to be exogenous and does not study stability of the partitions.

The enforcement distorts payoffs inside the induced core (like a "a carrot") and outside the induced core (like a "a stick"). If the enforcement exists, then the corresponding induced equilibrium has the properties of a strong Nash equilibrium, even if there are more than one induced core inside.

The enforcement plan is organized as follows. The social planner distorts the payoffs inside the induced coalition partition and outside it. The players modify their actions in order to conform with the distorted system of payoffs. Thus the enforcement operates only through incentives of the players, using the old principle "a carrot and a stick". The carrot means a gain not in comparison to what a player could have without the enforcement, but in comparison to what the player will obtain if deviates from the coalition assigned for him. Credibility of the enforcement is supported through the property of self-financing.

The enforcement transforms the basic non-cooperative game into cooperative. This problem was mentioned by Myerson, ([275, p.370-371]): "how to transform non-cooperative game into a cooperative game using only non-coordinated behavior of players". The presented mechanism of coalition stability has the property of "local strict dominance". The enforcement distort payoffs of the players to make them stay only in the assigned coalitions.

We can think about the enforcement mechanism in terms of implementation theory. A (social) planner modifies payoffs and players need to adjust their strategies. The planner offers a coalition partition for the players, which prescribes for every player to which coalition to join. This takes may take place even if there is perfect information.

We demonstrate that there are cases when is it possible to construct induced coalitions different from the grand and trivial coalitions. The construction is based on self-financed enforcement mechanism. The mechanism is based on reallocation of payoffs between all players in the game, what is different from the view of transferable-utility cooperative games. An induced coalition is a core.

This approach does not guarantee an existence of an equilibrium, it significantly depends on the technology of enforcement. However the approach allows to apply all the tools the non-cooperative finite games for studying induced cores and induced partitions.

The novelties of the approach are:

1. Self-financed enforcement, which is based on transfer and operates as a combination of "carrots" and "sticks". "Carrots" - to support membership in the assigned coalition and " sticks" - to prevent deviations .

2. Introduction of induced cores and induced strong Nash equilibrium due to implementation of the enforcement. Possible existence of multiplicity of the induced cores in the induced coalition partition.

3. Introduction of a regret payoff, an outcome for a player when a coalition desired by the player fails to appears.

4. Explicit introduction of opportunistic strategy for every coalition into the strategy set of each player.

The paper is organized in the following way. After a brief literature survey a series of examples explain roles of each element from the novelty list. Then the detailed model and the equilibrium concept of induced strong Nash equilibrium are presented. A special section describes the relation of the paper to mechanism design literature.

4.2 Literature

Each of the approaches to stability of coalitions - non-cooperative and cooperative have different criteria. Cooperative approach looks for cores (Aumann, [20], also at

Aumann and Peleg, [29]) - α and β effectiveness) where players are considered as elements of sets and actually make no strategic decision.

More of that, Currarini and Marini, [89, p.6], noted that all players outside a selected coalition S, i.e. in the set $N \backslash S$, are "stick together", what means that there is any structure over the set of players beyond S. This remark is very important. It says that players outside S can not make strategic actions. Thus from one side all players in N are equal, from another those who are beyond S can not be distinguished. Thus the only way to eliminate the contradiction is to study the grand coalition.

from another side there is the concept of strong Nash equilibrium, (Aumann, [25]) which is defined in terms strategies and subsets of strategies. In terms of non-cooperative games it is considered to be very strict, almost never existing for the grand coalition. When a core is the grand coalition then it has the property of the strong Nash equilibrium.

View of the cooperative game theory is based on assumption of pre-play communication, which can hardly be credible in the general case. Assumption of an enforcement posses a question about credibility of the enforcement.

Non-cooperative view on coalition formation was formulated in $\gamma - \delta$ rules of Hart-Kurz, ([194,195]), Their mechanism has an implicit coordination to avoid cases like when a player wants a coalition, but it is not formed. Another drawback of the approach (mentioned by Bloch,([49]) is that individual deviations cannot be counted by subsequent moves - members of a coalition can not react to opportunistic action of other members.

For the last sixty years both of the approaches are investigated in the voluminous literature. However the open questions rests - what is a non-cooperative way to construct strong Nash equilibria, which consists of more than one coalition?

This paper offers the specially designed non-cooperative game, which avoids these problems. If a coalition can not be formed directly from the "local dominance of strategies" then we suggest self-financed enforcement mechanism.

It follows the standard way of the general equilibrium theory. The target coalition partition - can it be reached by reallocation of payoffs and threats between *all the players* in the game. This is the different view on reallocation, used in cooperative games with transferable utility, where transfer takes place only inside a coalition and due to strategic actions of the players of this coalition.

4.3 Example of a game for five players to form a coalition partition

Examples in this section present

1. a simple game for non-cooperative coalition formation,

2. a simple self-financed enforcement mechanism with a social planner.

4.3.1 Preliminary comments and notation

We consider the coalition formation in a broad sense, as any collection of players or economic agents. Our main interest is to demonstrate how a coalition partition can be constructed with a credible self-financed enforcement.

Resulting stability of each coalition is based on individual strategies of all the players in the game. Stability of any coalition is understood in the sense of the strong Nash equilibrium, i.e. no one player wants to deviate individually or in a group.

The stability of the coalition implies that the process of formation has two-stages: the formation of a pre-coalition which identifies the actors which have an intention to be in the coalition and the following realization of strategies of these players inside the pre-coalition.

For the moment we assume that on the first stage, when players "enter into the coalition", actually a "pre-coalition" is formed. On the second stage it may transform into a stable coalition or may disintegrate. We assume that between the stages players do not have any communication and can not observe actions of each other.

The result depends on strategies of each player inside the pre-coalition.

Actions of the players which support transformation of the pre-coalition into the stable coalition are called cooperative strategies. Cooperative means that a player wants to stay in the coalition. Actions of players which *do not* transform the pre-coalition into the coalition are called opportunistic strategies.

The constructed game is the game with complete information. For simplify of analysis we can construct a one-period game, where each player has two strategies for each coalition: a cooperative strategy to support transformation of the pre-coalition into the stable coalition and an opportunistic strategy to prevent this transformation. A cooperative strategy is labeled as "good", an opportunistic strategy is labeled as "bad".

Let n_i be a coalition chosen by a player i. For each coalition where the player i may participate he has four payoffs.[2]

- α^i is the payoff for a player i if he (she) chooses a singleton coalition $n_i = \{i\}$. Realization of this payoff does not depend on strategies of all other players.

- ${\beta^i}_{n_i}$ is a coalition specific *regret* payoff of i if the coalition n_i is not formed. This case is realized in two cases: if n_i is not formed as a pre-coalition or if n_i is formed as a pre-coalition, but then disintegrates due to opportunistic actions of some members. In both cases the player i receives a singleton coalition, $\{i\}$.

[2] We will skip the notation of a coalition, if the coalition is clear from the context.

- $\delta^i_{n_i}$ is a *reservation payoff* of the player i if n_i is formed as a "pre-coalition" but then the player i ruins it by the opportunistic strategy. The player i receives a singleton coalition, $\{i\}$.

- $\gamma^i_{n_i}$ is a payoff if the coalition n_i is stable.

For a trivial coalition we assume that $\alpha^i = \beta^i_{\{i\}} = \gamma^i_{\{i\}} = \delta^i_{\{i\}}$.

In simple words the example is based on the simple idea - entering and supporting of stability of an coalition is based on Prisoner's Dilemmas for each player. One Dilemma - one to enter the pre-coalition or not and another - to be opportunistic or not. We assume that a coalition is formed from a unanimous agreement of the players to form it and not to be opportunistic.

4.4 Examples of coalition partition with self-financed enforcement.

Let there are 5 players, a set of the players is $N = \{A, B, C, D, E\}$. Each player $i \in \{A, B, C, D, E\}$ has two strategies for any coalition n_i he may participate. For example, for the player A the strategy $s^A_g(\{A, B, D\})$ means that the player A wants to be in the coalition with the players A, B, D and is interested in stability of this coalition. In the same way the strategy $s^A_b(\{A, B, D\})$ is the strategy of the player A to ruin the *pre-coalition* $\{A, B, D\}$ by choosing an opportunistic behavior.

In the same way we will use the notations for strategies: $s^i_g(n_i)$ and $s^i_b(n_i)$ for a player i, where n_i is a coalition, $i \in n_i$. We will write $s^i_g(n_i)$ for a strategy of i, when i is interested in stability of the coalition $n_i, i \in n_i$ and $s^i_b(n_i)$, when i is not interested in stability of the stability of n_i, $s^i_b(n_i)$ is the opportunistic strategy of i in the coalition n_i

We will use two notations to label a coalition, where a player i may participate.

1. n_i is a coalition of i inside a partition, assigned for him by the social planner,

2. m_i is any other coalition of i, different from n_i: $m_i \neq n_i$.

4.4.1 Construction of the game

Let the player $i \in \{A, B, C, D, E\}$ has utility function defined as

$$u^i = \begin{cases} \alpha^i & s^i = \{i\} \\ \beta^i_{n_i} & \exists j \in n_i \colon n_j \neq n_i, \text{ the regret payoff} \\ & \text{in the coalition } n_i \text{ there is a player } j \text{ who chooses coalition } n_j \neq n_i \\ \gamma^i_{n_i} & \forall j \in n_i \colon s^j_g(n_i), \\ & \text{every player } j \text{ in } n_i \text{ chooses } n_i \text{ and supports stability of } n_i \\ \delta^i_{n_i} & \begin{cases} \forall r \in n_i \colon n_r = n_i \\ s^i = s^i_b(n_j) \end{cases} \\ & \text{the coalition } n_i \text{ may be formed but } i \text{ is opportunistic} \\ \beta^i_{n_i} & \begin{cases} \forall j \in n_i \colon n_j = n_i \\ \exists r \in n_i \colon s^r = s^r_b(n_i) \end{cases} \\ & \text{the regret payoff} \\ & \text{the coalition } n_i \text{ can not be formed as some player } r \text{ is opportunistic} \end{cases}$$

with numerical payoffs in the Table 4.1.

In the definition of the utility we do not discriminate between the two cases why a coalition is not formed. For the construction of the example it is not significant. But this can be the direction for further investigation for non-cooperative coalition formation for games in extensive form.

Let $P = \{\{A, E\}, \{B, D, E\}\}$ be a socially desirable coalition partition which comes from the social planner. A general coalition for the a $i \in N$ is $n_i, n_i \in P$.

We will define coalition formation process following rules of non-cooperative games: from strategies profile to a coalition partition. Thus payoffs in a coalition depend on strategies of all players in the game, not only of those who are inside the coalition.

For example, we can describe all cases of coalition partition for the player A. Case 1. The player A chooses

$$\text{the strategy profile } (s^A(\{A\}), \ldots) \text{ what results in the coalition partition } (\{A\}, \ldots)$$

with the allocation of payoffs $(\alpha^A, \ldots)$

Case 2. The player A chooses

$$\text{the strategy profile } (s_g^A(\{A, E\}), \ldots, s^E) \colon \forall s^E \neq s_g^E(\{A, E\})$$

results in the coalition partition

$$(\{A\}, \ldots)$$

with the allocation of payoffs

$$(\beta^A(\{A, E\}), \ldots)$$

This case includes two outcomes - when the player E does not want to form the pre-coalition $\{A, E\}$ or chooses the opportunistic strategy inside this pre-coalition. In the last sub-case coalition partition will be $(\{A\}, \ldots, \{E\})$ and allocation of payoffs will be $(\beta^A(\{A, E\}), \ldots, \delta^E(\{A, E\}$. In the first sub-case we do not have enough information about strategies of E to make some conclusion about payoffs and coalitions of E.

Case 3. The player A chooses

$$\text{the strategy profile } (s_g^A(\{A,E\}),\ldots,s_g^E(\{A,E\}))$$

results in the coalition partition

$$(\{A,E\},\ldots)$$

with the allocation of payoffs

$$(\gamma^A(\{A,E\}),\ldots,\gamma^E(\{A,E\}))$$

Case 4. The player A chooses

$$\text{the strategy profile } (s_b^A(\{A,E\}),\ldots,s_g^E(\{A,E\}))$$

results in the coalition partition

$$(\{A\},\ldots,\{E\})$$

with the allocation of payoffs

$$(\delta^A(\{A,E\}),\ldots,\beta^E(\{A,E\}))$$

We can see that the coalition $\{A\}$ can come from different strategy profiles.

Table 4.1 presents payoffs for the players in the game. The payoffs in the coalition partition P are separated from other payoffs in other coalitions.

From the Table 4.1 we can see that only player A is directly interested in the partition P. All other players are interested in opportunistic behavior if the coalition partition P takes place. The coalition partition P can not be implemented through

Player	Coalition	γ	β	δ
Player A, $\alpha^A = 1$	$\{A, E\}$	10	1	5
	$m_A \neq \{A, E\}$	1	1	1
Player E, $\alpha^E = 1$	$\{A, E\}$	3	2	3.5
	$m_E \neq \{A, E\}$	3	3	1
Player $i \in \{B, C, D\}$, $\alpha^i = 1$	$\{B, C, D\}$	2.5	2	3
	$m_E \neq \{B, C, D\}$	3.6	2	3

Table 4.1: Payoffs in the game

any pre-play agreements and requires the enforcement. Thus P can appear only as an *induced* partition.

4.4.2 Construction of the enforcement

There is a social planner, who can collect donations and transform them into transfers and if necessary is able to implement punishments.

The enforcement is self-financed, i.e. - a reallocation of payoffs between *all* players in the game with the properties:

1. There is one player who voluntarily donates a resource to the central planner for enforcement. The donation does not make the player to reject the coalition assigned to him.

2. It induces each player to accept a coalition exogenously assigned to him ("a carrot")

3. It threats and punishes a player, who decide strategically deviate from the assigned coalition ("a stick").

4. The enforcement is credible.

5. The central planner has enough resource to react to any individual or a group either following his instructions or deviating from them.

All the conditions to motivate players to accept the *induced* coalition partition P are based on incentive compatibility of Hurwitz ([212]).

The player A is the only donor of resources to form P. Let $x^A = 4$ be the size of his donation. The player A donates 4 units of his utility in order to support the partition P chosen by the central planner.[3] Then there is:

$$\gamma^A(\{A,E\}) - x^A = 10 - 4 > \max\{\alpha^A, \delta^A(\{A,E\}), \beta^A(\{A,E\})\} = \max\{1,5,1\} = 5$$

The donation of A does not distort his incentive to be in the coalition $\{A,E\} \in P$ [4]

The gain of A from the coalition $\{A,E\}$ is still bigger after the donation than his gain from any other coalition:

$$\gamma^A(\{A,E\}) - x^A = 10 - 4 = 6$$
$$> \max\{\gamma^A(m_A), \beta^A(m_A), \delta^A(m_A)\} = 1 \quad (4.1)$$

for any coalition $m_A \neq \{A,E\}$.

Now let's discuss structure of the distorted payoffs for other players.

Let $y^i(n_i)$ be a transfer to the player $i \in \{B,C,D,E\}$ to make him accept the coalition $n_i \in P, i \in n_i$. This transfer operate like "a carrot" to motivate the player i to accept the *induced* coalition n_i assigned for him in the partition P.

Let $z^i(m_i)$ be a punishment for the player i if he chooses a coalition $m_i \colon m_i \notin P^*$.

Let

$$y^E := z^E := y^B := z^B := y^C := z^C := y^D := z^D = 1$$

[3] All the numbers are taken from the Table 4.1.

[4] At this stage of research we ignore possible free-riding between donors, possible resistance of players against punishments or competition between different social planners.

be values of transfers (which can be also called as promotions or benefits) and punishments, equal for the players B, C, D, E. We can describe motivations of the player E to accept P as two inequalities. In terms of the transfers for him:

$$\gamma^E(\{A, E\}) + y^E = 3 + 1 = 4 > \max\{\alpha^E, \delta^E(\{A, E\}), \beta^E(\{A, E\})\} = \max\{3.5, 2\} = 3.5 \quad (4.2)$$

and in terms of the punishments for him

$$\gamma^E(\{A, E\}) = 3 > \max\{\gamma^E(m_E), \delta^E(m_E), \beta^E(m_E)\} - z^E = \max\{1, 3, 3\} - 1 = 2, \quad (4.3)$$

$\forall m_E \neq \{A, E\}$

In words this means - for a player E the payoff in the induced coalition $\{A, E\}$ is greater than a payoff from his deviation to any coalition $m_E \neq \{A, E\}$.

The threat and the punishment construct local dominations in strategies for the induced coalition $\{A, E\}$ in comparison to other coalitions for the player E.

Distortion of payoffs for all other players is based on the same idea. Distorted payoffs for a player $i \in \{B, C, D\}$ satisfy the conditions. In terms of the transfers:

$$\gamma^i(\{B, C, D\}) + y^i = 2.5 + 1 = 3.5 > \max\{\alpha^i, \delta^i(\{B, C, D\}), \beta^i(\{B, C, D\})\} = \max\{1, 3, 2\} = 3 \quad (4.4)$$

and in terms of the punishments

$$\gamma^i(m_i) = 3 > \max\{\alpha^i, \beta^i(m_i), \gamma^i(m_i), \delta^i(m_i), \} - z^E = \max\{1, 3.6 - 1, 3 - 1, 2 - 1\} = 2.6, \quad (4.5)$$

$\forall m_i \neq \{B, C, D\}$.

So the enforcement vectors $x = (x^1, \ldots, x^5)$, $y = (y^1, \ldots, y^5)$ and $z = (z^1, \ldots, z^5)$ with the structure

$$\begin{cases} x = (4, 0, 0, 0, 0) \\ y = (0, 1, 1, 1, 1) \\ z = (0, 1, 1, 1, 1) \end{cases}$$

support the coalition partition P. The induced coalition partition has two induced cores $\{A, E\}$ and $\{B, C, D\}$ and all the properties of the induced strong Nash equilibrium: neither a single player nor a group of players is motivated to deviate.

4.4.3 Credibility of the enforcement

To summarize construction of the enforcement. It contains three parts - voluntarily donation, transfers and punisments/threats. Donation supplies resources for the enforcement. Transfer operates as "a carrot". The threat operates as "a stick" for the player who deviates. If a player does deviate, then the threat becomes the punishment for him. The punishment supplies credibility of losses for deviations. Credibility is supported by the self-financing of the enforcement.

We can say that a player receives either "a carrot" or "a stick', but not both.

For simplicity of the example we assumed one-to-one mapping between expenditures on punishment z^i and a loss in utility for the player i, if he is exposed for the punishment.

The difference with transferable utility in cooperative games is that in the presented model resources can be reallocated between all the players in the game, not only inside a coalition. The difference from reallocations in market games of Shapley and Shubik, ([337]), is that there is no market and more than one coalition is possible.

In terms of a non-cooperative game the enforcement changes *opportunity costs* between entering the induced coalition and deviation. This makes players to respond by choices of strategies.

Credibility of the self-financed enforcement demands that the social planner has enough resources to react to any action of a player or a group of players to deviate or to participate in the partition P.

The enforcement in the example is credible as the total donations is enough to implement any transfer or a punishment for any player or a group of players simultaneously:

$$x^A = 4 \geq \sum_{i=B,C,D,E} \max\{y^i, z^i\} = 4$$

.

The left-hand side is the total donation (from the player A), the right-hand side is the sum of all potential transfers or potential punishments.

We do not study the important questions appearing in the context of the example - an optimal enforcement, allocation of transfers, optimal punishments, optimal donations/taxes or technologies of enforcement etc. We just demonstrate that it is possible to construct a self-financed enforcement mechanism which supports existence of an induced coalition partition with two induced cores, which can not appear from pre-play agreements of the players.

from the construction of the mechanism we can see that neither player individually or in a group is motivated to deviate from P. The reason is the local strict dominance of distorted payoffs. Thus the enforced coalition partition $P = \{\{A, E\}, \{B, C, D\}\}$ has the properties of the strong Nash equilibrium.

In this example the credibility of the pre-play agreements is substituted by the credibility of the self-financed enforcement. The induced allocation P in the example can not be supported only by agreements: we can see from the Table 4.1 that all agreements to accept for P are cheap-talks. But the constructed enforcement allows to implement P, which has two induced cores.

4.5 The model

This section presents the non-cooperative model for construction of coalition partitions. The model is a finite game.

4.5.1 Individual actions and payoffs of players

There are N players, $\#N \geq 2$ with a general element i. $\tilde{N}^i$ is a set of all coalitions i may want to be in, $\tilde{N}^i = \{n_i \colon i \in n_i, n_i \subset \tilde{N}^i\}$. Every player can make two actions inside a coalition n_i- either to "be good" or to "be bad" - $\{g, b\}$.

To "be good" means to support stability of the pre-coalition and transform it into the coalition. To "be bad" means to take an opportunistic strategy and to destroy the pre-coalition, if it is formed.

A strategy set of i is

$$S^i = \tilde{N}^i \times \{good, bad\}$$

with a general element $s^i \in S^i$.

The strategy $s_g^i(n_i)$ means that the player i wants to be in the stable coalition $n_i, i \in n_i$. The strategy $s_b^i(n_i)$ means that the player i wants to take his private benefit from the coalition $n_i, i \in n_i$ and then to leave it ("take the cream" strategy or an opportunistic strategy).

Let $S^{-i} = \times_{r \neq i} S^r$ with a general element s^{-i}.

Utility function of i is $u^i(s^i, s^{-i}) \colon S^i \times S^{-i} \mapsto R$ such that

$$
u^i(s^i, s^{-i}) = \begin{cases}
\alpha^i & \text{if } s^i \in \{i\} \times \{g, b\} \\
\beta^i_{n_i} & \text{if } \exists j \in n_i \colon n_j \neq n_i, \text{ regret payoff from } n_i \\
 & \exists \text{ a player in } n_i \text{ who wants a coalition } n_j \neq n_i \\
\gamma^i_{n_i} & \text{if } \forall j \in n_i \colon s^j = n_i \times \{good\} \\
 & \text{every player in } n_i \text{ is interested to make the coalition } n_i \\
\delta^i_{n_i} & \text{if } \begin{cases} \forall j \in n_i \colon n_j = n_i \\ s^i = n_i \times \{bad\} \end{cases} \quad \text{reservation payoff from } n_i \\
 & \text{the player } i \text{ chooses an opportunistic strategy } s^r = n_i \times \{bad\} \\
\beta^i_{n_i} & \text{if } \begin{cases} \forall j \in n_i \colon n_j = n_i \\ \exists r \in n_i \colon s^r = n_i \times \{bad\} \end{cases} \\
 & \text{the coalition } n_i \text{ is formed as a pre-coalition but} \\
 & \text{some } r \text{ chooses the opportunistic strategy } s^r = n_r \times \{bad\}
\end{cases}
$$

We assign the same regret values and do not discriminate cases when a coalition is not formed due to opportunistic behavior of some player or when a pre-coalition was not formed.

The game in normal form is $(N, (S^i, u^i)_{i \in N})$.

4.5.2 Enforcement

Enforcement serves to support stability of an exogenous coalition partition P. It comes from a central planner. Enforcement requires resources, which can come only from all players in the game (no credits from outside, etc). This is different from the notion of transferable payoffs within a coalition.

We assume that enforcement is self-financed: it can exist only due to redistribution of payoffs among all the players in the game. This is the simplest possible assumptions in order to organize enforcement not as a free activity.

Let P be a desirable coalition partition of the set of players N, which comes from a social planner.

The source of resource is a player i who chooses his coalition n_i inside P. The utility of this player must satisfy the condition:

$$u^i(s^i_{n_i} \times \{good\}, s^{-i}) - x^i > u^i(s^i, s^{-i})$$

for any $s^i \neq s^i_{n_i} \times \{good\}$. This player strictly prefers the coalition n_i in P in comparison to any other coalition.

Let $h^i_{n_i}(x)$ be a transfer to the player i when $i \in n_i$, $n_i \in P$, where $h^i(x)$ is some technology of this reallocation, a continuous function.

Thus the distorted payoffs for i inside the coalition $n_i \in P$ is

$$\gamma^i(n_i) - x^i + h^i_{n_i}(x), \forall n_i \in P$$

.

Let $k^i_{m_i}(x)$ be a punishment to the player i when he deviates to the coalition $m_i, m_i \neq n_i$, where $k^i(x)$ is some technology of the punishment, a continuous function. The distorted (enforced by the punishment) payoffs for any coalition m_i which is not in P must satisfy the conditions , $m_i \neq n_i$:

- $\alpha^i - k^i_{m_i}(x) < \gamma^i_{n_i} - x^i + h^i_{n_i}(x)$,
- $\beta^i_{m_i} - k^i_{m_i}(x) < \gamma^i_{n_i} - x^i + h^i_{n_i}(x)$
- $\gamma^i_{m_i} - k^i_{m_i}(x) < \gamma^i_{n_i} - x^i + h^i_{n_i}(x)$
- $\delta^i_{m_i} - k^i_{m_i}(x) < \gamma^i_{n_i} - x^i + h^i_{n_i}(x)$.

The difference between punishments and transfers is that a transfer is what a player i can obtain only if he is in the recommended coalition n_i in P. A punishment is what a player will be imposed only if he chooses any other coalition. Thus a player receives either a transfer or a punishment. The punishment is realized only if the player deviates. In the general case the punishment depends on a coalition m_i to which the player deviates instead of the assigned coalition n_i.

An enforcement must be credible - the central planner must have enough resources to be able to react to any strategy(s) of any player individually or in group. We assume that the central planner may have cross-financing of his operations between players.

Let $x = \sum_{i \in N} x^i$ is the total resource available for the enforcement from all players. Let $(h^i_{n_i}(x))^{-1}$ be a volume of resource used to make the transfer $h^i_{n_i}(x)$ for the player i. Let $(k^i_{m_i}(x))^{-1}$ be a volume of resource used to make the punishment $k^i_{m_i}(x)$ for the player i. The condition for the self-financed enforcement is:

$$x \geq \sum_{i \in N} (\max\{(h^i_{n_i})^{-1}, \max_{\forall m_i}((k^i_{n_i})^{-1}\})$$

4.5.3 Game and equilibrium

Let P be a socially desirable partition. Let $\Gamma = ((N, (S^i, u^i)_{i \in N}), P, (k^i, h^i)_{i \in N})$ be a game with an exogenous coalition partition P and enforcement where

1. $(N, (S^i, u^i)_{i \in N})$ - a game in a normal form,
2. P - an externally given coalition partition,
3. $(k^i)_{i \in N}$ - a vector of punishment technologies for every player in N,
4. $(h^i)_{i \in N}$ - a vector of transfer technologies for every player in N

In the general case technologies of punishments and transfers depend on the coalition partition P.

We can think about Γ as a simultaneous game with complete information of the players *and* the central planner or as three step game of the central planner and the players.

1. Step 1. The central planner announces P.

2. Step 2. Players make actions $(s^i)_{i\in N}$.

3. Step 3. Central planner implements his enforcement plan.

Induced Nash equilibrium with enforcement for the induced coalition partition P is

$$(P, (\hat{s}^{i*})_{i\in N}, (x^{i*}, h^{i*}, k^{i*})_{i\in N})$$

where

1. P - an externally given coalition partition which can be reached with a strategy profile $(\hat{s}^{i*})_{i\in N}$

2. $(x^{i*})_{i\in N}$ - an equilibrium vector of donations,

3. $(h^{i*}(\sum_{i\in N} x^{i*}))_{i\in N}$ - an equilibrium vector of reallocations,

4. $(k^{i*}(\sum_{i\in N} x^{i*}))_{i\in N}$ - an equilibrium vector of threats (punishments),

such that for every player $i \in N$ and any $s^i \neq \hat{s}^i$ there is

$$u^i(\hat{s}^{i*}, s^{-i*}) - x^{i*}_{n_i} + h^i_{n_i}(\sum_{i\in N} x^{i*}) \geq u^i(s^i, s^{-i*}) - k^i_{m_i}(\sum_{i\in N} x^{i*})$$

,

$$\sum_{i\in N} x^{i*} \geq \sum_{i\in N}(\max\{h^i_{n_i}(\sum_{i\in N} x^{i*}), \max_{\forall m_i} k^i_{m_i}(\sum_{i\in N} x^{i*})\})$$

And there is a player i who can supply resources for the enforcement:

$$u^i(\hat{s}^{i*}, s^{-i*}) - x^{i*}_{n_i} \geq u^i(s^i, s^{-i*})$$

without the loss of the interest in $n_i \in P$ for any $s^i \neq \hat{s}^{i*}$.

Restrictions in the definition of the induced Nash equilibrium are organized in the way that if the equilibrium exists then the players do not have motivation to deviate either individually or in group. In other words transfers and punishments construct strict dominance of strategies which generate the partition P. Thus the induced Nash equilibrium with enforcement is the induced strong Nash equilibrium.

Further research will make the restrictions more flexible.

4.6 Relation to mechanism design

The mechanism design view on the problem consists of

1. A finite set of player N.
2. An outcome space, Ω a set of all coalition partitions for the set of players.
3. An environment, $(S^i, u^i)_{i \in N}$ a set of strategies and payoffs of the players.
4. A social choice rule, $\psi(K)\colon (\times_{i \in N} S^i) \times R^{3N}_{+} \mapsto \Omega$, which transforms an environment into set of outcomes given the enforcement technology K

Investigation of the relation between mechanism design and non-cooperative formation of coalitions with enforcement will be presented in the next papers paper.

4.7 Conclusion

The presented paper studies the question - how to construct a coalition partition from a non-cooperative game using a self-financed enforcement mechanism. We present an example for this construction.

The essence of the example is that if a coalition partition can not be formed from voluntarily participation of the players then it still can be formed as an induced partition if there is enough resources to make the enforcement self financed.

The enforcement is based on two technologies - technologies of the transfer of payoffs and of the punishments. The punishment is organized as a confiscation of utility. Coalitions which can be supported with the enforcement are called the induced coalitions. The example has two induced coalitions with unequal number of players,

The paper presents the construction of a non-cooperative game for coalition formation with an enforcement and an equilibrium concept for this game. Formation of induced coalitions is based on the tools of non-cooperative game theory.

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Estratto per riassunto della tesi di dottorato

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Titlo della tesi: Essays on Trade and Cooperation

Abstract: In this dissertation, we study two questions of strategic behavior.

One is related to the trade in strategic market games. The paper studies strategic market games with wash sales. This class of games posses best response correspondences that in turn generate non-uniqueness of pure strategy equilibria. We introduce a mixed strategy equilibrium that resolves the aforementioned indeterminacy, therefore, results into a unique equilibrium. Finally we provide an example that illustrates our equilibrium concept.

Another paper offers non-cooperative mechanism for coalition formation. It includes a special non-cooperative game and a self-financed enforcement. Enforcement is performed by an external central planner. The enforcement operates as a reallocation between all players in the game. In order to support stability of the induced coalition partition the central planner distorts payoffs in an equilibrium ("a carrot") and outside the equilibrium ("a stick"). Induced cores of the distorted game can not exist without the enforcement. If the enforcement exists, then the corresponding induced equilibrium has the properties of a strong Nash equilibrium. Credibility of the enforcement is supported by the balance of used resources from one side and reallocations and punishments from another.

Estratto: In questa dissertazione, si studiano due domande di comportamento strategico.

Un saggion e' legato al commercio di giochi strategici di mercato. Il documento studia giochi strategici di mercato con un fatturato di lavaggio. Questa classe di giochi possiede corrispondenze di risposta migliori che a loro volta generano non-unicitA' degli equilibri di strategia pura. Introduciamo un equilibrio misto DI strategia che risolve l'indeterminatezza di cui sopra, pertanto, i risultati in un equilibrio unico. Infine, forniamo un esempio che illustra il nostro concetto di equilibrio. . Un altro saggio offre un meccanismo non cooperativo per la formazione della coalizione. Esso comprende uno speciale gioco non cooperativo e un'applicazione autofinanziata . L'applicazione viene eseguita da un pianificatore esterno centrale. L'applicazione funziona come una ridistribuzione tra tutti gli attori in gioco. Al fine di sostenere la stabilit? della coalizione di partizione indotta, il pianificatore centrale distorce i profitti in un equilibrio ("a carota") e fuori l'equilibrio ("a bastone"). Nuclei indottI della distorsione del gioco non pu? esistere senza l'applicazione. Se l'applicazione esiste, allora l'equilibrio corrispondente indottO ha le propriet? di un equilibrio forte Nash. La credibilite della esecuzione supportata dal bilanciamento delle risorse utilizzate da un lato e riassegnazioni e punizioni da un altro.