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Dynamic Factor Model with Non-Linearities: Application to the Business Cycle Analysis

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Summary

This thesis is dedicated to the study of a particular class of non-linear Dynamic Factor Models, the Dynamic Factor Models with Markov Switching (MS-DFM). Combining the features of the Dynamic Factor model and the Markov Switching model, i.e. the ability to aggregate massive amounts of information and to track recurring processes, this framework has proved to be a very useful and convenient instrument in many applications, the most important of them being the analysis of business cycles.

In order to monitor the health of an economy and to evaluate policy results, the knowledge of the current state of the business cycle is essential. However, it is not easy to determine since there is no commonly accepted dataset and method to identify turning points, and the official institutions announce a new turning point, in countries where such practice exists, with a structural delay of several months. The MS-DFM is able to resolve these issues by providing estimates of the current state of the economy in a timely, transparent and replicable manner on the basis of the common component of macroeconomic indicators characterizing the real sector.

The thesis contributes to the vast literature in this area in three directions. In Chapter 3, I compare the two popular estimation techniques of the MS-DFM, the one-step and the two-step methods, and apply them to the French data to obtain the business cycle turning point chronology. In Chapter 4, on the basis of Monte Carlo simulations, I study the consistency of the estimators of the preferred technique - the two-step estimation method, and analyze their behavior in small samples. In Chapter 5, I extend the MS-DFM and suggest the Dynamical Influence MS-DFM, which allows to evaluate the contribution of the financial sector to the dynamics of the business cycle and vice versa, taking into consideration that the interaction between them can be dynamic.

Keywords: Markov-Switching, Dynamic Factor Model, business cycle, financial cycle, turning point analysis, two-step method, consistency, small-sample performance, Monte Carlo simulations, dynamical interaction, systemic risk.

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To my brother Maxim

Chapter 1

General Introduction

1.1 General Introduction

This thesis is dedicated to a particular class of non-linear Dynamic Factor Models, the Dynamic Factor Models with Markov Switching (MS-DFM). Combining the features of the Dynamic Factor model and the Markov Switching model, i.e. the ability to aggregate massive amounts of information and to track fluctuating processes, this framework has proved to be a very useful and convenient instrument in many applications. Among them: tracking of labor productivity (Dolega (2007)), modeling the joint dynamics of yield curve and GDP (Chauvet and Senyuz (2012)), examination of fluctuations in the employment rates (Juhn et al. (2002)) and many others. However, the main application of the MS-DFM is the analysis of the business cycle turning points (see, for example, Kim and Yoo (1995), Darné and Ferrara (2011), Camacho et al. (2012), Chauvet and Yu (2006), Wang et al. (2009)). For this reason, business cycle analysis is in the center of this study.

To monitor the health of an economy and to evaluate policy results, the knowledge of the current state of the business cycle is essential. However, it is not easy to determine. The first problem comes from the fact that there is a structural delay in the announcement of the current state by the official institutions (the official dating, if published, appears with a lag of several months). Secondly, it is not obvious which dataset and method should be used to determine turning points. Several procedures exist, and the results they provide do not completely coincide. Finally, it is important to identify and to quantify the relation between the financial sector on the real business cycle, which proved to be particularly important during the 2008 crisis and after. Markov-Switching Dynamic Factor Model (MS-DFM), introduced by Diebold and Rudebusch (1996) as a multivariate extension of the Markov-Switching model of Hamilton (1989), has become a convenient instrument for identification of turning points of an unobservable common process governing many observable series.

In order to better understand why MS-DFM is particularly well suited for the business cycle analysis, let us consider this phenomenon in greater detail.

1.1.1 Business Cycle

1.1.1.1 History

The notion of the business (or economic) cycle appeared for the first time in "Nouveaux Principles d'économie politique" by Simonde de Sismondi (1819), who was the first to suggest the notion of periodic crises. According to Sismondi, the main reasons for this regularity are underconsumption and overproduction, i.e. *internal* reasons. This idea was in contrast to the widespread understanding of that time that crises have *external* reasons, such as wars or poor harvest. Robert Owen expressed similar views on the subject in his "Report to the Committee of the Association for the Relief of the Manufacturing Poor" (see Owen (1817)). Later, Charles Dunoyer adapted these ideas into his theory of alternating cycles introducing the concept of cycling from one crisis to another (see Benkemoune (2009)). Juglar (1862) was the first to identify the fluctuations of economic activity of 7 to 11 years long. Afterwards, Schumpeter (1939) defined four stages of the business cycle, as we know them now: expansion, crisis, recession, recovery.

It is interesting to note that the term "cycle" is misleading, since cyclicality implies fixed periodicity, which is not true for the economic crises. The term "fluctuations" or "oscillation" would be more precise, but the historical tradition was formed using shorter and better-sounding "cycles".

The periodicity of crises defines the type of the business cycle. Joseph Schumpeter and his contemporaries distinguished four types of cycles: the Kitchin inventory cycle (3-5 years), Juglar fixed-investment cycle (7-10 years), Kuznets infrastructural investment cycles (15-25 years) and finally, Kondratiev long technological cycles (45-60 years). However, it's the second type of cycles that got the major attention of the researchers, as the other were either contested (as in the case of Kondratiev cycle), neglected (as in the case of Kitchin cycle) or not in the main focus (in case of Kuznets cycle).

1.1.1.2 Definition

The financial press often refers to the movement of the GDP while discussing the business cycle. Although widely accepted as the major indicator of the economic activity, it appears to be not comprehensive enough to describe the business cycle. Indeed, if the industrial production has taken off while the unemployment is not decreasing but, on the opposite, increasing (as was the case after the Great Recession), should we consider this situation as beginning of an expansion? Probably, not. If the expansion started in a few sectors, but the rest of the economy is still stagnating, can this be qualified as the beginning of expansion? Probably neither. For this reason, the now standard definition of business cycles provided by Burns and Mitchell (1946) states the following: *"Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; in duration, business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar characteristics with amplitudes approximating their own"*.

There are several important points that we would like to underline in this definition. First, it does not depend on the underlying mechanism of the business cycles. This is an important remark because the MS-DFM can identify recessions, but can not classify them by cause. Secondly, it stresses the importance of synchronicity: a business cycle is something common to many economic activities.

Another important point to mention is that the definition above can be applied not only to raw levels economic indicators, but to transformed series as well. This defines another typology of business cycles: business cycles (fluctuations in levels), growth cycles (fluctuations around a long-term trend) and growth rate cycles (fluctuation of growth rates). Theoretically, there is no preference of one type to another. In empirical works, the choice of the type of cycle depends on the research problem. Although quite often used interchangeably under a generic term "business cycles", these three cycles can be slightly different, and it's important to keep in mind this difference especially while comparing different turning point datings. In the context of this thesis, we will focus on growth rate cycles for two reasons: first, the MS-DFM requires stationary data, so it is not designed to analyze pure business cycles; secondly, since the contribution of all chapters is mainly methodological, we will try to eliminate the impact of series pre-treatment procedures, such as de-trending for growth cycles extraction, as much as possible.

1.1.1.3 Existing explanations

Although this thesis does not have an objective to explain the mechanism of the business cycles, it seems important to present a brief overview of the existing explanations for business cycles.

Nowadays, there are two major groups of approaches to the explanation of the sources of business cycle: mainstream (Keynesian and Real Business Cycle theory) and alternative (heterodox economics: credit-based theories such as financial instability hypothesis and debt deflation).

According to Keynesian approach, the fluctuations of aggregate demand lead to the equilibrium below or above full employment. Due to Keynesian multiplicator and accelerator, the responses of output on initial shocks in this case are cyclical. Real Business cycle theory implies that the fluctuations in the economy happen due to the external shock (in contrast to the Keynesian approach), such as technological shock. The difference in the approach to the cause of the fluctuations - endogenous or exogenous - is important for policy-makers. If the cycles can not be generated by endogenous shocks, as suggested by the RBC theory, then any countercyclical policy is not efficient, and the policy measures should better focus on improving the conditions for the long-term growth instead of mitigating medium-term fluctuations of economic activity.

Credit-based theories suggest that the business cycle is dependent on the credit cycle, so that the expansion or contraction of credit causes expansion or recession in the business cycle, respectively. One of the mechanisms of this interaction, the debt-deflation theory, was suggested by Fischer (1933). He states that, when an economy faces deflation, this leads to the overall accumulation of debt making economic agents default on their credits. The assets of banks is contracting since the value of collateral is falling, leading to massive bank insolvencies. This prevents banks from lending which impacts the consumption negatively. Another mechanism, Financial instability hypothesis by Minsky (1992), suggests that the cycles are induced by misestimation of the ability of the firms to repay the debt. During the expansion, the banks are willing to issue loans to firms considering that they will easily pay them back given the favourable economic conditions. At a certain point, the firms get over-indebted, reduce their investment and production, and the recession starts. Given the high interest to the relation between financial and business cycles arisen after the 2008 crisis, we study this question in Chapter 5.

1.1.1.4 Dating of the turning points

Whatever the type of the business cycle we are referring to and whatever its underlying mechanism, as we have mentioned in the beginning, it is important to be able to determine the current phase of the cycle, to *nowcast* it, i.e. to obtain the estimate of the current state for the current point.

Different techniques can be used for this task. The most widely used non-parametric method is the Bry-Boschan turning point algorithm and its counterpart for quarterly data BBQ algorithm by Harding and Pagan (2002). The Bry-Boschan routine (see Bry and Boschan (1971) for details) identifies the peak and the trough of a cycle as local maximum and minimum under a set of constraints (on the duration of the cycle, minimal duration of each phase, etc). Practical and clear, unfortunately it has a disadvantage that most of non-parametric methods have: it is quite sensitive to censoring parameters. A moderate change in the parameter controlling, for example, the minimal duration of each phase may completely change the sequence of turning points. Subsequently, the Bry-Boschan requires a lot of fine-tuning, where the expert's hypothesis on the timing of the major turning points plays a crucial role.

Popular parametric methods include binomial regressions (standard or dynamic) and Markov-Switching models. The two major advantages of the parametric methods are the following: 1) the estimate of the current phase of the cycle has a form of probability thus providing more information 2) it is possible to make forecasts of future states. The binomial (usually probit) regressions are estimated to find the relation between the existing reference dating (such as of NBER, for example) and leading or coincident indicators or factors (see, for example, Estrella and Mishkin (1996), Chauvet and Potter (2010) and Fossati (2015)). In contrast, Markov-Switching model does not require a reference dating, as it allows to make inference on from the unobserved cycle using observable economic indicators, which is an important advantage for the analysis of business cycles in developing economies where a reference dating may be nonexistent. Although both models produce probabilities of recession, their output can be quite different qualitatively. Probit models generate high probabilities during recessions, but are rather volatile. Markov-Switching models, on the opposite, give a clear and stable signal during recessions, but tend to identify turning points with a small delay (which we observe in

Chapter 3).[1] The final choice of the model thus depends on the preference for false positives and availability of reference dating. The absence of the latter in case of many countries is the key reason to study Markov-switching models in this dissertation.

In case of the analysis of the growth cycle (i.e. in terms of deviation from the trend), the task of dating is complicated by the preceding task of identification of the growth cycle. Similarly to the case of seasonal adjustment, in order to extract the cycle one has to solve a problem of decomposition of series into trend, cycle and noise components. The main difficulty of the task lies in the fact that there is no formal definition for each of them, therefore there exist numerous ways to perform the decomposition. In practice, most of the other existing methods of business cycle extraction can be divided into two groups: frequency extraction and signal extraction. Since we focus on growth rate cycles rather than growth cycles, we do not discuss each of these methods in detail. It is important to underline, however, that none of the above-mentioned methods can be proclaimed as best in any situation, the choice depends greatly on the underlying process. For a comprehensive description of each of the two methods as well as guidelines for the choice of the filter see Estrella (2007).

1.1.2 Markov-Switching Dynamic Factor Model

1.1.2.1 General assumptions

The MS-DFM is applied to the business cycle analysis on the basis of two non-technical assumptions. First, it is supposed that the business cycle of an economy can be approximated by an unobservable factor which aggregates information on a certain number of economic indicators. This assumption comes from the idea of comovement of economic indicators mentioned in the introduction and the fact that the official dating committees (such as in NBER, OECD and others) take into consideration several economic series when declaring a turning point. Secondly, the dynamics of this factor is governed by a Markov chain, which means that this process can be characterized as having several distinct regimes and a matrix of probabilities of transition between these regimes. For the purpose of business cycle analysis, these states are usually recession and expansion, although the number of states can be higher (so that a regime of stagnation and fast growth, for example, can be identified). It is therefore assumed that the switch between regimes happens instantaneously, without any transition period (as considered, for example, in Smooth Transition Autoregression family models). We motivate this assumption by the fact that the transition period before crises is normally short enough to be omitted.

[1] see Fossati (2011) for more details.

A comment on factor extraction

The extraction of the unobserved factor can be performed in several ways: with the help of the Kalman filter, two-step method by Doz et al. (2011), quasi-maximum likelihood method by Doz et al. (2012), PCA and other. Whatever the method is, the question of the composition of the database that is used for the DFM part is very important. In order to obtain a reasonable approximation of the business cycle, it is crucial to carefully select informative series in case of a small dataset (as in one-step method in Chapter 3) or to have a well-balanced database which would not over- or under-represent certain sectors or types of series in case the database is large (as in two-step method used in Chapters 3-5). This questions has been much discussed in the literature and among practitioners. To avoid deviating too much into this direction, we use the commonly accepted Stock-Watson database and its twin for French data in our empirical applications in Chapter 3 and Chapter 5, and try to keep the composition of the database for the financial indicator in the Chapter 5 as diversified as possible.

Another important question concerning construction of the factors is the dynamics of factor loadings. Indeed, since the time-span under consideration in case of business cycle analysis is usually quite long, it is natural to assume that some series may start to have more contribution to the business cycle than the others as the structure of an economy evolves or structural breaks take place. Possible solutions include Markov-switches in factor loadings or periodic re-estimation of factor loadings, for example. In this study, we consider loadings to be static and we leave the broad question of their dynamics outside the scope of the thesis.

1.1.2.2 Advantages of the MS-DFM

Among the major advantages of MS-DFM over the other methods of business cycle turning point identification are timeliness (the MS-DFM estimates are available as soon as the dataset is updated), informativeness (contrary to non-parametric methods, MS-DFM is capable to distinguish the switch in mean from the switch in variance) and transparency (it is a fully replicable econometric technique and not an expert opinion). The primary output of the MS-DFM is therefore the estimate of the current state of the cycle, i.e. its nowcast (detection). Due to its high performance, it has been used in the several papers analyzing various economies, such as Darbha (2001) for India, Mills and P. (2003) for UK, Watanabe (2003) for Japan, Bandholz and Funke (2003) for Poland and Hungary, Bai and Ng (2013) Germany, Chauvet and Senyuz (2012) and Kim and Yoo (1995) for the US, Darné and Ferrara (2011) for France. The literature is very dynamic, and many extensions to the baseline model exist. Among them, of particular interest is the bi-factor MS-DFM, introduced by Chauvet (1998), which studies the comovement of the financial and the business cycles in order to identify whether the former causes the latter, the fact which can further be used for the construction of early warning indicators and prediction of recessions in the business cycles.

1.1.2.3 Brief literature review

The Dynamic Factor Model with Markov Switching (MS-DFM) was first suggested by Diebold and Rudebusch (1996)[2]. This paper relies on the seminal paper by Hamilton (1989) which applies a univariate Markov-Switching model to business cycle analysis. It was then formalized for the multivariate case by Kim (1994) and by Kim and Yoo (1995) and used afterwards by Chauvet (1998), Kim and Nelson (1998), Kaufmann (2000). While the original model assumes switches in mean, other types of non-linearity were proposed by Kholodilin (2002a), Kholodilin (2002b), Dolega (2007), Bessec and Bouabdallah (2015) where the slope of factors or exogenous variables is state dependent; or by Chauvet (1998), Chauvet (1999), Kholodilin (2002a), Kholodilin (2002b), Kholodilin and Yao (2004), Billio et al. (2007) where the variance of the idiosyncratic component is state dependent; and lastly by Chauvet and Potter (1998) and Carvalho and Lopes (2007) where the authors allow for structural breaks in factor loadings.

According to Google Scholar, about 15 new papers on Markov-Switching Dynamic Factor models applied to the business cycle analysis appear every year.[3] Current research in this area is led is several directions. Besides rapidly growing number of empirical applications, the most important are

- MS-DFM with mixed frequencies, allowing to consider even larger datasets (see Camacho and Martinez-Martin (2015) for the latest paper) and other improvement of the DFM part of the model (see Hindrayanto et al. (2016));
- multivariate MS-DFM studying the interaction between the cycles (see Leiva-Leon (2017), Koopman et al. (2016));
- highly dimensional Markov-switching models and their estimation methods (see Droumaguet et al. (2017), for example);

[2] The working paper version appeared in 1994 in NBER Working Papers 4643.

[3] The query "Markov-switching" "Dynamic Factor Models" "business cycle" (search by exact phrases) gives 129 results, while the query "Markov-switching Dynamic Factor Models business cycle" (search for results that content the words of the query in any order) gives 1510 results. These numbers include publications available online only.

- time-varying MS-DFM (see Bazzi et al. (2017)),

Of course, this list is by no means exhaustive, but it still helps to better understand the contribution of this thesis to the literature.

1.1.3 Contribution

The thesis contributes to the vast literature on the business cycle turning point identification in three directions. In the first chapter, the two popular estimation techniques of MS-DFM are compared and applied to the French economy. In the second chapter, on the basis of Monte Carlo simulations, the estimators of the preferred technique - the two-step estimation method - are shown to be consistent and their small-sample behavior is analyzed. Thus, the second chapter validates the results of the first chapter, as well as previously obtained results with the two-step method. In the third chapter, I extend the MS-DFM and I suggest the Dynamical Influence MS-DFM, which allows to model the financial and the business cycles simultaneously taking into consideration that the interaction between them can be dynamic. The DI-MS-DFM is estimated with a two-step approach. Thus, each chapter of the thesis is a logical continuation of the previous chapter.

The chapter "Dating business cycle turning points for the French economy: a MS-DFM approach" is dedicated to the comparison of the two estimation methods of MS-DFM. The one-step and two-step methods are applied to French data and their performance is compared. While one-step maximum likelihood estimation is confined to small data sets, the two-step approach is based on the use of principal components and can thus accommodate much bigger information sets. The one-step method implies estimation of the parameters of the model and the factor simultaneously, under specific assumptions on the dynamics of the factor. The two-step method consists of 1) extraction of a composite indicator reflecting the economic activity (the factor) from a large dataset; 2) estimation of the parameters of the univariate Markov-Switching model on the factor series. We find that both methods give qualitatively similar results and agree with the OECD dating of recessions on a sample of monthly data covering the period 1993-2014, however, the one-step method requires careful selection of data, which can be quite time-consuming. Moreover, the two-step method is more precise in determining the beginnings and ends of recessions as given by the OECD. Both methods indicate additional downturns in the French economy that were too short to enter the OECD chronology. We come to the conclusion that two-step method is preferred to one-step method since it is computationally easier and allows to avoid the difficult question of data selection.

The chapter "On the consistency of the two-step estimates of the MS-DFM: a Monte-Carlo study" is dedicated to the study of the two-step estimates under different size of the underlying dataset. Despite the fact that the two-step method has been used in several papers, to our knowledge, no study has ever shown that the two-step estimates actually converge to the true values of the parameters. With the help of Monte Carlo simulations we find that, under reasonable conditions on the number of series, the sample size and and under parameters of the DGP corresponding to the ones usually observed in the empirical studies (close to the ones obtained in the first chapter, for example), the two-step estimates are consistent. We find, however, that the estimates and their standard errors tend to be biased in small samples, so the statistical inference must be done with caution. Moreover, we observe that the variance of the error term of the factor tends to be overestimated even when the number of series and observations is high, calling for the use of some additional techniques, such as bootstrapping, for example. Notwithstanding these distortions, the performance of the MS-DFM in terms of state identification is satisfactory. These findings are robust to changes in the data generating process which can commonly take place: different degree of noise in the series, high variance of the factor dynamics, large or small difference between the two states of the process, etc.

The chapter "Dynamical Interaction between financial and business cycles" studies a popular question of mutual influence between the financial and the real sectors of the economy. We build on the Dynamical Influence model by Pan et al. (2012) from computer science and merge it with Markov-Switching Dynamic Factor Model. The resulting model, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-FM), allows to reveal the pattern of interaction between business and financial cycles in addition to their individual characteristics. More specifically, with the help of this model we are able to identify and describe quantitatively the existing regimes of interaction in a given economy, and we allow them to switch over time, which seems a reasonable assumption given that business cycle analysis usually implies the use of data spanning at least 20 years. We are also able to determine the direction of causality between the two cycles for each of the regimes. The model estimated on the US data demonstrates reasonable results, identifying the periods of higher interaction between the cycles in the beginning of 1980s and during the Great Recession, while in-between the cycles evolve almost independently. The output of the model can be useful for policymakers since it provides a timely estimate of the current interaction regime, which allows to adjust the timing and the composition of the policy mix. Moreover, it allows to evaluate the impact of government policies on the duration of each of the interaction regimes.

Each of the three chapters being an individual and independent research paper, the directions for further research are described in the end of each chapter, correspondingly. Nevertheless, the linear logic of this thesis suggests that the most fruitful and promising direction consists in further development of its last chapter. Indeed, the generalization of the DI-MS-FM to the case of multiple interacting cycles with several states each and having a richer range of modes of interaction opens door to many interesting applications. Among them are the analysis of the interaction of international business cycles, the study of the role of credit cycle and equity cycle separately and many other.

Chapter 2

Introduction Générale

Cette thèse est dediée à une classe particulière de modèles à facteurs dynamiques non linéaires, les modèles à facteurs dynamiques à changement de régime markovien (MS-DFM). Par la combinaison des caractéristiques du modèle à facteur dynamique et celui du modèle à changement de régimes markoviens (i.e. la capacité d'agrégation des quantités massives d'information et le suivi des processus fluctuants.), ce cadre s'est révélé très utile et convenable pour plusieurs applications. Parmi eux, le suivi de la productivité du travail (Dolega (2007)), la modélisation de la dynamique conjointe de la courbe de rendement et du PIB (Chauvet and Senyuz (2012)), l'étude des fluctuations des taux d'emploi (Juhn et al. (2002)) et plein d'autres peuvent être exemples. Pourtant, l'application principale des (MS-DFM) concerne l'analyse des points de retournement des cycles économiques (voir, par exemple, Kim and Yoo (1995), Darné and Ferrara (2011), Camacho et al. (2012), Chauvet and Yu (2006), Wang et al. (2009)). Pour cette raison-ci, l'analyse des cycles économiques est au coeur de cette thèse.

La connaissance de l'état actuel des cycles économiques est crucial afin de surveiller la santé économique et d'évaluer les résultats des politiques économiques. Néanmoins, ce n'est pas une tâche facile à réaliser. Le premier problème vient du fait qu'il existe un retard structurel dans l'annonce de l'état actuel de la part des institutions officielles (la date officielle s'annonce avec un retard de quelques mois). Deuxièmement, ce n'est pas évident quelle base de données et quelle méthode il faut adopter pour déterminer les points de retournement. Certaines procédures existent mais leurs résultats ne sont pas tout à fait sur la même lignée. Finalement, il est important d'identifier et de quantifier la relation entre le secteur financier et les cycles économiques, qui s'est avérée particulièrement importante durant la crise de 2008 et après. Le modèle à facteur dynamique à changement de régime markovien introduit par Diebold and Rudebusch (1996) comme une extension multivariée du modèle à changement de régime markovien de Hamilton

(1989)devint un instrument adéquat pour l'identification des points de retournement du processus commun non observable régissant plusieurs séries observables.

Afin de comprendre mieux la raison pour laquelle (MS-DFM) est bien adapté pour l'analyse du cycles économique, considérons ce phénomène plus en détail.

2.0.1 Cycle économique

2.0.1.1 Histoire

La notion de cycle économique (business cycle) apparut pour la première fois dans le livre "Nouveaux Principles d'économie politique" par Simonde de Sismondi (1819)qui était le premier à suggérer la notion des crises périodiques. D'après Sismondi, la recrudescence des crises est liée à la consommation et à la surproduction (i.e les raisons *internes*). Cette idée était en opposition avec la compréhension répandue de cette époque que les crises avaient des raisons *externes* telles que les guerres et la mauvaise récolte. Robert Owen exprima un point de vue similaire sur le sujet concerné dans son "Rapport au comité de l'Association pour le secours aux fabricants pauvres." (Committee of the Association for the Relief of the Manufacturing Poor, Owen (1817)). Ultérieurement, Charles Dunoyer adopta ces idées dans sa théorie d'alternance des cycles, introduisant le concept de "passer d'une crise à l'autre" (Benkemoune (2009)). Juglar (1862) était le premier à identifier les fluctuations de l'activité économique de sept ans à onze ans. Ensuite, Schumpeter (1939) définit quatre étapes du cycle économique, comme nous les connaissons maintenant : l'expansion, la crise, la récession et la reprise.

C'est intéressant de noter que le terme "cycle" est trompeur étant donné que la cyclicité implique la périodicité fixe, laquelle n'est pas vraie pour les crises économiques. Le terme "fluctuation" ou "oscillation" serait plus précis. En revcanche, le déroulement historique manifesta sa préférence pour une appelation plus concise qui est "cycles".

La périodicité des crises définit le type du cycle économique. Joseph Schumpeter et ses contemporains distinguèrent quatre types de cycle économique : cycle d'inventaire de Kitchin (de 3 à 5 ans), cycle d'investissement fixe de Juglar (de 7 à 10 ans), cycle d'investissement en infrastructure de Kuznets (de 15 à 25 ans) et finalement, cycles technologiques longs de Kondratiev (de 45 à 60 ans). Cependant, c'est le deuxième type de cycle qui a attira l'attention des chercheurs vu que les autres types de cycle furent remis en question (comme dans le cas de cycle Kondratiev), négligé (le cycle de Kitchin) ou ne furent pas le point d'intérêt central (le cycle de Kuznets).

2.0.1.2 Définition

La presse financière se réfère souvent au changement de PIB au sein des discussions sur le cycle économique. Malgré le fait que PIB est considéré comme l'indicateur majeur de référence pour l'activité économique, il s'avère que celui-ci n'est pas assez satisfaisant afin d'écrire le cycle. En effet, si la production industrielle décolle lorsque le taux de chômage ne diminue pas mais au contraire, il augmente, doit-on considérer cette dernière comme le début de l'expansion ? Probablement pas. Si l'expansion commence dans quelques secteurs mais le reste de l'économie est en état de stagnation, pourrait-on percevoir cela comme le début de l'expansion ? Probablement pas non plus. Pour cette raison-ci que la définition standard du cycle économique fournie par Burns and Mitchell (1946) indique ce qui suit : "Les cycles économiques représentent un type de fluctuation présente dans l'économie agrégée des nations qui organisent leurs activités au sein des entreprises commerciales : le cycle consiste en expansions ayant lieu en même temps dans plusieurs activités économiques, suivies généralement par des récessions, des contractions et de la relance économique qui se fusionnent avec la phase d'expansion du cycle suivant. En termes de durée, les cycles économiques varient de plus qu'une année à dix ou douze ans ; ils ne sont pas divisible en cycles plus courts à caractère similaire avec des amplitudes approximant les leurs".

Il y a certains points importants sur lesquels on aimerait mettre l'accent pour cette définition. Premièrement, elle ne dépend pas de mécanisme sous-jacent du cycle économique. Ceci est un point important vu que MS-DFM est en mesure d'identifier les récessions mais ne parvient pas à les classer par ses causes sous-jacentes. Deuxièmement, cette définition souligne l'importance de la synchronicité : le cycle économique est un phénomène commun à la plupart des activités économiques.

Un autre point à préciser est que cette définition mentionnée ci-dessus est non seulement valable pour les indicateurs économiques de niveau brut mais aussi pour les séries transformées. Ceci définit un autre typologie de cycle économique : les cycles économiques (fluctuations en niveau), les cycles de croissance (fluctuations autour d'une tendance à long terme) et les cycles de taux de croissance (fluctuation en taux de croissance). Théoriquement, il n'y a pas de préférence d'un type à l'autre. Dans les travaux empiriques, le choix du type de cycle dépend de la question de recherche. Bien que souvent utilisé de façon interchangeable sous un terme générique "cycles économiques", ces trois cycles peuvent être légèrement différents, et il est important de garder à l'esprit ces differences, notamment lorsqu'on compare la datation des différents points de retournement. Dans ce contexte de la thèse, on se focalisera sur les cycles de taux de croissance pour deux raisons : premièrement, le MS-DFM requiert des données stationnaires. En conséquence, ce modèle n'est pas destiné à analyser les cycles économiques pures. Deuxièmement, comme la contribution de tous les chapitres est principalement méthodologique, on tâchera d'éliminer autant que possible l'impact de procédures de prétraitement des séries comme la procédure d'élimination de la tendance pour le taux de croissance.

2.0.1.3 Les explications existantes

Bien que cette thèse n'ait pas l'objectif d'expliquer le mécanisme derrière le cycle économique, il semble important de présenter un bref aperçu des explications existantes pour les cycles économiques.

Récemment, il existe deux groupes d'approche pour l'explication des sources du cycle économique : un courant dominant (keynésien et la théorie du cycle économique réel) et un courant alternatif (l'économie hétérodoxe : des théories basées sur le crédit telles que l'hypothèse de l'instabilité financière et la déflation de la dette.)

D'après l'approche keynésienne, les fluctuations de la demande agrégée conduisent à un équilibre au- dessous ou au-dessus du plein emploi. En raison du multiplicateur keynésien et de l'accélérateur, les réactions de l'output sur les chocs initiaux sont cycliques dans ce cas-ci. La théorie du cycle économique réel implique que les fluctuations dans l'économie ont lieu du fait des chocs externes (contrairement à l'approche keynésienne), comme le choc technologique. La différence dans l'approche de la cause des fluctuations endogène ou exogène est important pour les décideurs politiques. Si les cycles ne peuvent pas être générés par les chocs endogènes, comme le suggère la théorie du cycle économique réel, toute politique anticyclique n'est pas efficace et les mesures politiques doivent se concentrer davantage sur l'amélioration des conditions de croissance de long terme, au lieu d'atténuer les fluctuations de l'activité économique à moyen terme.

Les théories basées sur le crédit prétendent que le cycle économique est dépendant du cycle du crédit. Par conséquent, l'expansion ou la contraction du crédit suscitent l'expansion ou la contraction respectivement dans le cycle économique. L'un des mécanismes de cette interaction, la théorie de la déflation par la dette, fut suggéré par Fischer (1933). Il indique que lorsqu'une économie fait face à la déflation, cette dernière conduit à l'accumulation globale de la dette, poussant les agents à faire défaut sur leurs crédits. Les actifs des banques contractent du fait que la valeur des garanties diminuent, entraînant des insolvabilités bancaires massives. Cela empêche les banques de faire des prêts, en influançant négativement la consommation. Un autre mécanisme, l'hypothèse de d'instabilité financière de Minsky (1992) suggère que les cycles sont induits par la mauvaise estimation de la capacité des entreprises pour rembourser leurs dettes. Au cours de l'expansion, les banques sont disposées à accorder des prêts aux entreprises vu qu'elles les rembourseront facilement compte tenu des conditions économiques favorables. A un certain point, les entreprises sont surendettées et se mettent à réduire leur investissement et leur production. Ainsi, la récession commence. Compte tenu des taux d'intérêts élevés, causés par la relation entre les cycles financiers et économique après la crise de 2008, on traitera cette question dans le chapitre 5.

2.0.1.4 La datation des points de retournement

Quel que soit le type de cycle économique auquel nous nous référons et quel que soit le mécanisme sous-jacent, Comme nous l'avons mentionné au début de cette introduction, il est important de pouvoir déterminer la phase actuelle du cycle, pour *nowcast*, c'est-à-dire, pour obtenir l'estimation de l'état actuel pour le point actuel.

Différentes techniques peuvent être utilisées pour cette tâche. La méthode non paramétrique la plus utilisée est l'algorithme de point de retournement, élaboré par Bry and Boschan (1971) et son homologue pour l'algorithme de base de données trimestrielles par Harding and Pagan (2002). La routine de Bry-Boschan identifie le pic et le creux d'un cycle économique comme le local maximum et minimum sous un ensemble de contraintes (sur la durée du cycle, durée minimale de chaque phase, etc. Bien que pratique et transparent, cet algorithme a, malheureusement, un inconvénient inhérent à la plupart des méthodes non paramétriques : il est très sensible aux paramètres de censure. Un changement modéré dans le paramètre de contrôle (par exemple, la durée minimale de chaque phase) peut modifier complètement la séquence des points de retournement. Par consequent, le Bry-Boschan nécessite beaucoup de réglages où l'expertise joue un rôle crucial.

Les méthodes paramétriques populaires comprennent des régressions binomiales (standard ou dynamiques) et des modèles à changement de régime markovien. Les deux principaux avantages des méthodes paramétriques sont les suivants: 1) pour chaque point d'observation, on obtient *une probabilité* de recession, ce que est plus informatif; 2) il est possible de faire des prévisions d'états futurs. Les régressions binomiales (habituellement probit) cherchent à expliquer la relation entre la datation de référence existante (par exemple, de NBER) et des indicateurs (avancés ou coïncidents) ainsi que des facteurs (voir, par exemple, Estrella and Mishkin (1996), Chauvet and Potter (2010) et Fossati (2015)). En revanche, le modèle à changement de régime markovien ne nécessite pas de datation de référence, car il permet de faire une inférence sur le cycle non observé en utilisant des indicateurs économiques observables, ce qui constitue un avantage important pour l'analyse des cycles économiques dans les économies en développement où une datation de référence peut ne pas être disponible. Bien que les deux modèles produisent des probabilités de récession, leur estimations peuvent être qualitativement très différentes. Les modèles probit génèrent des probabilités élevées mais volatiles pendant les récessions. Les modèles à changement de régime markovien, au contraire, donnent un signal clair et stable pendant les récessions, mais ont tendance à identifier les points de retournement avec un retard (ce que nous observons dans le chapitre 3).[1] Le choix final du modèle dépend donc de la préférence pour les faux positifs et de la disponibilité des datations de référence. L'absence de ces-derniers dans le cas de nombreux pays est la principale raison d'étudier les modèles à changement de régime markovien dans cette thèse.

En cas de cycle de croissance (c'est-à-dire le cycle en termes de déviation par rapport à la tendance), l'identification des points de retournement est compliquée par la préalable tâche d'extraction du cycle de croissance. De même que dans le cas d'ajustement saisonnier, afin d'extraire le cycle, il faut résoudre un problème de décomposition des séries en composantes de tendance, de cycle et de bruit. La principale difficulté de la tâche réside dans le fait qu'il n'y a pas de définition formelle pour chacune d'entre elles : il n'y a pas d'unicité de la décomposition. Dans la pratique, la plupart des méthodes existantes d'extraction du cycle économique peuvent être divisées en deux groupes: extraction de fréquences et extraction de signaux. Étant donné que nous nous concentrons sur les cycles des taux de croissance plutôt que sur les cycles de croissance, nous ne discutons pas en détail chacune de ces méthodes. Il est important de souligner, cependant, qu'aucune des méthodes mentionnées ci-dessus ne peut être n'apparait piviligiée, et le choix dépend beaucoup du processus en question. Pour une description complète de chacune des deux méthodes ainsi que des conseils pour le choix du filtre, voir Estrella (2007).

2.0.2 Le modèle MS-DFM (Markov-Switching Dynamic Factor Model)

2.0.2.1 Hypothèses générales

Le modèle MS-DFM est appliqué à l'analyse du cycle économique sur la base de deux hypothèses non techniques. Premièrement, on suppose que le cycle économique peut être approximé par un facteur inobservable qui agrège l'information d'un certain nombre

[1]see Fossati (2011) pour plus de détails.

d'indicateurs économiques. Cette hypothèse vient de l'idée de co-mouvement des indicateurs économiques mentionnée dans la définition et du fait que les comités de datation (comme ceux du NBER ou de l'OCDE, par exemple) prennent en compte plusieurs séries économiques lorsqu'ils déclarent un point de retournement. Deuxièmement, on suppose que la dynamique de ce facteur est gouvernée par une chaîne de Markov, ce qui signifie que ce processus peut être caractérisé par plusieurs régimes distincts et une matrice de probabilités de transition entre ces régimes. Pour l'analyse des cycles économiques, ces états sont généralement la récession et l'expansion, mais le nombre d'états peut être plus élevé (de sorte que des régimes de stagnation et de croissance rapide, par exemple, puissent être identifiés). On suppose donc que le changement entre les régimes se produit instantanément, sans aucune période de transition (*a conrario* des modèles Smooth Transition Autoregression, par exemple). Cette hypothèse est motivée par le fait que la période de transition avant des crises profondes est en général suffisamment courte pour être omise.

2.0.2.2 Avantages du modèle MS-DFM

Les avantages principaux du modèle MS-DFM par rapport aux autres méthodes d'identification des points de retournement du cycle économique, sont l'absence de délai (les estimation du modèle MS-DFM sont disponibles dès que les données sont actualisées), la qualité informative (contrairement aux méthodes non paramétriques, le modèle MS-DFM est capable de distinguer un changement de moyenne d'un changement de variance) et la transparence (il s'agit d'une technique économétrique entièrement réplicable et non d'une opinion d'expert). Le premier résultat du modèle MS-DFM est donc l'estimation de l'état actuel du cycle, c'est-à-dire sa détection (nowcast). En raison de ses grandes performances, ce modèle a été utilisé dans plusieurs articles analysant diverses économies, comme les travaux de Darbha (2001) sur l'Inde, Mills and P. (2003) sur le Royaume-Uni, Watanabe (2003) sur le Japon, Bandholz and Funke (2003) sur la Pologne et la Hongrie, Bai and Ng (2013) et Chauvet and Senyuz (2012) sur l'Allemagne, Kim and Yoo (1995) sur les Etats-Unis, et Darné and Ferrara (2011) sur la France.

2.0.2.3 Littérature

The Dynamic Factor Model with Markov Switching (MS-DFM) was first suggested by Diebold and Rudebusch (1996)[2]. This paper relies on the seminal paper by Hamilton (1989) which applies a univariate Markov-Switching model to business cycle analysis. It was then formalized for the multivariate case by Kim (1994) and by Kim and Yoo (1995)

[2]The working paper version appeared in 1994 in NBER Working Papers 4643.

and used afterwards by Chauvet (1998), Kim and Nelson (1998), Kaufmann (2000). While the original model assumes switches in mean, other types of non-linearity were proposed by Kholodilin (2002a), Kholodilin (2002b), Dolega (2007), Bessec and Bouabdallah (2015) where the slope of factors or exogenous variables is state dependent; or by Chauvet (1998), Chauvet (1999), Kholodilin (2002a), Kholodilin (2002b), Kholodilin and Yao (2004), Billio et al. (2007) where the variance of the idiosyncratic component is state dependent; and lastly by Chauvet and Potter (1998) and Carvalho and Lopes (2007) where the authors allow for structural breaks in factor loadings. The literature is very dynamic, and many extensions to the baseline model exist. Among them, of particular interest is the bi-factor MS-DFM, introduced by Chauvet (1998), which studies the comovement of the financial and the business cycles in order to identify whether the former causes the latter, the fact which can further be used for the construction of early warning indicators and prediction of recessions in the business cycles.

Le modèle dynamique à changement de régime markovien (MS-DFM) fut suggéré par Diebold and Rudebusch (1996)[3]. Cet article est basé sur le document séminal de Hamilton (1989) qui propose un modèle markovien univarié pour analyser le cycle économique. Il a ensuite été généralisé pour le cas multivarié par Kim (1994) et Kim and Yoo (1995), puis utilisé par Chauvet (1998), Kim and Nelson (1998), Kaufmann (2000). Bien que le modèle d'origine suppose des changements qu'en moyenne, d'autres types de non-linéarité ont été proposés par Kholodilin (2002a), Kholodilin (2002b), Dolega (2007), Bessec and Bouabdallah (2015) où ce sont les coefficients des facteurs ou des variables exogènes qui dépendent de l'état; en outre, Chauvet (1998), Chauvet (1999), Kholodilin (2002a), Kholodilin (2002b), Kholodilin and Yao (2004), Billio et al. (2007) utilisent la variante de MS-DFM avec variance du composant idiosyncratique dépendant de l'état; Chauvet and Potter (1998) et Carvalho and Lopes (2007) introduisent les changement de régimes dans les loadings des facteurs. La littérature est très dynamique, et il existe de nombreuses extensions du modèle de base. Parmi celles-ci, le modèle MS-DFM bi-facteur introduit par Chauvet (1998) est particulièrement intéressant, car il étudie la dynamique jointe des cycles financier et économique afin d'identifier si le premier cause le second, ce qui peut par la suite être utilisé pour la mise en place d'indicateurs d'alerte et pour la prédition des récessions dans le cycle économique.

Une recherche sur Google Scholar permet de constater qu'en moyenne 15 articles sur le MS-DFM appliqué à l'analyse du cycle économique apparaissent chaque année.[4] La

[3] La version du document de travail a été publiée en 1994 dans les documents de travail NBER 4643.

[4] La requête "Markov-switching" "Dynamic Factor Models" "cycle économique" (recherche par phrases exactes) donne 129 résultats, tandis que la requête "Markov-switching Dynamic Factor Models

recherche actuelle dans ce domaine est menée dans plusieurs directions. Hormis le nombre croissant d'applications empiriques, la recherche se focalise sur plusieurs axes tels que:

- MS-DFM aux fréquences mixtes, permettant de considérer des bases de données plus grandes (voir Camacho and Martinez-Martin (2015)) et d'autres améliorations de la partie DFM du modèle (voir Hindrayanto et al. (2016));
- MS-DFM multivarié étudiant l'interaction entre les cycles (voir Leiva-Leon (2017), Koopman et al. (2016));
- les modèles à changement de régime markovien à haute dimension et les méthodes d'estimation associées (voir Droumaguet et al. (2017), par exemple);
- MS-DFM variable dans le temps (voir Bazzi et al. (2017)),

Bien sûr, cette liste n'est pas exhaustive, cependant elle permet de mieux comprendre la contribution de cette thèse à la littérature.

2.0.3 Contribution

Cette thèse contribue à la vaste littérature sur l'identification des points de retournement du cycle économique dans trois directions. Dans le Chapitre 3, on compare les deux techniques d'estimation de MS-DFM et on les applique aux données françaises. Dans Chapitre 4, sur la base des simulations de Monte Carlo, on montre que les estimateurs de la technique préférée - la méthode d'estimation en deux étapes - sont convergents et on étudie leur comportement en échantillon fini. Dans le Chapitre 5, on propose une extension de MS-DFM - le MS-DFM à l'influence dynamique (DI-MS-DFM) - qui permet de modéliser la dynamique jointe du cycle financier et du cycle économique tout en tenant compte du fait que l'interaction entre eux peut être dynamique. Le DI-MS-DFM est estimé avec une approche en deux étapes. Ainsi, chaque chapitre de la thèse consititue une suite logique du chapitre précédent.

Le chapitre "Dating business cycle turning points for the French economy: a MS-DFM approach" est consacré à la comparaison des deux méthodes d'estimation de MS-DFM. Les méthodes en une étape et en deux étapes sont appliquées aux données françaises et leur performance est comparée. Tandis que l'estimation de la vraisemblance maximale en une étape est limitée aux petits ensembles de données, l'approche en deux étapes est basé sur l'utilisation de composants principaux et peut donc accueillir des ensembles d'informations beaucoup plus importants. La méthode en une étape implique

business cycle" (recherche des résultats qui contiennent les mots de la requête dans n'importe quel ordre) donne 1510 résultats. Ces chiffres comprennent les publications disponibles en ligne uniquement.

l'estimation des paramètres du modèle et du facteur simultanément, sous des hypothèses spécifiques sur la dynamique du facteur. La méthode en deux étapes consiste en 1) extraction d'un indicateur composite reflétant l'activité économique (le facteur) à partir d'un grand ensemble de données; 2) l'estimation des paramètres du modèle à changement de régime univarié sur la série de facteurs. Nous trouvons que les deux méthodes donnent des résultats qualitativement similaires et sont en accord avec la datation des récession de l'OCDE. Sur un échantillon de données mensuelles couvrant la période 1993-2014, cependant, la méthode en une étape nécessite une sélection soigneuse des données, ce qui peut être coûteux en termes de temps. En outre, la méthode en deux étapes est plus précise à détécter des débuts et des fins des récessions. Les deux méthodes indiquent des récessions supplémentaires dans l'économie française, qui, étant trop courts, n'entrent pas la chronologie officielle de l'OCDE. On coclut que la méthode en deux étapes est préférable à la méthode en une étape, car elle est moins intense en termes de calcul et permet d'éviter la difficile question de la sélection des données.

Le chapitre "On the consistency of the two-step estimates of the MS-DFM: a Monte-Carlo study" est consacré à l'examen du comportement des estimateurs en deux étapes sous la taille différente de l'échantillion. Malgré le fait que la méthode en deux étapes a été utilisée dans plusieurs études, à notre connaissance, aucune étude n'a montré que les estimations en deux étapes convergent vers les vraies valeurs des paramètres. Avec l'aide des simulations de Monte Carlo, nous constatons que, dans les conditions raisonnables concernant le nombre de séries et d'observations, et avec les paramètres de la DGP correspondant à ceux habituellement observés dans les études empiriques (par exemple dans le chapitre précédent), les esimateurs à deux étapes sont convergents. Cependant, les estimateurs et leurs écart-types ont tendance à être biaisés en petit échantillon, par consequent dans ce cas l'inférence statistique doit être effectuée avec beaucoup de prudence. De plus, nous observons que la variance du terme d'erreur du facteur a une tendance à être surestimée ce qui nécessite l'utilisation de certaines techniques supplémentaires telles que la méthode de bootstrap, par exemple. Nonobstant ces distorsions, la performance du MS-DFM en termes d'identification de l'état de la chaine markovienne sous-jacente est satisfaisante. Ces résultats sont robustes aux changements de paramétage de DGP qui sont censés à simuler les bases des données avec certaines particularités souvent rencontrées dans les données: par exemple, séries avc beaucoup de bruit, grande variance de la dynamique des facteurs, grande ou petite différence entre les deux états du processus, etc.

Dans le chapitre «Dynamical Interaction between financial and business cycles» on étudie la question de l'interaction entre le cycle de conjoncture et le cycle de finance. On développe le modèle d'influence dynamique proposé par Pan et al. (2012) et provenant du domaine des sciences informatiques en le fusionnant avec le MS-DFM. Le modèle résultant, le modèle d'influence dynamique aux facteurs et au changements de regime markoviens (DI-MS-FM) permet de révéler la dynamique d'interaction entre les cycles économiques et financiers en plus de leurs caractéristiques individuelles. Plus précisément, avec l'aide de ce modèle on est en mesure d'identifier et de décrire quantitativement les régimes d'interaction existants (déterminer la direction de la causalité entre les deux cycles) dans une économie donnée, et on est capable de tracer les périodes quand chaqu'un de ces régimes est activé. Le modèle estimé sur les données américaines donne des résultats plausibles. On identifie les périodes d'interaction plus élevée au début des années 1980 et après 2008, alors que entre ces deux périodes les cycles évoluent presque indépendamment. L'output du modèle peut être utile pour les décideurs politiques, car il fournit une estimation du régime d'interaction actuel, ce qui permet d'ajuster le calendrier et la composition de la politique publique. En outre, il permet d'évaluer l'impact des politiques gouvernementales sur la durée de chacun des régimes d'interaction et chaqu'une des phase de cycles.

Chacun des trois chapitres étant un document de recherche individuel et indépendant, les pistes de future recherche sont décrites à la fin de chaque chapitre. Néanmoins, la logique linéaire de cette thèse suggère que la direction la plus prometteuse consiste à développer davantage le dernier chapitre. En effet, la généralisation du DI-MS-FM pour le cas de cycles multiples avec plusieurs états chacun et ayant une plus riche gamme de modes d'interaction ouvre la porte à de nombreuses applications intéressantes. Parmi celles-ci figurent l'analyse de l'interaction des cycles économiques internationaux, l'étude du rôle du cycle du crédit et du cycle de l'équité séparément et plusieurs autres.

Chapter 3

Dating Business Cycle Turning Points for the French Economy: an MS-DFM approach

Abstract

Several official institutions (NBER, OECD, CEPR, and others) provide business cycle chronologies with lags ranging from three months to several years. In this chapter, we propose a Markov-switching dynamic factor model that allows for a more timely estimation of turning points. We apply one-step and two-step estimation approaches to French data and compare their performance. One-step maximum likelihood estimation is confined to relatively small data sets, whereas two-step approaches that use principal components can accommodate much bigger information sets. We find that both methods give qualitatively similar results and agree with the OECD dating of recessions on a sample of monthly data covering the period 1993-2014. The two-step method is more precise in determining the beginnings and ends of recessions as given by the OECD. Both methods indicate additional downturns in the French economy that were too short to enter the OECD chronology.

3.1 Introduction

The knowledge of the current state of the economic cycle is essential for policymakers. However, it is not easy to determine. The first problem is that a certain time is to pass before the official institutions announce the state of today. NBER and CEPR produce the reference economic cycle dating for the USA and Europe, respectively, on a basis of a consensus of expert opinions with a lag of several months or years. The OECD dating for

Europe also appears with a lag of up to 3 months as it is based on the quarterly GDP series. Other institutions, such as ECRI[1], provide dating with at least one year lag. Besides the timing, the second complicated issue is the identification of the list of series which can serve as indicators of the economic cycle. Finally, it is not obvious which method should be used to determine turning points. Several procedures exist, and the results are likely to differ. In this paper, we attempt to tackle these three problems in case of the French economic cycles on the basis of the Markov Switching Dynamic Factor Model.

The Dynamic Factor Model with Markov Switching (MS-DFM) was first suggested by Diebold and Rudebusch (1996) [2]. This paper relies on the seminal paper by Hamilton (1989) which applies a univariate Markov-Switching model to business cycle analysis. It was then formalized for the multivariate case by Kim (1994) and by Kim and Yoo (1995) and used afterwards by Chauvet (1998), Kim and Nelson (1998), Kaufmann (2000). The model allows to consider two features of an economic cycle as described by Burns and Mitchell (1946), namely the comovement of individual economic series and the division of an economic cycle into two distinct regimes, recession and expansion. Thus, the common factor of the economic series contains the information on the dynamics of the economic activity, while the two-regime pattern is captured by allowing the parameters of the factor dynamics to follow a Markov-chain process. While the original model assumes switches in mean, other types of non-linearity were proposed by Kholodilin (2002a), Kholodilin (2002b), Dolega (2007), Bessec and Bouabdallah (2015) where the slope of factors or exogenous variables is state dependent; or by Chauvet (1998), Chauvet (1999), Kholodilin (2002a), Kholodilin (2002b), Kholodilin and Yao (2004), Billio et al. (2007) where the variance of the idiosyncratic component is state dependent; and lastly by Chauvet and Potter (1998) and Carvalho and Lopes (2007) where the authors allow for structural breaks in factor loadings.

The MS-DFM model can be estimated either in one or two steps. The one-step method implies estimation of the parameters of the model and the factor simultaneously, under specific assumptions on the dynamics of the factor. The two-step method consists of 1) extraction of a composite indicator reflecting the economic activity (the factor); 2) estimation of the parameters of the univariate Markov-Switching model on the factor series. As usual, each method has its advantages and disadvantages. The one-step approach is given more favor in the literature since, within this method, the extracted factor is designed so that it has Markov-switching dynamics. On the other hand, the one-step approach is subject to convergence problems and is more time-consuming, since

[1] Economic Cycle Research Institute, private organization.

[2] The working paper version appeared in 1994 in NBER Working Papers 4643.

the number of parameters to estimate is much larger than in the case of the two-step procedure and increases with the number of series in the database. Thus, it is necessary to choose a set of variables that would reflect the oscillations of the economic activity correctly. The two-step procedure is much easier to implement, it is flexible in the model specification and does not put any restrictions on the number of series by default. This is why it has been used in a number of papers, for example by Chauvet and Senyuz (2012), Darné and Ferrara (2011), Bessec and Bouabdallah (2015) and others. However, Camacho et al. (2012) argued that this method may face misspecification issues, as the factor extracted in the first step is not supposed to have non-linear dynamics. More precisely, the authors argued that, when estimated with a linear DFM, the factor may give too much weight to the past values of the underlying series, thus being too slow to reflect the most recent changes. In this paper we analyze and compare the results of these two estimation methods to identify the turning points of the growth rate cycle of the French economy. We estimate the MS-DFM for the period May 1993 - March 2014 via the two-step method on a large database containing 151 series and via the one-step method on 4 series, as suggested by the original paper of Kim and Yoo (1995). We show evidence that, when the factor is estimated by PCA on the first step, and when the number of series is sufficiently large, the two-step estimation method can, in fact, provide satisfactory results.

We determine the key economic indicators that are able to give early and accurate signals on the current state of the growth rate cycle for the one-step method. We then compare the results obtained via the one-step results to the two-step results. This analysis is a contribution to the existing literature on the comparison of the two methods, notably the paper by Camacho et al. (2012), who argued that the one-step method is preferable to the two-step one, although its marginal gains diminish as the quality of the indicators increases and as more indicators are used to identify the non-linear signal. Their result was illustrated on four series of the Stock-Watson coincident index for the US, while we perform the comparison on an extensive dataset of 151 French series. Secondly, we decrease the degree of subjectivity regarding the choice of variables for the one-step method by testing all possible combinations of 25 main economic indicators. This is a contribution to existing works on the alternative economic cycle chronologies for France estimated on a small dataset by Kaufmann (2000), Gregoir and Lenglart (2000), Kholodilin (2006), Chen (2007), Chauvet and Yu (2006), Dueker and Sola (2008), Darné and Ferrara (2011) Finally, we conclude that although both methods provide valid results and outperform the reference dating in timing of the announcement of a current state of the business cycle, the two-step method has the advantage to be easy to implement and to detect quickly the temporal deterioration in an economy.

The structure of the paper is as follows: in the Section 3.2 we describe the baseline Markov Switching Dynamic Factor model and its two estimation methods. In the Section 3.3 we discuss the dataset and the measures of quality that we use to compare the approaches. The Section 3.4 is devoted to the description of one-step and two-step estimation results and to their comparison. Section 3.5 concludes.

3.2 The model and the estimation methods

3.2.1 The model

The general framework for Markov switching factor models has been first proposed by Kim (1994) and was then used by Kim and Yoo (1995) to study the US business cycle. In the present paper, we take the same kind of specification as in Kim and Yoo (1995), and we assume that the growth rate cycle of the economic activity has only two regimes (or states), associated with its low and high levels. The economic activity itself is represented by an unobservable factor, which summarizes the common dynamics of several observable variables. It is assumed that the switch between regimes happens instantaneously, without any transition period (as is considered, for example, by STAR family models). This assumption can be motivated by the fact that the transition period before deep crises is normally short enough to be omitted. For example, the growth rate of French GDP fell from 0.5% in the first quarter of 2008 to -0.51% in the second quarter of the same year, and further down to -1.59% in the first quarter of 2009.[3]

The model is thus decomposed into two equations, the first one defining the factor model, and the second one describing the Markov switching autoregressive model which is assumed for the common factor. More precisely, in the first equation, each series of the information set is decomposed into the sum of a common component (the common factor loads on each of the observable series with a specific weight) and an idiosyncratic component:

$$y_t = \gamma f_t + z_t, \tag{3.1}$$

where y_t is a $N \times 1$ vector of economic indicators, f_t is a univariate common factor, z_t is a $N \times 1$ vector of idiosyncratic components, which is uncorrelated with f_t at all leads

[3] INSEE, France, Gross Domestic Product, Total, Contribution to Growth, Calendar Adjusted, Constant Prices, SA, Chained, Change P/P

and lags, γ is a $N \times 1$ vector. In this equation all series are supposed to be stationary, so that some of the components of y_t may be the first differences of an initial non-stationary economic indicator.

The second equation describes the behavior of the factor f_t, which is supposed to follow an autoregressive Markov Switching process with constant transition probabilities.[4] We consider, in most of the paper, that the change in regime affects only the level of the constant with the high level corresponding to the expansion state and the low level to the recession state. Following Kim and Yoo (1995), we also suppose that the lag polynomial $\phi(L)$ is of order 2 so that:

$$f_t = \beta_{S_t} + \phi_1 f_{t-1} + \phi_2 f_{t-2} + \eta_t, \tag{3.2}$$

where $\eta_t \sim i.i.d.\ \mathcal{N}(0,1)$, and ϕ_1 and ϕ_2 are the autoregressive coefficients.

The switching mean is defined as:

$$\beta_{S_t} = \beta_0(1 - S_t) + \beta_1 S_t, \tag{3.3}$$

where S_t follows an ergodic Markov chain, i.e.

$$P(S_t = j|S_{t-1} = i, S_{t-2} = k, ...) = P(S_t = j|S_{t-1} = i) = p_{ij}$$

.

As it is assumed that there are two states only, S_t switches states according to the transition probabilities matrix $\begin{bmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{bmatrix}$, where

$$P(S_t = 0|S_{t-1} = 0) = p_0,$$

$$P(S_t = 1|S_{t-1} = 1) = p_1.$$

There is no restriction on the duration of each state, and the states are defined pointwise, i.e. a recession period may last one month only.

[4] Kim and Yoo (1995) showed that although the assumption of the time dependent probabilities improves the quality of the model, the gain in terms of loglikelihood is not very large.

Following Kim and Yoo (1995), we also assume that the idiosyncratic components z_{it} are mutually uncorrelated at all leads and lags, that each of them follows an autoregressive process with a lag polynomial $\psi_i(L)$, and that the degree of this polynomial is 2. Thus:

$$z_t = \psi_1 z_{t-1} + \psi_2 z_{t-2} + \varepsilon_t, \tag{3.4}$$

where ψ_1 and ψ_2 are diagonal matrices of coefficients, $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$, and Σ is a is diagonal matrix.

The model can be cast into state-space form:

$$y_t = B\alpha_t, \tag{3.5}$$

$$\alpha_t = T\alpha_{t-1} + \mu_{S_t} + Rw_t, \tag{3.6}$$

where α_t is the state variable,

$$\alpha_t = (f_t, f_{t-1}, z_t', z_{t-1}')', \text{ with } z_t = (z_{1t}, ..., z_{Nt})'$$

$$w_t = (\eta_{t,} \varepsilon_t')', \text{ with } \varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$$

$$E(w_t w_t') = Q = diag\{1, \sigma_1^2, ..., \sigma_N^2\},$$

$$\mu_{s_t} = (\beta_{s_t}, 0'_{(2N+1)\times 1})'$$

and B, T and R are corresponding coefficient matrices.

More explicitly, the state-space representation takes the form:

$$y_t = \begin{pmatrix} \gamma & 0 & I_N & 0 \end{pmatrix} \begin{pmatrix} f_t \\ f_{t-1} \\ z_t \\ z_{t-1} \end{pmatrix}, \tag{3.7}$$

$$\begin{pmatrix} f_t \\ f_{t-1} \\ z_t \\ z_{t-1} \end{pmatrix} = \begin{pmatrix} \phi_1 & \phi_2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \psi_1 & \psi_2 \\ 0 & 0 & I_N & 0 \end{pmatrix} \begin{pmatrix} f_{t-1} \\ f_{t-2} \\ z_{t-1} \\ z_{t-2} \end{pmatrix} + \begin{pmatrix} \beta_{s_t} \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & I_N \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \varepsilon_t \end{pmatrix}. \tag{3.8}$$

3.2.2 One-step estimation method

In this section, we recall the estimation method which has been introduced by Kim (1994) and Kim and Yoo (1995): it is a one-step method, but it can be employed only for a small set of observable series. Using the state-space representation of the model, which is given by equations (7) and (8), the Kalman filter can be written conditionally on the realizations of the state variable at time t and $t-1$. If $X_{t|t-1}^{(j,i)}$ denotes the predicted value of the variable X_t conditional on the information available up to $t-1$ and on the realizations $S_t = j$ and $S_{t-1} = i$, the Kalman filter formulas are the following:

Prediction step:

$$\alpha_{t|t-1}^{(j,i)} = T\alpha_{t-1|t-1}^{(i)} + \mu_{S_t}^{(j)}, \tag{3.9}$$

$$P_{t|t-1}^{(j,i)} = TP_{t-1|t-1}^{(i)}T' + RQR'. \tag{3.10}$$

Error step:

$$v_{t|t-1}^{(j,i)} = y_t - B\alpha_{t|t-1}^{(j,i)}, \tag{3.11}$$

$$Var(v_{t|t-1}^{(j,i)}) = H_{t|t-1}^{(j,i)} = BP_{t|t-1}^{(j,i)}B'. \tag{3.12}$$

Updating step:

$$\alpha_{t|t}^{(j,i)} = \alpha_{t|t-1}^{(j,i)} + K_t^{(j,i)}v_{t|t-1}^{(j,i)}, \tag{3.13}$$

$$P_{t|t}^{(j,i)} = (I_{(2N+2)} - K_t^{(j,i)}B)P_{t|t-1}^{(j,i)}. \tag{3.14}$$

The Kalman gain $K_t^{(j,i)}$ is given by

$$K_t^{(j,i)} = P_{t|t-1}^{(j,i)}B'(H_{t|t-1}^{(j,i)})^{-1}. \tag{3.15}$$

As mentioned in Kim (1994) or Kim and Yoo (1995), it is possible to introduce some approximations in order to make the Kalman filter implementable in practice. Instead of producing four sets of values $\alpha_{t|t}^{(j,i)}$ and $P_{t|t}^{(j,i)}$ at each step t, according to the four possible values of (i,j), the idea is to approximate $\alpha_{t|t}$ and $P_{t|t}$ by taking weighted averages over states at $t-1$, which allows to collapse these four sets of values into two. Thus, the following approximations are used:[5]

$$\alpha^j_{t|t} = \frac{\sum_{i=0}^{1} P(S=i, S_t=j|I_t,\theta)\alpha^{(j,i)}_{t|t}}{P(S_t=j|I_t,\theta)}, \tag{3.16}$$

$$P^j_{t|t} = \frac{\sum_{i=0}^{1} P(S_{t-1}=i, S_t=j|I_t,\theta)(P^{(j,i)}_{t|t} + (\alpha^j_{t|t} - \alpha^{(j,i)}_{t|t})(\alpha^j_{t|t} - \alpha^{(j,i)}_{t|t})')}{P(S=j|I_t,\theta)}. \tag{3.17}$$

The filtered probability of being in state $j \in \{0;1\}$ in period t conditional on the information available up to t can then be computed using Hamilton's filter (see Hamilton (1989)) and the previous Kalman filter formulas.

More precisely, if $\theta = (\phi_1, \phi_2, \text{diag}(\psi_1), \text{diag}(\psi_2), \gamma, \sigma_1^2, ..., \sigma_N^2, \beta_0, \beta_1, p_0, p_1)'$ is the vector of unknown parameters, if $f(\cdot)$ is the Gaussian density function, and if I_t is the information set available at t, it is possible to compute the filtered probability $P(S_t=j|I_t,\theta)$ through the following equations (based on Bayes' theorem):

$$P(S_t=j|I_t) = \sum_{i=0}^{1} P(S=j, S_{t-1}=i|I_t,\theta), \tag{3.18}$$

where

$$\begin{aligned} P(S_t=j, S_{t-1}=i|I_t,\theta) &= \frac{f(y_t, S_t=j, S_{t-1}=i|I_{t-1},\theta)}{f(y_t|I_{t-1},\theta)} \\ &= \frac{f(y_t|S_t=j, S_{t-1}=i, I_{t-1},\theta) \times Pr(S_t=j, S_{t-1}=i|I_{t-1},\theta)}{f(y_t|I_{t-1},\theta)}, \end{aligned} \tag{3.19}$$

$$\begin{aligned} f(y_t|S_t=j, S_{t-1}=i, I_{t-1},\theta) &= (2\pi)^{-N/2}|H^{(j,i)}_{t|t-1}|^{-1/2} \\ &\times exp\{-\frac{1}{2}(y_t - B\alpha^{(j,i)}_{t|t-1})'(H^{(j,i)}_{t|t-1})^{-1}(y_t - B\alpha^{(j,i)}_{t|t-1})\}, \end{aligned} \tag{3.20}$$

$$P(S=j, S_{t-1}=i|I_{t-1},\theta) = Pr(S_t=j|S_{t-1}=i,\theta) \times P(S_{t-1}=i|I_{t-1},\theta), \tag{3.21}$$

[5] For further details see Kim (1994) and the references therein

$$f(y_t|I_{t-1},\theta) = \sum_{j=0}^{1}\sum_{i=0}^{1} f(y_t, S_t = j, S_{t-1} = i|I_{t-1},\theta). \tag{3.22}$$

When $P(S_{t-1} = i|I_{t-1},\theta)$ is given, every term in equation (3.19) is known, due to the Markovian assumption on S_t. Thus, for any given value of θ, the associated filtered probability $P(S_t = j|I_t)$ can be computed recursively through equations (18) to (22).

The recursion is initialized with the steady state probability of being in state $j \in \{0;1\}$ at time $t = 0$:

$$P(S = 1|I_0,\theta) = \tfrac{1-p_0}{2-p_0-p_1}, \tag{3.23}$$

$$P(S_0 = 0|I_0,\theta) = 1 - P(S_0 = 1|I_0,\theta). \tag{3.24}$$

The previous formulas are also used to compute the log-likelihood function for the whole sample for any given value of θ, since the log-likelihood function for the sample can be written as:

$$\mathscr{L}(y,\theta) = ln(f(y_T, y_{T-1}, ..., y_0|I_T,\theta) = \sum_{t=1}^{T} ln(f(y_t|I_{t-1},\theta)), \tag{3.25}$$

and $f(y_t|I_{t-1},\theta)$ can be computed using formulas (18) to (22).

The likelihood function can thus be maximized through a numerical optimization algorithm.[6] Then, if $\hat{\theta}$ is the maximum likelihood estimator of θ, the Kalman filter formulas and Hamilton's filter can be used to compute the associated estimated factor and the associated filtered probabilities. In practice, the use of a numerical search algorithms appears to be relatively costly in terms of time and imposes limitations on the number of series included into the model. For instance, the use of four classic series (industrial production index, employment, retail sales and real income of households) already implies estimation of 22 parameters. Every additional series brings at least four more coefficients

[6]For our estimations we used Nelder-Mead simplex direct search with maximum function evaluations set to 2000, and tolerance for both function and dependent variables set to 0.001. We set the initial values of the parameters to the estimates of the same state-space model but without switch, i.e. the estimates of Stock and Watson (1989) DFM. The latter is, in turn, initialized with the OLS estimates of the system of equations where the first principal component is used as a proxy for the latent common component.

to estimate, which extends the estimation time and increases the complexity of the optimal point search. For this reason, we'll mainly apply this method using four series, as it was done by Kim and Yoo (1995) and as it is often done in case of one-step estimation. We also assume that the transition probabilities are time-independent, and in most of the paper we assume that the switch happens in the constant only, as described in (3.2).

Within this method it is thus assumed that the growth rate cycle of the economic activity is described as a common component of just a few series, so the choice of variables is essential and will be discussed in section 3.

3.2.3 Two-step estimation method

As we just mentioned, the main drawback of the one-step method is that, due to computational constraints, it can only be used with a small set of data. Another possible approach is to proceed in two steps in the following way:

1. The factor f_t is extracted from a large database of economic indicators according to equation (3.1) without taking its Markov-Switching dynamics into account. In the present paper, we use principal component analysis and we consider that the first principal component $\hat{f}_t$ gives a good approximation of the factor.

2. The parameters of the autoregressive Markov-Switching model described by equations (3.2) and (3.3) are estimated by maximum likelihood, with f_t replaced by $\hat{f}_t$. This amounts to fit the univariate model of Hamilton (1989) to the estimated factor $\hat{f}_t$, which is taken as if it were an observed variable. The filtered probability of recession $Pr(S_t = 1|I_{t,})$ is then calculated as in (3.18).

Let us recall that if a Markov switching model is estimated with Hamilton's method for an observable variable, say z_t, then the log-likelihood

$$ln\, f(z_T, z_{T-1}, ..., z_0|I_T, \theta) = \sum_{t=1}^{T} ln\, f(z_t|I_{t-1}, \theta) \tag{3.26}$$

is computed along the same lines as in equations (18)-(22) and has to be maximized through a numerical optimization algorithm too. However, as the number of parameters that have to be estimated is small in this case, the maximization of the loglikelihood through a numerical procedure does not raise any specific problem. This is one of the reasons why the two-step procedure is attractive: in the second step, the number of parameters that have to be estimated is small.

Another attractive feature of the two-step procedure is that it allows to consider a large amount of series, which are used to build the estimated factors in the first step. Here we take the first principal component of 151 economic indicators concerning the production sector, financial sector, employment, households, banking system, international trade, monetary indicators, major world economic indicators, business surveys and others. This large set of series is more likely to reflect the business cycle than a small set of series, as it is used in the one-step procedure (as we said before, many authors use only four series when they want to estimate this kind of model).

Finally, as the second step of the two-step procedure is easily tractable, it is possible to introduce additional switching parameters, and to estimate richer models this way. For instance, it is possible to consider a switching variance and to replace σ_η^2 with $\sigma_{\eta S_t}^2$.

The two-step procedure has been employed in several papers (see the non-exhaustive list given in the introduction) but in most of them, the number of series under study is small or moderate. Further, Camacho et al. (2012) argue that this two-step procedure faces misspecification problems, since the Markov-switching dynamics are not taken into account in the first step. We expect that, under standard assumptions, the two-step procedure gives in fact consistent estimators of the parameters. The complete proof of this consistency is addressed in a companion paper (which is still in progress at this time), but the main idea is that, under these standard assumptions, the first principal component consistently estimates the factor. Indeed, as (S_t) is supposed to be a stationary ergodic Markov chain, (f_t) is a stationary process and all the usual sets of assumptions which are commonly used to assert the consistency of PCA for large N and large T (see Bai (2003), Stock and Watson (2002) for instance) can be employed in the present setting.

To conclude this section, let us also mention that PCA is not the only way to get a consistent estimator of the factor in the first step. In future work, we intend to extract the factor in the first step either with the two-step estimator based on Kalman filtering, which has been proposed by Doz et al. (2011), or with the QML estimator Doz et al. (2012). It seems indeed promising to use these two methods in the first step of the present framework, as they may provide more efficient estimators and as they allow for mixed frequency, missing data, and data with ragged ends.

3.3 Data, reference dating and quality indicators

3.3.1 The dataset

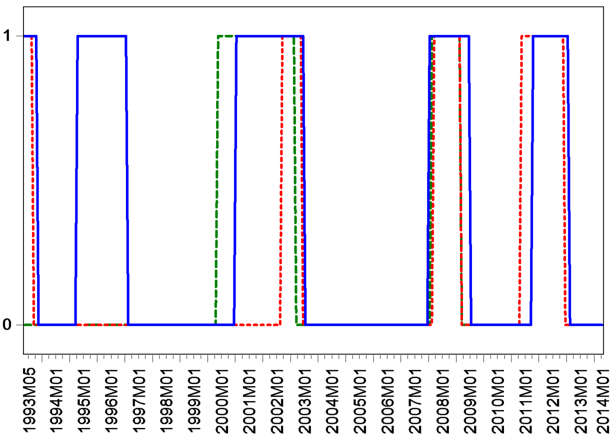
For the purpose of comparison one would like to run the two estimation methods on the same dataset. However due to the different requirements on the number of series in the database for each method (large dataset for the two-step method in order to get the consistency of the PCA factor estimate, small dataset for the one-step method to obtain the convergence of the algorithm), we are unable to perform this kind of analysis. We therefore use a separate dataset for each method.

The database for the two-step procedure is constructed following Stock and Watson (2014) for the US and Bessec and Doz (2014) for France. It contains 151 monthly series spanning the period May 1993 - March 2014.[7] The data cover information on the production sector, financial sector, employment, households, banking system, business surveys, international trade, monetary indicators, major world economic indicators, and other indicators.

For the one-step method it is crucial to select series properly. The series must be an indicator of the economic cycle and should be available in monthly frequency. We choose 25 series out of the 151 series of the database for the two-step method and to use them in combinations of four, overall $C_{25}^4 = 12650$ combinations. The strategy of trying all possible combinations of 4 out of 25 may seem too bulky and inelegant, however we deliberately avoided any data selection technique in order to minimize its possible impact on the output of the one-step results. The selection was made on the basis of the existing literature on the one-step method applied to business cycle analysis. To the four classical indicators for business cycle dating of the US economy (total personal income, total manufacturing and trade sales, number of employees on nonagricultural payrolls, total industrial production index) we added series used in Kholodilin (2006) (French stock market index CAC40, interest rates on the 3 months and 12 months government bonds, imports and exports), selected series of business surveys proved to be useful by Bessec and Doz (2014), the components of the OECD Composite Leading Indicator, as well as several series characterizing the dynamics of the major trade partners (Germany, USA, Asia). Since almost all of these series have been already used in the analysis of the French business cycle (and the others are likely to comove with it), we suppose that the common component of each combination can be considered as an estimate of the

[7] The trade-off between the sample size and the number of cross-sections made us restrict the dataset to just 21 years of observations. A longer period (starting with 1990) would reduce the number of cross-sections to 97, while the full original balanced database (213 series) starts in February 1996.

FIGURE 3.1: Economic cycles chronologies according to OECD, ECRI and CEPR



Note: The blue solid line - OECD dating, the green dashed line - CEPR dating, red dotted line - ECRI dating. The recession phase corresponds to 1, the expansion phase corresponds to 0.

business cycle.

All series are seasonally adjusted, tested for the presence of unit roots and transformed to stationarity if necessary, then centralized and normalized. Detailed lists of series for both methods are given in Tables A.1 and A.2 of Appendix A.

3.3.2 Reference dating

In order to measure the quality of the results of each of the two methods, we need to compare it to some reference business cycle chronology. The choice is not obvious, as the true dating is unknown, whereas the estimates of the true dating provide different sets of turning points. To our knowledge, there are at least three open source dating chronologies for the European countries: OECD[8], CEPR[9] and ECRI[10]. Note that INSEE does not publish any official business cycle dating. Figure 3.1 below shows that these chronologies indeed do not coincide in the starting and the final points of recessions and in the duration of economic cycle phases. Moreover, OECD detects a recession of April 1995 - January 1997 which other institutions do not identify.

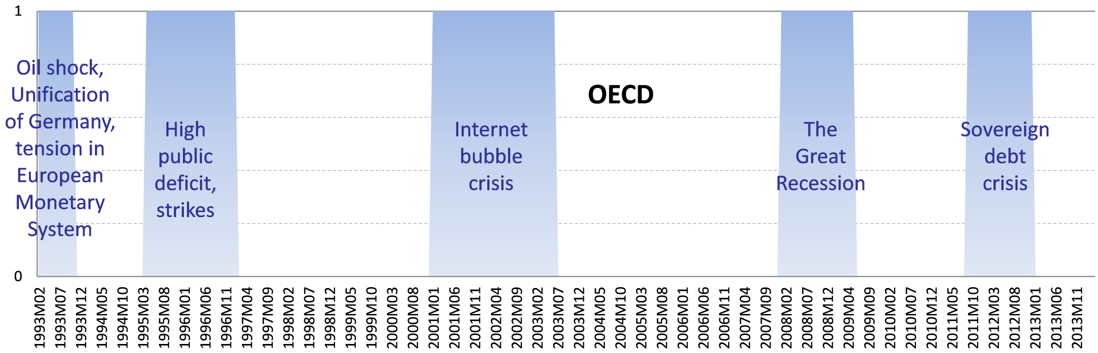
The difference obviously lies in the methodology and the data taken into consideration. The OECD dating is the output of the Bry-Boschan algorithm (see Bry and Boschan (1971)) applied to the Composite Leading Indicator (CLI), which is an aggregate of a

[8]http://stats.oecd.org/mei/default.asp?rev=2
[9]http://www.cepr.org/content/euro-area-business-cycle-dating-committee
[10]https://www.businesscycle.com/

fixed set of nine series, highly correlated to the reference series (industrial production index or GDP series).[11] The turning point chronologies of CEPR are obtained from the balance of expert opinions on the basis of series selected by the experts involved. The ECRI index is the output of an undisclosed statistical tool on the undisclosed (but probably the most information-rich) dataset.

In this paper we take the OECD dating as a benchmark as it relies on a clear and replicable algorithm. Therefore, the time sample that we consider covers 5 crises in the French economy as determined by OECD (see Figure 3.2):

Figure 3.2: OECD reference turning points for the French economy, 1993-2013



Note: 1 corresponds to recession, 0 - to expansion

- March 1992 - October 1993: the crisis caused by the oil shock following the first Gulf War, German reunification and tensions in European Monetary system;

- April 1995 - January 1997: rather a slowdown in economic growth rates than a real recession, with only one quarter of slightly negative (-0.011) growth rate, caused by the decrease of high public deficit and the consequent strikes throughout the country;

- January 2001 - June 2003: the Internet bubble crisis;

- January 2008 - June 2009: the Great Recession, the global financial crisis;

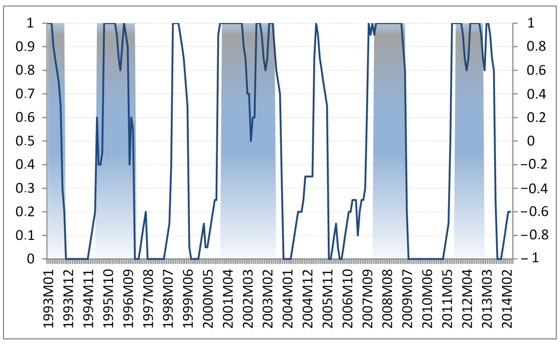
- October 2011 - January 2013: the sovereign debt crisis.

[11] The CLI components are: 1. New passenger car registrations (number) 2. Consumer confidence indicator (% balance) 3. Production (Manufacturing): future tendency (% balance) 4. SBF 250 share price index (2010=100) 5. CPI Harmonized All items (2010=100) inverted 6. Export order books (Manufacturing): level (% balance) 7. Selling prices (Construction): future tendency (% balance) 8. Permits issued for dwellings (2010=100) 9. Expected level of life in France (Consumer Survey) (% balance). All series are detrended, and seasonally, calendar- and noise-adjusted. They are selected so that they have a cycle pattern similar and coincident (or leading) to the one of the reference series. Until April 2012 the industrial production index was taken as a reference series, replaced by monthly estimates of GDP growth afterwards.

It can be argued that OECD dating can not be used as a reference because it represents the chronologies of the growth cycle, whereas we use MS-DFM to identify the growth rate cycle (for most series, in order to achieve stationarity in data we use differences of logarithms). In our exercise, we avoid cyclical component extraction on purpose as it implies additional complications inherent to the definition of a trend. However, we support our choice by the fact that the OECD chronology is the closest to the other cyclical indicators calculated for France. In the working paper by J. and Tallet (2008) (and in a similar paper Bardaji et al. (2009)), the authors propose a reference dating on the basis of the cyclical component of GDP extracted with the Christiano-Fitzgerald filter (see Christiano and Fitzgerald (2003)). We reproduce these estimates on the basis of monthly interpolated GDP growth data. The dating we obtain is indeed very close to OECD results. At the same time, it is rather close to the dating obtained by Billio et al. (2007) for Eurostat (see Figure 3.3).

Note that the dating on the basis of the Christiano-Fitzgerald filter has two additional recessions (in 1998-1999 and 2004-2005) which are not present in the OECD dating. Interestingly, the Reversal Index also detects these additional recessions, having spikes of high probability of recession in 1998 and 2005 (see Figure 3.4).[12] This discrepancy might be due to an important feature of the Bry-Boschan procedure, which is the existence of a lower bound of phase duration (15 months). Consequently, short recessions or expansions do not appear in the OECD chronology.

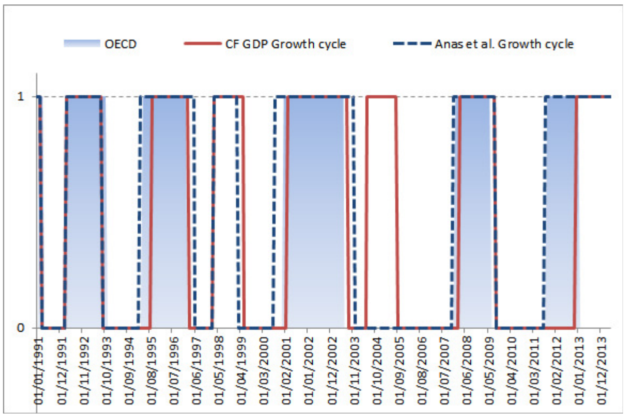
FIGURE 3.4: The index of reversal and OECD reference dating



Note: The index of reversal (solid blue line, right axis) and OECD reference dating (shaded areas, left axis).

[12]The Reversal Index (l'Indicateur de Retournement) published by INSEE is the index comprised between -1 and 1 which shows the difference between the probability to be in expansion in the current period and the probability to be in recession in the current period. The index is based on the business surveys about the current, past and future perceptions of the economic situation.

Figure 3.3: Turning point chronology of the French economy



Note: Shaded areas correspond to the OECD dating, the blue dashed line corresponds to the dating for the French economy produced for Eurostat by Billio et al. (2007), the red solid line corresponds to the GDP growth cycle extracted with Christiano-Fitzgerald filter. 1 to recession, while 0 corresponds to expansion. A Christiano-Fitzgerald filter is applied to the series of French GDP in levels (bandwidth 6 to 40 quarters), the turning points are considered to take place in the second month of a quarter.

Indeed, in both cases INSEE detected a temporary deterioration of the economic activity due to different reasons. In 1998-1999 France experienced a significant decline in the net external trade. Undermined by the Asian and Russian crises, the external demand from Japan, China, and Russia, as well as other developing Asian countries and even the UK, Belgium, and Italy, fell dramatically - from 10% growth rate in 1997 to only 4% in 1998. The depreciation of yen and dollar contributed to the appreciation of the real effective exchange rate of franc. In general, the external balance of France decreased by 7.1%, which resulted into negative contribution to the GDP growth (-0.4 pp).[13] The producers were pessimistic about future activity (also worried about the financial crisis and reducing prices for energy and oil, which threatened to turn into disinflation), decreasing their investment and limiting the inventories.[14] In 2005 the external demand of France decelerated substantially due to uncertainty in the economic situation in the US and Japan caused by the oil price shock. Producers in manufacturing and service acted with caution: the prices for raw materials were rising, and the euro was appreciating in real terms, the saving rate of households fell, the GDP quarterly growth was declining, too.[15,16]

To summarize, the OECD dating largely coincides with the other existing cyclical indicators for the major recessions, however some other indicators may detect additional shorter recession episodes.

[13]INSEE PREMIERE, N659 - June 1999
[14]INSEE CONJONCTURE, Note de conjoncture, December 1998
[15]INSEE CONJONCTURE, Note de conjoncture, Mars 2005
[16]Interestingly, Bruno and Otranto (2003) also find similar signals of 1998-1999 and 2005 for the chronology of the Italian economic cycle.

3.3.3 Measures of quality

To assess the quality of the results of each of the two methods we use the three following indicators:

- **Quadratic probability score.** This indicator shows the average error of filtered probability as an average quadratic deviation from the reference dating. A high QPS indicates a low quality of the fit of the model.

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(RD_t - P(S_t = 1|I_t))^2,$$

  where T is the number of periods in the sample, RD_t is the reference dating series of 0 and 1 (1 corresponding to recession, 0 to expansion), and $Pr(S_t = 1|I_t)$ is the filtered probability of being in a recession in period t.

- **False positives.** This indicator counts the number of wrongly predicted periods. Here we set the threshold probability on the intuitive level of 0.5.

$$FPS = \sum_{t=1}^{T}(RD_t - I_{P(S_{t=1}|I_t)>0.5})^2,$$

  where $I_{P(S_{t=1}|I_t)>0.5}$ is the indicator function equal to 1 if the estimated filtered probability is higher than 0.5 (determines recession) and 0 otherwise. The lower the FPS, the more qualitatively accurate the model.

- **Correlation.** An accurately estimated filtered probability should have a high correlation with the reference dating. We use a simple sample correlation $Corr$ between the two series.

3.4 Estimation results

3.4.1 One-step method

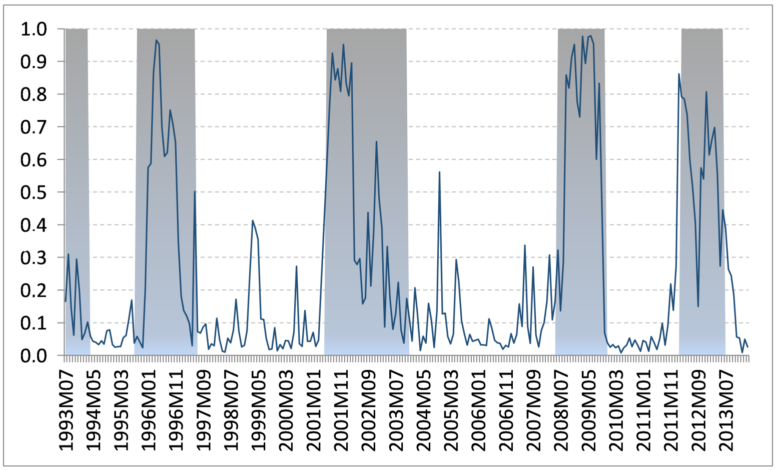
Informative series

The estimation of 12650 combinations did not produce 12650 outputs as for most of the combinations convergence was not achieved, or the series combination produced a factor that does not have a nonlinear structure. Therefore, only 575 combinations achieved convergence, and only 424 of them have interpretable filtered probabilities. Out of this number, we have retained 72 results that are informative in terms of signals of past recessions. Interestingly, the best candidate for the benchmark results - the combination of four series used by Kim and Yoo (1995), Kim (1994), Chauvet (1998) and others for business cycles of the US (total index of industrial production, employees in nonagricultural payrolls, total personal income less transfer payments, total manufacturing and trade) did not achieve convergence.

We construct the frequency rating of economic series (given in Table A.3 of Appendix B) for the integrity of all interpretable results of the one-step estimation. Some series turned out to have weak explanatory content, such as CPI index or CAC-40 financial index, the latter entering none of the successful combinations. Others did much better: the construction confidence indicator, capacity utilization, exports, the retail trade confidence index and the unemployment rate appear each in 22, 21, 20 and 19 combinations, respectively. This allows us to suggest that the contribution of these indicators is important for the final aggregate factor to follow bi-state dynamics. Interestingly, CPI and the stock market index both enter the OECD CLI, but they do not seem to be very informative for the turning points detection in our framework.

To illustrate the results that we considered as interpretable, we present the output of one of the plausible combinations in Figure 3.5. It consists of the four most frequent indicators that we mentioned above: unemployment rate, exports, retail trade and construction confidence indicator. The resulting filtered probability is one of the best in terms of fit to the OECD official dating.

FIGURE 3.5: The result of one-step estimation: the filtered probability of recession and the reference dating



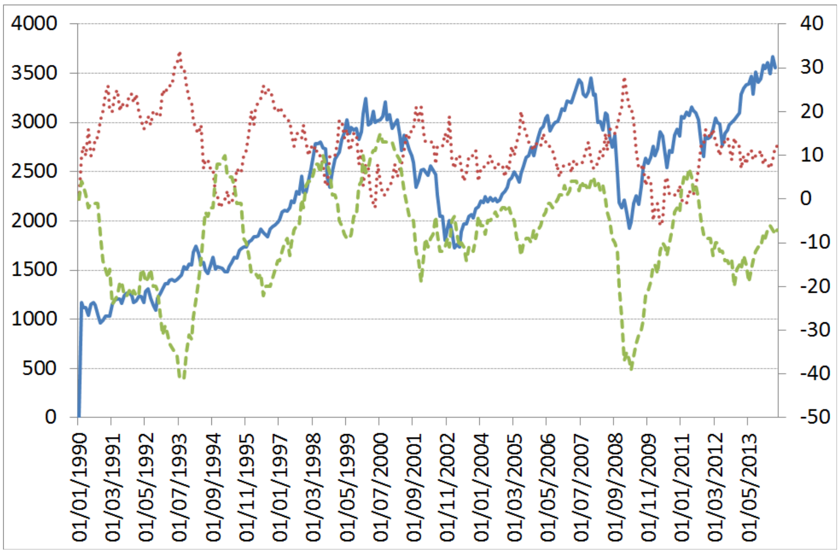
The estimates are obtained using data on unemployment rate, exports, retail trade and construction confidence indicator. The filtered probability of recession is depicted by the blue line, and the reference dating by OECD (1 corresponds to recession state) is marked by shaded areas.

As we can see from this figure, this combination produces a factor probability that captures four out of five crises if we consider the economy to be in recession if the filtered probability of a recession is higher than 0.5. One can notice two important features of this example: first, there is an extra signal in 2004; second, not all the crises are explained equally well. These pitfalls are often present in the other outcomes, so we discuss each of them in detail.

Extra signals

In general, out of 72 combinations only 27 do not produce any extra signals of recession. The other 45 combinations give an additional alert in the end of 1998, or another one around mid-2005, or both. Among the series with the highest loadings that appear relatively more often in such combinations than in the other ones are manufacturing finished goods stocks level, returns on the FTSE equity index, and the Manufacturing Industrial Confidence Indicator. Indeed, in Figure 3.6 we can see that all three series underwent significant downturn in 1998-1999, while in 2005 stocks of manufacturing finished goods and the manufacturing confidence index fell back to the levels of the end of the Internet bubble crisis. However, these events are not captured by the OECD dating.

FIGURE 3.6: False signals suspects



FTSE 100, All-Share, Index, Price Return, End of Period, GBP (solid line, left axis), Manufacturing Finished Goods Stocks Level (dotted line, right axis), Manufacturing Industrial Confidence Indicator (dashed line, right axis)

As we have mentioned above, these signals are not misleading in the sense of producing a false alert of recession when the economy is actually growing, and they correspond to a real deterioration of economic conditions. However, for the closest match to the OECD reference, these signals should be avoided. The one-step approach allows to do so, since one can exclude the series that are likely to produce extra signals from the dataset.

Different set of series for different crises

As for OECD detected recessions, it is important to keep in mind that none of them (at least the five recessions we consider here) had the same origins as any of the other, so it is possible that the determinants of economic activity evolved with time, and so it is likely that the common factor of a particular set of series does not reflect the Great recession as well as it reflected the crisis of 1992-1993. However, in order to construct a universal instrument, it is preferable to find series that would capture the recession in all cases, if possible. For this purpose we compare the quality indicators of 72 sets of variables for each crisis separately. Table A.4 in Appendix B summarizes the information on the best combinations by crisis. Here FPS shows the proportion of months of each crisis incorrectly determined as expansion, i.e. the lower FPS, the better a crisis is captured.

We can see that:

- the combination consisting of volume of total retail trade, unemployment rate, trade balance and order books in the building industry, with the highest loadings on unemployment, is the best to detect the first and the last crisis and captures the second crisis well, too;
- the combination consisting of new passenger cars sales and registration, retail trade orders intentions, export, confidence indicator in services, with the highest loadings on retail trade and Confidence Index in services, is leading in case of the second, the third and the fourth crises, being significantly superior to the other combinations for the third and the fourth recessions;
- although good during certain periods of the timeline, unfortunately none of these sets of variables could be used as a 'core' set due to their relatively poor performance on the expansion periods and non-detected crises.

The set of data contained in these two combinations appears to be sufficient to identify all five crises with a special role given to unemployment, retail trade orders intentions and confidence index in services.

The finally selected information set

Considering the observations on the effects of different series on the final filtered probability, we conclude that a good information set (relative to OECD reference) would:

1) contain the series that determine all fivecrises,

2) not contain the series that produce extra signals,

3) perform well in general in terms of QPS, FPS and $Corr$.

The top 25 combinations with the lowest QPS and FPS measures and the highest $Corr$ are given in Table A.5 in Appendix B. The first eight are in the best 10% by all three indicators, so seven of them (we exclude the second combination because of the presence of extra signals) could be candidates for the core sets of economic indicators that enable to match the OECD dating closely. The graphs of corresponding seven filtered probabilities are given in Figure B.1 of Appendix B.

It is not surprising that there are several "best" sets of variables, as the restriction of the model to comprise only four series is just a technical limitation, and the factor matching the dynamics of the economic activity is determined by many more series. The analysis of the factor loadings of these seven combinations can give us an idea of the economic indicators that play the most important role. According to our estimations, the heaviest factor loadings belong to (see Table A.5 in Appendix B):

- France, OECD MEI (Enquete de Conjoncture INSEE), Retail Trade Orders Intentions, SA;
- France, INSEE, Metropolitan, Unemployment, Job Seekers, Men, Total, Categories A, B & C, Calendar Adjusted, SA;
- France, OECD MEI (Enquete de Conjoncture INSEE), Manufacturing Business Situation Future, SA;
- France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA.

The first two of these indicators were also determined as components of the Growth Cycle Coincident indicator by Billio et al. (2007).

Among the other indicators contributing to the factors in the seven selected combinations are:

- France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Synthetic Index, SA,

- France, INSEE, Domestic Trade, Vehicle Sales & Registrations, New, Passenger Cars, Total, Calendar Adjusted, SA,
- France, OECD MEI, INSEE, Total Retail Trade (Volume), SA, Change P/P,
- France, OECD MEI, INSEE, Manufacturing Finished Goods Stocks Level, SA,
- France, INSEE, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR,
- France, INSEE, Foreign Trade, Export, Calendar Adjusted, SA, EUR,
- Japan, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, Nikkei 225, Balance,
- United States, Equity Indices, S&P, 500, Index (Shiller), Cyclically Adjusted P/E Ratio (CAPE),
- France, Service Surveys, DG ECFIN, Services Confidence Indicator, Balance, SA.

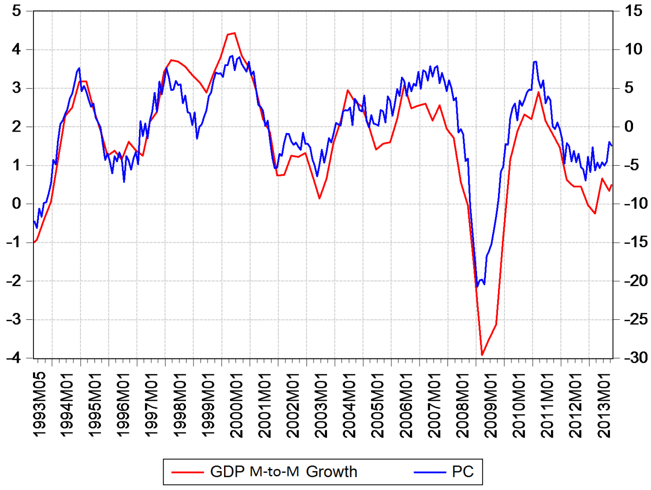
As an output of this analysis we have thus retained 13 out of 25 series which can be considered as essentially informative of the French business cycle. We tried to use the one-step method on these 13 series simultaneously, in order to take into account all the main information. Unfortunately, the optimization algorithm did not achieve convergence while searching for the likelihood-maximizing set of parameters, although, with the parameters set to their initial values, the filtered probability calculated at the initial values of parameters (obtained with OLS) captures all the five crises without detecting any extra recessions, as expected (see Figure A.2 in Appendix B). Therefore, since it seems unfeasible to use the information contained in the above listed 13 series simultaneously within the one-step approach, the results of the seven combinations could be used as complements.

3.4.2 Two-step method

First step: PCA

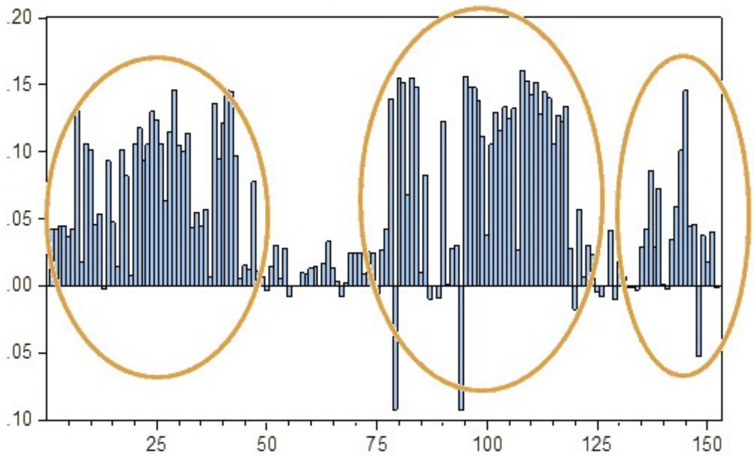
In the first step of the procedure we extract the first factor by principal component analysis. The first principal component that we use as a proxy for the factor in the two-step method describes 23.43% of the total variance, which is quite reasonable when considering the size and heterogeneity of the database. The dynamics of the first component and the factor loadings are presented below in Figure 3.7 and Figure 3.8. One can note that it is close to the dynamics of GDP growth, so the factor is relevant. Indeed, the correlation on the whole sample is equal to 0.91, while the correlation on the shorter period ending in December 2007 to eliminate the impact of the Great Recession is 0.895.

FIGURE 3.7: First principal component and monthly GDP growth rate



Note: the solid blue line corresponds to the dynamics of the first principal component of the full dataset (left axis), the dashed red line corresponds to the French GDP growth series (left axis). The quarterly GDP growth series were converted into monthly series via linear interpolation.

FIGURE 3.8: Factor loadings corresponding to the first principal component



The three groups of highest loadings of the first component correspond to (in circles, from left to right): 1) production and consumption series, disaggregated; 2) business surveys; 3) series on the world economy.

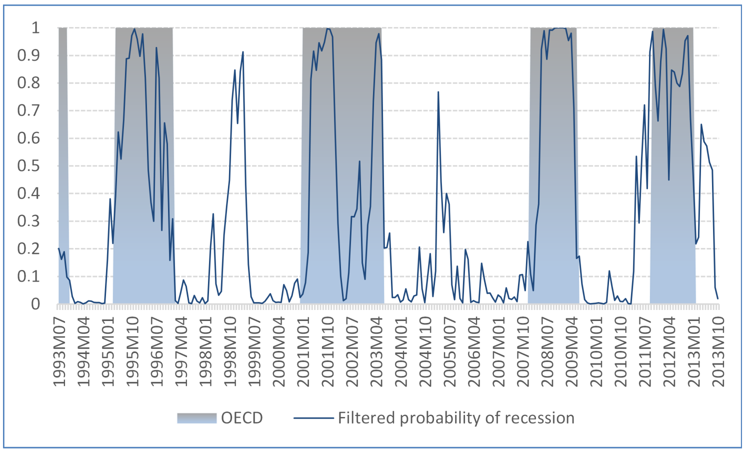
The three groups with the highest loadings corresponding to the first component belong to: 1) production and consumption series, disaggregated; 2) business surveys; 3) series concerning the world economy. The first component therefore captures the present behavior of firms and households (including their expectations about the short-term future) and the impact of foreign economies and pays less attention to the banking and financial sector, monetary aggregates, balance of payments and currency indicators.

Second step: estimation of a Markov-Switching model

Basic specification, switch in mean.

In the second step of the two-step estimation procedure, we estimate a Markov-switching model as defined in equation (3.2), with the unobservable factor replaced by the first principal component estimated in the first step.[17] The results are quite satisfactory, with the filtered probabilities capturing all the crises well (but the first one) and without sizable leads and lags (see Figure 3.9). As expected, the estimations provide a positive constant for the expansion periods, and a negative one for recessions: $\mu_0 = 1.04$, $\mu_1 = -1.77$, respectively (the estimates are significant at 1% level of significance). For this specification, $QPS = 0.1278$, $FPS/T = 0.1872$, $Corr = 0.75$, and the average lag of the identification of the beginning of a recession is 0.75 months, while the end of a recession is detected one month earlier. Note that this result is comparable to the average result of our one-step estimations ($QPS = 0.1346$, $FPS/T = 0.1779$, $Corr = 0.69$).

FIGURE 3.9: Filtered probability of recession, the two-step estimation



The specification of the MS-DFM includes switches in constant, non-switching autoregressive coefficients and variance, OECD reference dating is marked by shaded shaded areas.

The extra signals of 1998-1999 and 2004-2005 are clearly detected by the first component. This may be explained by the fact that the dataset includes the series that induce extra signals for the one-step method, as well as a number of other series that experienced shocks in these periods. We did a simple exercise by trying to eliminate these series from the dataset. It turned out to be impossible to get rid of all extra signals without

[17] Following Kim and Yoo (1995), we put two autoregressive lags in the baseline specification. This assumption turned out to be plausible: the correlogram of the first principal component has high partial autocorrelation for the first two lags. The choice of two lags was also confirmed by Akaike and Schwarz information criteria estimated on the model with one, two and three autoregressive lags.

deteriorating the signals on the OECD recessions. The removal of series undermines the performance of the two-step method and deprives it of its most valuable advantage - the large scale of the dataset.

Besides extra signals in 1998 and 2005, we can observe a transitory improvement in the middle of the Internet bubble crisis and the earlier detection of the beginning of the sovereign debt crisis, also omitted by the OECD. Similarly, the reasons for this amelioration can be tracked in the INSEE reports.[18]

Alternative specification, switches in mean and variance.

We take advantage of the possibility to introduce switches into other coefficients of the model to check whether this improves the detection of the turning points. Now we allow the variance of the error term in the factor dynamics to be state specific, too, so the model of factor dynamics becomes:

$$f_t = \beta_{S_t} + \phi_1 f_{t-1} + \phi_2 f_{t-2} + \eta_{S_t}, \tag{3.27}$$

where $\eta_{s_t} \sim N(0, \sigma^2_{S_t})$. While on average performing as well as the basic specification ($QPS = 0.1278$, $FPS/T = 0.1885$, $Corr = 0.67$), the alternative specification is slightly better in capturing the beginnings and ends of recessions (the identification lag is 0 and 1 months on average, respectively).[19] As before, the estimations provide a positive constant for the expansion period, and a negative one for recessions $\mu_0 = 1.22$, $\mu_1 = -1.52$. The volatility of the factor dynamics is estimated to be almost two times higher during recessions ($\sigma_0 = 0.4$, $\sigma_1 = 0.75$). The estimates of the other parameters are given in Table A.6 in Appendix C. Again, the filtered probabilities produced by this specification capture all the crises well (but the first one) and without sizable leads and lags. The dynamics of filtered probabilities for this specification resembles the one for the basic specification, so we do not report the graph here.

[18] INSEE observed the improvement of the business climate in 2001 primarily due to the subjective perception that the US had passed the trough of the business cycle; rebound growth in Asia, Germany and the negative oil price shock improved the expectations of investors and entrepreneurs, while the decrease in taxes gave an extra stimulus for household consumption, increasing their purchasing power (INSEE CONJONCTURE, Note de conjoncture, Mars 2002). The reasons for early peaks in 2011 are the deterioration of the business climate in France, the earthquake in Japan, anti-inflation policies in developing countries, as well as budget consolidation politics in the developed countries, positive price shocks for commodities (oil included) increased production costs. All this led to a certain pessimism among French investors (Point de conjuncture October 2011, INSEE).

[19] We also tried specifications with switching autoregressive coefficients and different combinations of switching parameters, but none of them were performing as well. To save space, we do not report the results here.

3.4.3 Comparison: one-step vs two-step

We compare the average performance of the one-step method and the two-step method in the baseline specification (lines "One-step method, average" and "Two-step method, full dataset" in Table 3.1). The difference in QPS and FPS is negligible ($QPS = 0.13$, $FPS/T = 0.18$ for the average one-step method versus $QPS = 0.13$, $FPS = 0.19$ for the two-step method), whereas the correlation with the OECD dating is only slightly higher for the one-step method ($Corr = 0.69$ versus $Corr = 0.67$). So, on average it is difficult to rank the performance of the methods. However, taking into account that the extra signals are responsible for part of the QPS and FPS/T of the two-step method, the two-step method is more precise in detecting OECD recessions. In particular, the two-step method is much more accurate with respect to the beginning and the end of recessions, with a tendency to indicate the beginning of a recession on average one quarter of a month earlier; the one-step method dates the beginning 2.5 months late and the end 2.6 months early, on average. In general, both methods produce early estimates: for the one-step method, the data in each of the retained combinations are updated with one month or even zero months lag. This means that the phase of the business cycle in January can be determined either in February or March, with no need to wait for the release of quarterly OECD dating in April. Though the gain in time is not very big, it may still be of a great important for policy makers. For the two-step method, the estimates are available in two months, which is still less than the timing of the OECD. In this respect, the estimation of the factor on the first step with the help of one of the procedures proposed by Doz et al. (2011) and Doz et al. (2012) is very promising since it allows to have the estimator of the factor based on the available information only, without waiting until all series in the database are updated. We leave this exercise for further research.

As for the parameter estimates, the two methods give qualitatively similar results in terms of values of coefficients (see Table A.6 in Appendix C): there are two distinct regimes, which are characterized by a negative constant in the recession state and a positive constant in the expansion state. The difference between the two constants varies in absolute value as the magnitude of factors is either determined by the underlying economic indicators (for the one-step method) or is estimated up to a constant (in case of the two-step method). The estimates of the transition probabilities are similar, too: the phases of the French growth rate cycle are very persistent, with the probability to stay in expansion (on average, $p_0 = 0.96$) a bit higher than the probability to stay in recession (on average, $p_1 = 0.91$). All other estimates of the Table A.6 cannot be interpreted directly as they refer to different series and are reported for completeness. The estimation

TABLE 3.1: The comparison of one-step and two-step estimation results

	QPS	FPS/T	Corr	Start lag	End lag	Timing
Benchmark Hamilton univariate MS-AR model						
Hamilton's AR-MS on IIP (benchmark)	0.3679	0.5231	0.0894	∞	∞	0M
MS-DFM (Kim and Yoo (1995))						
One-step method, average	0.1346	0.1779	0.6985	2.5	-2.6	1M
One-step, combination 1	0.1287	0.1383	0.7155	0.6	0.4	1M
One-step, combination 2	0.1254	0.1779	0.6899	1.8	-0.2	1M
One-step, combination 3	0.1328	0.1818	0.7431	3.4	-3.8	0M
One-step, combination 4	0.1412	0.1818	0.7006	5.2	-4.2	1M
One-step, combination 5	0.1184	0.1858	0.7082	3.6	-3.6	1M
One-step, combination 6	0.1493	0.1937	0.6815	3	-8	1M
One-step, combination 7	0.1492	0.1976	0.5607	1.8	0.6	1M
Two-step method on 13 series	0.3259	0.3287	0.1649	∞	∞	1M
Two-step method on 25 series	0.1207	0.2083	0.5703	2.5	-0.5	1M
Two-step method, full dataset	0.1315	0.1926	0.6712	0.75	-1	2M
Other specifications of MS-DFM						
Two-step method, full dataset, switching σ^2	0.0737	0.1885	0.6724	0	1	2M
Two-step method, switching μ and σ^2, MS-AR(4)	0.0658	0.1762	0.6751	0	0.8	2M
Two-step method, switching μ and σ^2+ pc2	0.2495	0.3648	0.3953	∞	∞	2M
Two-step method, switching μ and + pc4	0.1027	0.1803	0.6602	1.75	-1.75	2M
Two-step method, switching μ and σ^2+ pc4	0.0699	0.1721	0.7058	2	0.75	2M
Other results for the French economy						
Kaufmann (2000)	-	0.2151	-	-	-	-
Chauvet and Yu (2006)	-	0.3777	-	-	-	-
Chen (2007)	-	0.2839	-	-	-	-
Kholodilin (2006)	0.152	0.3333	-	-	-	-

For the composition of combination i see Table A.5 and Table A.2. Start lag - the number of lags between the estimated beginning of a recession and the OECD determined beginning; End lag - the number of lags between the estimated end of a recession and the OECD determined end; T is the number of periods in the sample

of the model with both methods on an expanding sample showed that the estimated coefficients and the resulting filtered probabilities are robust when the sample is up to 50 points shorter, however, the convergence is not always achieved for the one-step method.

The comparison to results in the preceding literature by Kaufmann (2000), Chauvet and Yu (2006), Chen (2007), Kholodilin (2006) shows the advantage of the MS-DFM in detecting business cycle turning points, although it should be considered with care since we compare the results on slightly different (although overlapping) time spans.

The final datings for both methods are similar in general, although there are some discrepancies to the OECD dating (see Table 3.2).

Table 3.2: Final dating produced by one-step procedure on 7 best sets of data, two-step procedure and OECD dating

		Comb 1	Comb 2	Comb 3	Comb 4	Comb 5	Comb 6	Comb 7	2step	OECD
1st crisis	P							1993m02		1992m02
	T							1993m10		1993m10
2nd crisis	P	1995m07	1995m06	1995m08	1995m08	1995m09	1995m07	1995m09	1995m01	1995m03
	T	1996m12	1996m12	1996m09	1996m10	1996m10	1997m01	1997m05	1997m01	1997m01
1st false signal	P								1998m09	
	T								1999m04	
3rd crisis	P	2001m01	2001m01	2001m02	2001m01	2001m02	2001m01	2001m04	2001m03	2000m12
	T	2003m07	2003m06	2003m04	2002m11	2003m03	2002m07	2003m12	2003m09	2003m06
2nd false signal	P								2005m02	
	T								2005m07	
4th crisis	P	2007m09	2007m09	2008m04	2008m04	2008m04	2008m04	2008m04	2008m04	2007m12
	T	2009m09	2009m11	2009m05	2009m06	2009m09	2009m04	2009m09	2009m08	2009m06
5th crisis	P	2011m09	2011m09	2011m09	2012m07	2012m05	2011m09	2011m06	2011m03	2011m09
	T	2013m07	2013m08	2012m10	2012m11	2013m07	2012m11	2013m07	2013m08	2013m01

Note: Comb i stands for Combination i. For the composition of Combination i see Table A.5 and Table A.2

Let us note that although the two methods provide rather similar results, we suggest using the two-step method as it is more robust and easier to estimate and allows to consider big datasets. Furthermore, since the factor is considered as observed, the baseline specification can be extended to include additional autoregressive lags and other explanatory variables. For example, we suggest the following possible extensions of the baseline model (section "Other specifications of MS-DFM" in Table 3.1): introduction of switching variance, use of two more lags in the autoregressive structure, inclusion of the second and the fourth principal component in the dynamics equation of the factor to take additional information into account. Some of these extensions increase the performance of the baseline specifications, although the improvement is rather minor if not negligible. Nevertheless this observation leaves room for further research in the direction of multifactor markov-switching dynamic factor models with a general VAR structure.

As an additional validity check, we make two more comparisons. The first one serves to evaluate the gain from the multivariate analysis. For this purpose we made a comparison with the results of a simple classical Hamilton (1989) model with two autoregressive terms and a switching constant estimated on the growth rate of the index of industrial production (see Table 3.1). One can see that, contrary to the United States, in the case of France this series contains much less information about the business cycles, at least for the period under consideration. The MS-AR model produces only one signal corresponding to the 2008 recession. This poor performance is reflected in our quantitative indicators as high QPS and FPS and very low correlation with the reference.

Secondly, to understand the role of the number of series for the two-step method, we analyze its performance on smaller datasets. A number of papers (see, for example, Boivin and Ng (2006) and Bai and Ng (2008)) state that using big datasets for factor analysis is not always better than using smaller datasets of appropriately selected series. To evaluate the role of the number of series for the two-step method we estimate the baseline specification on the subset of 25 series which were used for the one-step method as well as on the 13 series which were finally retained ("Two-step method on 25 series" and "Two-step method on 13 series", respectively, in Table 3.1). As we can see, the use of 13 series does not improve upon the results of the benchmark Hamilton (1989) model. The most likely reason for this is that the PCA estimate of the factor is not good enough to give meaningful results. However, when the number of series increases to 25, the results become much closer to the results on the full dataset ("Two-step method, full dataset"): QPS and FPS are almost identical ($QPS = 0.12$, $FPS/T = 0.21$ for 25 series, $QPS = 0.14$, $FPS/T = 0.19$ for the full dataset), the correlation with the OECD reference is much closer ($Corr = 0.57$ and $Corr = 0.67$ for 25 series and the full dataset, respectively), although the beginnings and the ends of recessions are estimated with less precision. To conclude, this exercise shows that the larger the dataset, the more accurate are the estimates of the factor, and therefore the better the quality of the extracted signal, although the marginal gain of a larger number of series decreases.

3.5 Conclusion

This paper focuses on the comparison of the two estimation methods of the MS-DFM model of the business cycle applied to French data. The Maximum Likelihood estimation of the model in one-step can be run only for a very small set of information, whereas the two-step estimation can accommodate much bigger information sets. In this paper we use an extensive dataset of French series covering the period March 1993 - October 2013. We estimate the MS-DFM on 151 series in two steps and on different subsets of four series of main economic indicators in one step. We show that the two-step estimation procedure produces good results in terms of turning points identification. The procedures are transparent and replicable. The model produces turning point estimates up to two months earlier than the reference OECD dating, which is an important gain in timing for economic agents and policymakers.

We find that both estimation methods provide qualitatively similar results: the common factor of several specific economic series (in case of one-step method) and the first principal component of a large set of series (in case of two-step method) can be characterized as having two distinct phases with low and high growth rates, correspondingly. The two-step method also allows to detect the difference in the magnitude of variance in the factor dynamics. For both methods, the periods of high filtered probabilities of recession match the OECD recessions. At the same time, the two-step method and several results of the one-step method identify short recessions in 1998 and 2005 that do not appear in the OECD dating, which is intended to indicate long-lasting phases. We show that these signals are not false, as the worsening of the economic situation was noted in the corresponding short-term INSEE reports, as well as captured by the Index of reversal by INSEE and the datings obtained with the help of the Christiano-Fitzgerald filter.

Both methods largely outperform the results of the univariate Hamilton (1989) model estimated on the index of industrial production, which shows the importance of the multivariate framework for business cycle turning point identification.

The results of the one-step method differ greatly depending on the composition of the four input economic series. We identify series with the highest explicative power (retail trade order intentions, number of job seekers, the survey on manufacturing business situation future and construction confidence index) and the series that produce extra signals (manufacturing finished goods stock level, price return on FTSE equity index and Manufacturing Industrial Confidence Indicator) and determine seven sets of series that perform best in terms of concordance of estimated turning points with the OECD chronology. Since the size of the dataset considered with the one-step method is generally limited to four series, it seems reasonable to use several sets (i.e. several results of the one-step estimation) as complements to overcome the information constraint.

Using a more comprehensive dataset with the two-step method allows us to obtain more accurate estimates of the beginning and the end of recessions. We show that the number of series plays an important role, with larger datasets leading to more accurate identification of the turning points. Introduction of additional autoregressive lags and other principal components further enhances the precision of the two-step results, although the improvements are minor.

We conclude that either method can be used to replicate the OECD dating. Nevertheless, we think the use of the two-step method is very appealing: it allows to get a valid dating of turning points without going through a complicated procedure of series selection, it is much less time-consuming and the numerical convergence problems are not frequent. Another advantage of the two-step method is that it opens the way to different extensions. First, the factor may be estimated within the first step using other methods like the two-step estimator proposed by Doz et al. (2011) or the QML estimator proposed by Doz et al. (2012): this will allow to use data of different frequencies, with missing observations or ragged ends. Second, multifactor Markov-switching models can be estimated. These extensions are left for future research.

Chapter 4

On the consistency of the two-step estimates of the MS-DFM: a Monte Carlo study

Abstract

The Markov-Switching Dynamic Factor Model (MS-DFM) has been used in different applications, notably in the business cycle analysis. When the cross-sectional dimension of data is high, the Maximum Likelihood estimation becomes unfeasible due to the excessive number of parameters. In this case, the MS-DFM can be estimated in two steps, which means that in the first step the common factor is extracted from a database of indicators, and in the second step the Markov-Switching autoregressive model is fit to this extracted factor. The validity of the two-step method is conventionally accepted, although the asymptotic properties of the two-step estimates have not been studied yet. In this paper we examine their consistency as well as the small-sample behavior with the help of Monte Carlo simulations. Our results indicate that the two-step estimates are consistent when the number of cross-section series and time observations is large, however, as expected, the estimates and their standard errors tend to be biased in small samples.

4.1 Introduction

Markov-Switching Dynamic Factor model (MS-DFM) has proved to be a useful instrument in a number of applications. Among them are tracking of labor productivity (Dolega (2007)), modeling the joint dynamics of the yield curve and the GDP (Chauvet and Senyuz (2012)), examination of fluctuations in the employment rates (Juhn et al. (2002)) and many others. However, the major application of the MS-DFM is the analysis of the business cycle turning points (see, for example, Kim and Yoo (1995), Darné and Ferrara (2011), Camacho et al. (2012), Chauvet and Yu (2006), Wang et al. (2009)). The initially suggested univariate Markov-switching model in the seminal paper by Hamilton (1989) was extended to the multivariate case, the MS-DFM, by Kim (1994). The model allows to obtain the turning points in a transparent and replicable way, and, importantly, in a more timely manner than the official institutions (OECD and NBER, for example).

The MS-DFM formalizes the idea of Diebold and Rudebusch (1996) that the economic variables comove and follow a pattern with alternating periods of growth and decline, this comovement essentially representing the business cycle. More precisely, the model assumes that the economic indicators have a factor structure, i.e. are driven by some common factor, which itself follows a Markov-switching dynamics with two regimes.

Depending on the number of economic series under consideration, the model can be estimated using different techniques. The original paper by Kim (1994) as well as some of the following applications is based on just a few economic indicators so the parameters and the factor can be estimated simultaneously with Kalman filter and Maximum Likelihood. However, when the number of series increases, convergence problems may arise and, besides, the estimation may become time-consuming since the number of parameters expands proportionally to the number of series. In the same time, the use of many economic indicators is desirable in order to consider as much information on the business cycle as possible. A natural solution to this trade-off between the size of the information set and the computational time is the the estimation of the Markov-Switching and Dynamic Factor parts of the MS-DFM separately, i.e. in two steps. Attractive in terms of applicability and information treatment, the two-step estimation method has been used in several studies (see, for example, Chauvet and Senyuz (2012), Darné and Ferrara (2011), Bessec and Bouabdallah (2015), Brave and Butters (2010), Davig (2008), Paap et al. (2009)). This method implies that the factor is extracted[1] on the first step, and then the classical univariate Markov-switching model à la Hamilton (1989) is fit to the estimated factor on the second step. The two-step procedure is much easier to implement[2] and does not impose any restrictions on the number of series by default.

Previous studies show the importance of the number of cross-sections N and the number of observations T for the accuracy of estimates on each of the steps. Connor and

[1]Different methods of factor extraction can be applied, Kalman filter (for a small number of series) and PCA are the most common ones.

[2]The procedures for the estimation of the Markov-Switching models are installed in some econometric software such as Eviews, Stata. The corresponding packages exist for Matlab and R.

Korajczyk (1986) proved the consistency of the principal components estimator $\hat{f}_t$ (commonly used in the first step) for fixed T and $N \to \infty$ under general assumptions used by Chamberlain (1983) and Chamberlain and Rothschild (1983) for the definition of the approximate factor structure. Stock and Watson (2002) examined conditions for the rates of N and T under which $\hat{f}_t$ can be treated as data for the OLS regression. More precisely, they show that the performance of the PCA is very good even when the sample-size and the number of series are relatively small, $N = 100$ and $T = 100$. Further on, Bai and Ng (2006) and Bai and Ng (2013) extended this result showing that, under a standard set of assumptions usually used in factor analysis, the $\hat{f}_t$ can be treated as data in subsequent regressions when $N \to \infty$, $T \to \infty$ and $N^2/T \to \infty$. As for the second step, Kiefer (1978), Francq and Roussignol (1997), Francq and Roussignol (1998), Krishnamurthy and Ryden (1998), Douc et al. (2004) and Douc et al. (2011) show that under particular conditions the maximum likelihood estimators of an autoregressive model with Markov regimes are consistent. Francq and Roussignol (1997) also propose a gaussian maximum pseudo-likelihood estimator, Krishnamurthy and Ryden (1998) derived the conditions under which the MLE are asymptotically Gaussian.

Even though the consistency of the estimates of the factor on the first step and the consistency of the ML estimates of the Markov-switching model on the second step has been already shown, it is not straightforward that the ML estimates of the Markov-switching model which is fit to the estimated (and not observed directly) factors are consistent. To the best of our knowledge, the consistency of the two-step estimates has not been shown yet. It is not very surprising since the asymptotic properties of Markov-switching autoregressive models are rather complicated and are difficult to derive analytically.

Another concern of the two-step approach are the small-sample properties of the estimates. Indeed, Hosmer (1973), Hamilton (1991) and Hamilton (1996) find that asymptotic approximations of the sampling distribution of the MLE may be inadequate in small-sample cases. In their Monte Carlo study, Psaradakis and Sola (1998) have shown that "the performance of the MLE was often unsatisfactory even for sample sizes as large as 400", pointing out the non-normality of the empirical distribution of the estimates and the bias that that takes place.

The purpose of this paper is thus twofold. First, we study the consistency of the two-step estimates, where the factor is estimated with PCA. Secondly, we would like to examine the behavior of the estimates in small samples and identify the minimum N and T required to obtain estimates with a reasonably small bias. In addition, we check whether the distribution of the two-step estimates approaches normal (as is the case for the Maximum Likelihood estimates of an MS-AR) given the amount of data usually available in macroeconomic applications. In this paper, we study the aforementioned questions with the help of Monte Carlo simulations. The analytic proof of consistency is being prepared in a companion paper to this work.

This paper thus contributes to the literature on the analysis of the two-step estimates of the MS-DFM. Previously, Camacho et al. (2015) studied the performance of the two-step estimates where the factor is extracted with a linear DFM on the first step. Their study focused on the quality of identification of states and was based on the use of a few series (N not higher than 8). Having compared the two-step results to regular (one-step) Maximum Likelihood estimates, the authors showed that the two-step results diverge from the one-step ones when the common factor is extracted with the help of a linear DFM while the data-generating process is a nonlinear MS-DFM, although the difference decreases when N rises or when the data are less noisy.

The paper is organized as follows. Section 4.2 describes the MS-DFM model. Section 4.3 presents the two-step estimation technique. In section 4.4 we describe the design of the Monte Carlo experiment and discuss the simulation results. Section 4.5 concludes.

4.2 Markov-Switching Dynamic Factor Model

In the present paper, we take the basic specification of the MS-DFM for the business cycle as in the seminal paper by Kim and Yoo (1995), and we assume that the growth rate cycle of the economic activity has only two regimes (or states), associated with its low and high levels. The economic activity itself is represented by an unobservable factor, which summarizes the common dynamics of several observable variables. It is assumed that the switch between regimes happens instantaneously, without any transition period (as is considered, for example, by STAR family models). This assumption can be motivated by the fact that the transition period before deep crises is normally short enough to be omitted. For example, the growth rate of French GDP fell from 0.5% in the first quarter of 2008 to -0.51% in the second quarter of the same year, and further down to -1.59% in the first quarter of 2009[3].

The model is thus decomposed into two equations, the first one defining the factor model, and the second one describing the Markov switching autoregressive model which

[3] INSEE, France, Gross Domestic Product, Total, Contribution to Growth, Calendar Adjusted, Constant Prices, SA, Chained, Change P/P

is assumed for the common factor. More precisely, in the first equation, each series of the information set is decomposed into the sum of a common component (the common factor loads each of the observable series with a specific weight) and an idiosyncratic component:

$$y_t = \lambda f_t + z_t, \tag{4.1}$$

where $t = 1, ..., T$, y_t is a $N \times 1$ vector of economic indicators, f_t is a univariate common factor, z_t is a $N \times 1$ vector of idiosyncratic components uncorrelated with f_t at all leads and lags, λ is a $N \times 1$ vector. In this equation all series are supposed to be stationary, so that some of the components of y_t may be the first differences of the initially non stationary economic indicator.

The idiosyncratic components z_{it}'s, $i = 1, ..., N$, are mutually uncorrelated at all leads and lags, and each of them follows an autoregressive process

$$\psi_i(L) z_{it} = \varepsilon_{it}, \tag{4.2}$$

where $\psi_i(L)$ is a lag polynomial such that $\psi_i(0) = 1$, $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_i^2)$ and $cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$ for all $i \neq j$.

The second equation describes the behavior of the factor f_t, which is supposed to follow an autoregressive Markov Switching process with constant transition probabilities[4]. In what follows, we consider that the change in regime affects only the level of the constant with the high level corresponding to the expansion state and the low level to the recession state:

$$\varphi(L) f_t = \beta_{S_t} + \eta_t, \tag{4.3}$$

where $\eta_t \sim i.i.d.\ \mathcal{N}(0, 1)$, $\varphi(L)$ is an autoregressive polynomial such that $\varphi(0) = 1$.

The switching mean is defined as:

$$\beta_{S_t} = \beta_0(1 - S_t) + \beta_1 S_t, \tag{4.4}$$

[4] Kim and Yoo (1995) showed that, in the business cycle applications, although the assumption of the time dependent probabilities improves the quality of the model, the gain in terms of loglikelihood is not very large.

where S_t takes a value 0 when the economy is in expansion and 1 otherwise, so $\beta_0 > \beta_1$. S_t follows an ergodic Markov chain, i.e.

$$P(S_t = j|S_{t-1} = i, S_{t-2} = k, ...) = P(S_t = j|S_{t-1} = i) = p_{ij}. \tag{4.5}$$

As it is assumed that there are two states only, S_t switches states according to a 2×2 transition probabilities matrix defined as $\begin{bmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{bmatrix}$, where

$$\begin{aligned} P(S_t = 0|S_{t-1} = 0) &= p_0, \\ P(S_t = 1|S_{t-1} = 1) &= p_1. \end{aligned} \tag{4.6}$$

There is no restriction on the duration of each state, and the states are defined point-wise, i.e. a recession period may last one period only.

The present framework can be generalized to the case of a higher number of states and/or to regime dependence in the other parameters of the model (the variance of the error term, the coefficients of the autoregressive polynomial). In our study, we consider the simplest case with two regimes and a switch in constant as in this specification it is easier to control the data generating process. It is also often selected by information criteria in the empirical applications.

4.3 Two-step estimation method

The model presented above can be cast into the state-space form and estimated with Maximum Likelihood. However, the estimation is complicated as the likelihood function has to take into account all possible paths of S_t, which is 2^T, and the number of parameters to estimate grows proportionally to the number of series in y_t. While the first issue can be solved with collapsing procedure suggested by Kim (1994), the second problem makes the solution unfeasible for large N. This is computationally challenging, and there are several ways to solve this issue. The one that we consider in this paper is estimating the model in two steps, where on the first step the factor is extracted from the data y_t, while on the second step the estimated factor is used as if it were the true factor in order to obtain the estimates of equation (4.3). More precisely, the procedure is the following.

Step 1

The factor f_t is extracted from a large database of economic indicators according to equation (4.1) without taking its Markov-Switching dynamics into account. In the present paper, we use principal component analysis to compute an approximation $\hat{f}_t$ of the true factor. Indeed, since (f_t) is a stationary process, as we have discussed in the introduction, under a mild set of assumptions, it is consistently estimated by $\hat{f}_t$. The factor can be extracted with a different method, for example, using the two-step estimator suggested by Doz et al. (2011) or Quasi-Maximum Likelihood estimator by Doz et al. (2012).

If we denote by $\hat{\Sigma} = \frac{1}{T}\tilde{y}'\tilde{y}$ the empirical correlation matrix of y (where $\tilde{y}$ is the standardized y), by $\hat{D}$ the $N \times N$ diagonal matrix of with the eigenvalues of $\hat{\Sigma}$ in decreasing order, by $\hat{V}$ the $N \times N$ matrix of the unitary eigenvectors corresponding to $\hat{D}$, then the matrix of the principal components $\hat{F}$ is defined as:

$$\hat{F} = y\hat{V},$$

and the corresponding matrix of loadings $\hat{\Lambda}$ is:

$$\hat{\Lambda} = \hat{V}',$$

The first column of the matrix $\hat{F}$ is then the estimate $\hat{f}_t$ of the true factor f_t, whereas the first column of $\hat{\Lambda}$ is the estimate $\hat{\lambda}$.

Step 2

The parameters of the autoregressive Markov-Switching model described by equations (4.3) and (4.5) are estimated by maximum likelihood, with f_t replaced by $\hat{f}_t$. This amounts to fit the univariate model of Hamilton (1989) to the estimated factor $\hat{f}_t$, which is taken as if it were an observed variable:

$$\varphi(L)\hat{f}_t = \beta_{S_t} + u_t, \tag{4.7}$$

where $u_t \sim N(0, \sigma^2)$. Suppose that $\theta = (\varphi(L),\psi_1(L),...,\psi_N(L),\lambda_1, ...,\lambda_N,\sigma_1^2,...,\sigma_N^2,\beta_0,\beta_1, p_0, p_1,\sigma^2)'$ is the vector of unknown parameters, $g(\cdot)$ is the Gaussian density function. The log-likelihood function takes the following form:

$$\mathcal{L}_T(\hat{f},\theta) = ln\, l(\hat{f}_1, \hat{f}_2, ..., \hat{f}_T, \theta) = \sum_{t=1}^{T} ln\, g(\hat{f}_t|I_{t-1},\theta), \tag{4.8}$$

where we denote $\hat{f} = (\hat{f}_1, ..., \hat{f}_T)$ and $I_{t-1} = \{\hat{f}_1, ..., \hat{f}_{t-1}\}$. The density

$$g(\hat{f}_t|I_{t-1},\theta) = \sum_{j=0}^{1}\sum_{i=0}^{1} g(\hat{f}_t, S_t = j, S_{t-1} = i|I_{t-1},\theta), \tag{4.9}$$

is computed using filtered probability $P(S_t = j|I_t, \theta)$ on the basis of Bayes' theorem:

$$g(\hat{f}_t, S_t = j, S_{t-1} = i|I_{t-1},\theta) = g(\hat{f}_t|S_t = j, S_{t-1} = i, \theta) \times P(S_t = j, S_{t-1} = i|I_{t-1},\theta), \tag{4.10}$$

where

$$g(\hat{f}_t|S_t = j, S_{t-1} = i, I_{t-1}, \theta) = (2\pi\sigma^2)^{-1/2} exp\left\{-\frac{1}{2}\frac{(\varphi(L)\hat{f}_t - \beta_{S_t})^2}{\sigma^2}\right\} \tag{4.11}$$

$$P(S = j, S_{t-1} = i|I_{t-1},\theta) = P(S_t = j|S_{t-1} = i, \theta) \times P(S_{t-1} = i|I_{t-1},\theta), \tag{4.12}$$

and

$$P(S_t = j|I_t,\theta) = \sum_{i=0}^{1} P(S = j, S_{t-1} = i|I_t,\theta) \tag{4.13}$$

$$\begin{aligned} P(S_t = j, S_{t-1} = i|I_t,\theta) &= \frac{g(\hat{f}_t, S_t = j, S_{t-1} = i|I_{t-1},\theta)}{g(\hat{f}_t|I_{t-1},\theta)} \\ &= \frac{g(\hat{f}_t|S_t = j, S_{t-1} = i, I_{t-1},\theta) \times P(S_t = j, S_{t-1} = i|I_{t-1},\theta)}{g(\hat{f}_t|I_{t-1},\theta)} \end{aligned} \tag{4.14}$$

The recursion is initialized with the steady state probability π of being in state $j \in \{0; 1\}$ at time $t = 0$:

$$\pi = P(S_0 = 1|I_0,\theta) = \frac{1-p_0}{2-p_0-p_1}, \tag{4.15}$$

$$P(S_0 = 0|I_0,\theta) = 1 - P(S_1 = 1|I_0,\theta) = 1 - \pi. \tag{4.16}$$

The two-step estimates $\hat{\theta}(\hat{f})$ are obtained as the maximum of the likelihood function $\mathcal{L}_T(\hat{f}, \theta)$ using numerical optimization algorithms. Then, for $\hat{\theta}(\hat{f})$ given, we can infer the associated filtered probability $P(S_t = j|I_t, \hat{\theta})$[5] with formulas (4.9)-(4.14). Also, it is possible to compute the smoothed probabilities of each state $P(S_t = 1|I_T, \hat{\theta})$ using backward filtering (see Hamilton (1989)).

In the majority of studies on business cycle fluctuations analyzed with MS-DFM, the filtered probability of recession is the main focus. Since it allows to have an estimate of the state at time t on the basis of the information available up to the moment t, it is used for the purposes of nowcasting. The smoothed probabilities are often used to establish business cycle dating retrospectively on the basis of the full information set, i.e. a posteriori. It is therefore important to verify if the two-step estimates provide good quality of identification of states in terms of both filtered and smoothed probability.

4.4 Monte Carlo simulations

We use Monte Carlo simulations to examine the consistency of the two-step estimates as well as their small-sample properties. We first discuss the experimental design. The numerical results follow.

4.4.1 Experimental setup

4.4.1.1 The DGP

The data generating process (DGP) used in simulations is described in equations (4.1)-(4.5). We assume for simplicity that the order of $\varphi(L)$ is one and the order of $\psi_i(L)$ is zero, $i = 1, ..., N$. The autoregressive polynomials of higher order would complicate the control over the variance of the factor and the idiosyncratic components without changing the essence of the dynamics of the underlying processes (unless it renders the dynamics nonstationary). The DGP is therefore:

$$y_{it} = \lambda_i f_t + \varepsilon_{it}, \tag{4.17}$$

$$(1 - \varphi L) f_t = \beta_{S_t} + \eta_t, \tag{4.18}$$

$$\beta_{S_t} = \beta_1 S_t + \beta_0 (1 - S_t) = \beta_0 + (\beta_1 - \beta_0) S_t, \tag{4.19}$$

$$P(S_t = 0|S_{t-1} = 0) = p_0,$$

[5]To simplify notations, we denote $\hat{\theta} = \hat{\theta}(\hat{f})$.

$$P(S_t = 1|S_{t-1} = 1) = p_1, \tag{4.20}$$

where $i = 1, ..., N$, $t = 1, ..., T$, $\eta_t \sim N(0, 1)$. The factor loadings λ_i are generated from a normal distribution and are normalized to have a unit sum of squares, i.e. $\lambda_i = \frac{\gamma_i}{\sqrt{\gamma'\gamma}}$, $\gamma_i \sim N(0, 1)$.[6] The idiosyncratic disturbance terms ε_{it} are cross-sectionally independent and have a Gaussian distribution $\varepsilon_{it} \sim N(0, \sigma_i^2)$. The state variable S_t is a Markov switching process with two states $S_t \in \{0; 1\}$ (0 corresponds to expansion, and 1 to recession) and transition probability matrix $\begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}$.

We put the unconditional mean of the factor to zero, a classical assumption for the factor models. Since $ES_t = \pi$, this imposes a fixed relation between β_0 and β_1:

$$\beta_1 = \beta_0(1 - \frac{1}{\pi}). \tag{4.21}$$

4.4.1.2 The parameters of control

Intuitively, besides the size of N and T, there are four aspects of the dynamics of the DGP that might affect the quality of the estimates of the MLE. These are:

1. The persistence of each regime. For a given sample size, a more persistent regime is better identified since it is activated during longer periods of time.

2. The noise-to-signal ratios $s_i = \frac{\sigma_i^2}{V(y_{it})}$. When the data are less noisy, the estimate of the factor is more precise.

3. The persistence of the autoregressive dynamics of the factor. Presumably, the closer the root of the autoregressive polynomial is to one in absolute value, the more difficult it is to distinguish between the change in regime and the long-lasting effect of a shock in the error term.

4. The share of variance of the factor due to the switch. If most of the variance of the factor is generated by the error term η_t, the states are more difficult to identify.

Under the assumption that the unconditional mean of the factor is zero, it is possible to show that (see Appendix for the details) that the unconditional variance of the true factor f_t is:

$$V(f_t) = \frac{1}{1-\varphi^2}\left(\sigma^2 + \beta_0^2\left(\frac{1-p_1}{1-p_0}\right)\left(\frac{1+\varphi(p_0+p_1-1)}{1-\varphi(p_0+p_1-1)}\right)\right). \tag{4.22}$$

[6] The vector λ is unitary in order to provide the same scale to the generated factor and the estimated factor obtained by PCA with normalized loadings.

We control the first item directly by changing p_0, p_1. We suppose that the s_i's are uniformly distributed: $s_i \sim \mathcal{U}[u; 1-u]$, and we control the noisiness by choosing u. We control the persistence of the factor by changing the autoregressive term φ. The share of variance of the factor due to the error term is varied by changing the ratio $c = \frac{V(f_t)}{\sigma^2}$. Therefore, the free parameters of the simulation are c, p_0, p_1, u, φ. The parameters to be estimated are $\theta = (\beta_0, \beta_1, \varphi, \sigma^2, p_0, p_1)$.

In order to examine behavior of the two-step method under different conditions, we run Monte Carlo simulations for the following scenarios:

1. baseline scenario: $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.3$;
2. noisy factor: $c = 2$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.3$;
3. persistence of the factor dynamics:
    3.1. high autocorrelation: $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.9$;
    3.2. medium autocorrelation: $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.6$;
4. persistence of states:
    4.1. impersistent states: $c = 5$, $p_0 = 0.5$, $p_1 = 0.5$, $u = 0.1$, $\varphi = 0.3$;
    4.2. very persistent states: $c = 5$, $p_0 = 0.95$, $p_1 = 0.95$, $u = 0.1$, $\varphi = 0.3$;
5. homogeneous data: $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.5$, $\varphi = 0.3$;

The values of the parameters in the baseline scenario have been taken from existing empirical studies using MS-DFM. Thus, φ close to 0.3 has been obtained by Chauvet (1998), Kim and Yoo (1995), Kim and Nelson (1998), whereas the ratio of factor variance and variance of the error term varies between 2 in Kim and Yoo (1995) and 7.5 in Kim and Nelson (1998). The transition probabilities p_0 and p_1 are often estimated to be around 0.9 and 0.8, respectively, i.e. the two states are very persistent and the probability of staying in expansion is always higher than the probability to stay in recession. We generate $K = 2000$ replications of each scenario and estimate the MS-DFM with the help of the two-step method for all combinations of $N \in \{25, 50, 100, 150, 300\}$ and $T \in \{25, 50, 100, 150, 300\}$.

As the consistency properties of the PCA estimates have already been shown in previous literature, we take as given that, with high values of N and T, we obtain a good estimate

$\hat{f}_t$ for the factor f_t in the first step. Interestingly, our simulations[7] show that the factor can be estimated well even for $N = 50$ and $T = \{25, 50\}$. In their Monte-Carlo study, Stock and Watson (2002) confirm this result.For this reason, we also report the behavior of the two-step estimates samples as small as $N = 25$ and $T = 25$.[8]

4.4.1.3 Estimation

It is known that the PCA estimates of the factors are identified up to a sign change. We manually control the sign of the estimated factor by multiplying the $\hat{f}$ by -1 if its correlation with the true factor is negative. In practice, is it also usually possible to recover the sign of the true factor.

For each replication, the likelihood function $\mathcal{L}_T(\hat{f}, \theta)$ is maximized under constraints on transition probabilities (to insure that they lie in the open unit interval) and variance (to insure that it is positive). The maximum likelihood estimate is obtained with SQP[9] (sequential quadratic programming) method, which is essentially a version of Newton's method for constrained optimization. At each iteration, the Hessian of the function is updated using the Broyden-Fletcher-Goldfarb-Shranno (BFGS) method. The variance-covariance matrix $\hat{\Omega}_T^{-1}$ of the estimates $\hat{\theta}(\hat{f})$ is then estimated as

$$\hat{\Omega}_T = -\frac{1}{T}\left(\frac{\partial^2 \mathcal{L}_T(\theta, \hat{f})}{\partial\theta\partial\theta'}\right)\Bigg|_{\theta=\hat{\theta}}.$$

4.4.1.4 Measures of quality

In order to quantify the impact of the use of $\hat{f}$ instead of f on the ML estimates of the second step, we compare the empirical distributions of the two-step estimates to the MLE obtained on the observed factor. We denote by $\hat{\theta}(f)$ (resp. $\hat{\theta}(\hat{f})$) the vector estimates obtained with equation (4.3) (resp. equation (4.7)), and by $\hat{\theta}_i(f)$ (resp. $\hat{\theta}_i(\hat{f})$) the i-th component of $\hat{\theta}(f)$ (resp. $\hat{\theta}(\hat{f})$). For each pair of elements $\hat{\theta}_i(f)$ and $\hat{\theta}_i(\hat{f})$, we compute two following measures:

[7]To save space, we do not report these results here, but they are surely available on request.

[8]This setting may be interesting for the empirical studies of the business cycle in countries with limited availability of data.

[9]which corresponds to the *sqp* optimization algorithm of the *fmincon* optimizer in Matlab R2015a.

1. the Kullback-Leibler divergence

$$D_{KL}(F_{\hat{f}}||F_f) = \sum_j F_f^i(j) \ln \frac{F_{\hat{f}}^i(j)}{F_f^i(j)},$$

   where $F_{\hat{f}}^i$ and F_f^i are the empirical cumulative distribution functions of $\hat{\theta}_i(f)$ and $\hat{\theta}_i(\hat{f})$, and $F_f(j)$ and $F_{\hat{f}}(j)$ are probability measures on a bin j.[10] This is a measure of the information loss when $F_{\hat{f}}^i$ is used to approximate F_f^i and corresponds to a proxy of the expectation of the logarithmic difference.

2. the Kolmogorov-Smirnov statistic

$$KS_i = \sup_{\hat{\theta}_i} |F_f(\hat{\theta}_i) - F_{\hat{f}}(\hat{\theta}_i)|,$$

   where $\sup_{\hat{\theta}_i}$ is the supremum of the set of distances between $F_{\hat{f}}^i$ and F_f^i at different values of $\hat{\theta}_i$. KS_i shows the maximum deviation of $F_f(\hat{\theta}_i)$ from $F_{\hat{f}}(\hat{\theta}_i)$. The statistic is used for the Kolmogorov-Smirnov test with the null hypothesis that $\hat{\theta}_i(f)$ and $\hat{\theta}_i(\hat{f})$ come from the same distribution. The null is rejected at 5% when $KS_i > 0.043$.[11] The KS statistic and the corresponding test thus show whether the two empirical distributions are statistically different. It's important to underline that the test requires that the two empirical distributions correspond to independent samples. For this reason, we compute $F_f(\hat{\theta}_i)$ and $F_{\hat{f}}(\hat{\theta}_i)$ using two disjoint subsets of 1000 replications, i.e. using the first 1000 replications for $F_f(\hat{\theta}_i)$ and the last 1000 replications for $F_{\hat{f}}(\hat{\theta}_i)$.

To study the small-sample behavior and consistency of the two-step estimates, we report the ratio $\frac{\hat{\theta}_i(\hat{f})}{\theta_{0i}}$, where θ_{0i} is the genuine value of the parameter, and the ratio between the mean estimated standard error and sampling standard error of $\hat{\theta}_i(\hat{f})$.

To measure the ability of the model to identify states we compare the obtained estimates of filtered probability $P(S_t = 1|I_t)$ to the true sequence of states. To simplify notations, we use $FP_t(\hat{f}) = P(S_t = 1|\hat{f}_t, \hat{f}_{t-1}, ..., \hat{f}_1)$ and $FP_t(f) = P(S_t = 1|f_t, f_{t-1}, ..., f_1)$ for the filtered probability corresponding to equations (4.7) and (4.3), respectively. We use the following quality indicators:

[10] In order to render the KL distances of the parameters comparable between each other, we set the width of a bin to 0.25, the minimum value which guarantees non-emptiness of $F_f^i(j)$ for the distributions of each element of $\hat{\theta}_i(\hat{f})$.

[11] In the general case, the null is rejected when $KS_{n,n'} > c(\alpha)\sqrt{\frac{n+n'}{nn'}}$ where $c(\alpha)$ is the quantile of the Kolmogorov distribution ($c(\alpha) = 1.36$ for $\alpha = 0.05\%$), n and n' are the sizes of first and second sample respectively.

1. the quadratic probability score by Brier (1950), which measures the average quadratic deviation of the filtered probability from the true state and is defined as

$$QPS = \frac{1}{T}\sum_{t=1}^{T}(S_t - FP_t(\hat{f}))^2;$$

2. false positives score, which measures the average number of wrongly identified states in the sample, under the assumption that $S_t = 1$ when $FP_t(\hat{f}) > 0.5$[12] and $S_t = 0$ otherwise; FPS is defined as

$$FPS = \frac{1}{T}\sum_{t=1}^{T}(S_t - I_{FP_t(\hat{f})>0.5})^2;$$

3. the correlation between the true state and $FP_t(\hat{f})$

$$r_1 = corr(FP_t(\hat{f}),\, S_t);$$

4. the correlation between the true state and the filtered probability of recession inferred from the dynamics of the true factor

$$r_2 = corr(FP_t(f),\, S_t)$$

   which allows to evaluate the performance of the Markov-Switching model for the identification of the state S_t in finite samples. By comparing r_1 and r_2, we can assess the impact of the use of the proxy of the factor $\hat{f}_t$ instead of the factor f_t itself.

While the correlations measure how well the filtered probability follows the business cycle, the QPS and FPS show how reliable it is about the estimate of the state and how often it fails. The same indicators QPS, FPS, r_1 and r_2 are computed for the smoothed probability of recession.

Finally, it is interesting to study whether the distribution of the two-step estimates has the same properties as the MLE of the MS-AR model. Indeed, in case of a regular Markov-Switching autoregressive model, is often assumed that under sufficient regularity conditions,[13] the MLE $\hat{\theta}$ is Gaussian and so $\sqrt{T}(\hat{\theta} - \theta_0)$ converges in distribution to $N(0, \Omega_0^{-1})$ as $T \to \infty$, where

[12]The cut-off threshold of 0.5 for the filtered probability of recession is chosen arbitrary, however, it is quite common in the literature.

[13]see, for example, Kiefer (1978).

$$\Omega_0 = \lim_{T\to\infty} \frac{1}{T} \left(\frac{\partial^2 \mathcal{L}_T(\hat{f}, \theta)}{\partial\theta\partial\theta'} \right) \Bigg|_{\theta=\theta_0},$$

and Ω_0 is the information matrix.

In order to verify whether the two-step estimates have normal distribution (or tend to it asymptotically), we study the conventional t-statistics corresponding to the elements of $\hat{\theta}_i$:

$$t_i = \frac{\hat{\theta}_i(\hat{f}) - \theta_{0i}}{\hat{\sigma}_{\hat{\theta}_i(\hat{f})}}.$$

If the two-step estimates are asymptotically normal, the t-statistics should also have asymptotically Gaussian distribution. In this case, the use of Wald-type tests (including significance tests) when interpreting the results of the MS-DFM is justified. We examine this hypothesis by analyzing the mean, the skewness and the excess kurtosis of the distribution of t_i, as well as run the Kolmogorov-Smirnov normality test. As an additional indicator of gaussianity, we also compute the empirical rejection rates of the test with the null $H_0 : E(\hat{\theta}_i(\hat{f})) = \theta_{0i}$. If the empirical rejection rate coincides with the theoretical one, this is regarded as an additional sign of normality of the distribution.

4.4.2 Simulation results

In this section we provide simulation results for the baseline scenario. The experiments were performed on SCSCF, a multiprocessor cluster system owned by Università Ca'Foscari Venezia.

4.4.2.1 The impact of the first step

Figure 4.1 and Table 4.1 provide the information on how the first step - the use of estimated factor instead of the true one - modifies the ML estimates of the Markov-Switching autoregressive model. For each matrix in Figure 4.1, the change in the color columnwise corresponds to the effect of the increase of the number of series N, while the change rowwise to the increase of the number of observations T. As expected, the Kullback-Leibler distance between the empirical distributions of $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$ decreases when N rises. However, we observe the distance increase when T rises for a given N. This intuitively contradictory finding is connected to the presence of replications with aberrant results, i.e. replications with estimated transition probabilities very close to 0 or 1 (which is an implausible result since it implies that the underlying Markov chain is not irreducible), or with $\hat{\beta}_0$ is very close to $\hat{\beta}_1$ (in this case, the states are not identified, neither are the parameters). In most cases, these estimates correspond to the convergence of the maximum likelihood optimizer to a wrong local maximum. Present in both $\hat{\theta}(\hat{f})$ and $\hat{\theta}(f)$, this kind of estimates form an additional mode which distorts the distributions. Since these obviously abnormal values of the estimators can be easily identified as such while working with the real data, we discard them from our analysis. Typically, their amount is not large (5%-10% of replications, depending on the parameter), so the remaining number of replications is still large enough to analyze the properties of the two-step estimator.[14]

Figure 4.2 reports the Kullback-Leibler divergence between $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$ when the implausible replications are discarded. In this case we observe convergence both in N and T. Importantly, for all parameters the distance is high for $N < 100$. When $N > 150$, little improvement can be achieved by increasing the number of series even more, so the major factor of proximity between the two distributions is the number of observations. This observation is validated by the Kolmogorov-Smirnov tests reported in Table 4.1, where the null that $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$ come from the same distribution is not rejected in the majority of cases.

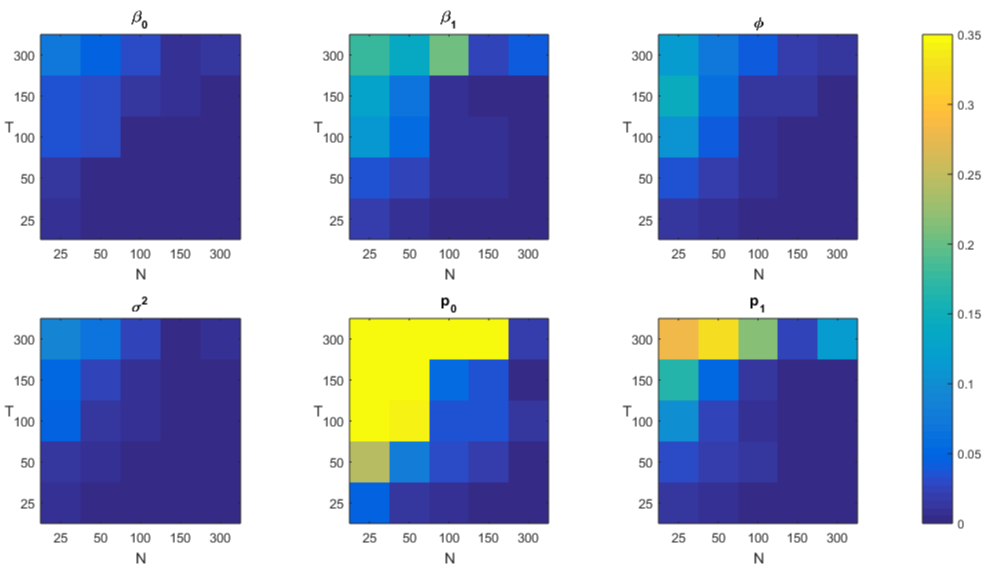


FIGURE 4.1: Kullback-Leibler distance between $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$

Note: $\theta = (\beta_0, \beta_1, \varphi, \sigma^2, p_0, p_1)$

[14] For the purpose of comparison, we compute several tables reported in this section for all replications (see Appendix B.3). Other tables are available on request.

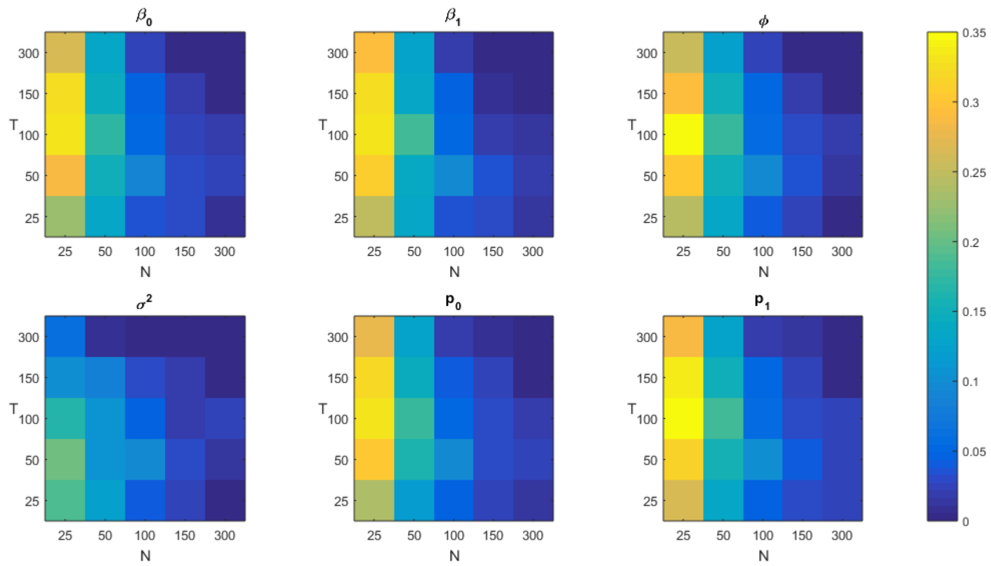


FIGURE 4.2: Kullback-Leibler distance between $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$, aberrant replications excluded

Note: $\theta = (\beta_0, \beta_1, \varphi, \sigma^2, p_0, p_1)$

Table 4.1: Test statistic of the Kolmogorov-Smirnov test

N	T	β_0	β_1	φ	σ^2	p_0	p_1
25	25	0.05	0.07	0.08	0.10	**0.04**	**0.04**
25	50	0.05	0.05	**0.03**	0.11	**0.04**	**0.04**
25	100	0.08	0.07	0.06	0.14	0.05	**0.03**
25	150	0.09	**0.03**	**0.04**	0.15	**0.03**	**0.04**
25	300	0.09	**0.03**	0.07	0.15	**0.04**	**0.04**
50	25	**0.04**	0.05	**0.04**	**0.04**	0.05	0.06
50	50	**0.03**	0.06	**0.03**	0.10	**0.04**	**0.04**
50	100	**0.03**	**0.04**	**0.03**	0.10	**0.04**	**0.04**
50	150	0.07	0.06	0.06	0.09	0.06	**0.03**
50	300	**0.04**	0.05	**0.03**	0.12	0.06	**0.03**
100	25	0.06	**0.03**	**0.04**	0.05	**0.04**	0.05
100	50	**0.03**	**0.03**	**0.03**	0.05	0.05	0.05
100	100	**0.03**	0.06	**0.03**	**0.04**	**0.04**	**0.04**
100	150	**0.03**	**0.03**	**0.03**	0.07	**0.04**	0.06
100	300	**0.03**	**0.03**	**0.03**	0.08	**0.04**	**0.04**
150	25	**0.04**	0.06	0.05	**0.03**	0.06	0.06
150	50	0.07	**0.02**	0.03	**0.03**	0.05	0.06
150	100	**0.02**	0.05	0.04	**0.04**	**0.04**	0.05
150	150	**0.03**	**0.03**	**0.03**	**0.04**	0.06	**0.03**
150	300	**0.02**	0.05	**0.04**	0.07	**0.02**	0.06
300	25	**0.03**	0.05	0.05	**0.02**	0.06	0.05
300	50	**0.03**	**0.04**	**0.03**	**0.03**	**0.04**	**0.04**
300	100	**0.03**	**0.03**	**0.02**	0.06	**0.04**	**0.04**
300	150	**0.04**	**0.03**	**0.02**	**0.04**	**0.03**	**0.03**
300	300	**0.04**	**0.03**	**0.03**	**0.04**	**0.03**	**0.04**

The null hypothesis of the test is that $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$, with $\theta = (\beta_0, \beta_1, \varphi, \sigma^2, p_0, p_1)$, are from the same continuous distribution. The null is rejected when $KS > 0.043$. The cases when the null is not rejected are marked with bold font.

4.4.2.2 Consistency and small-sample performance of the two-step estimates $\hat{\theta}(\hat{f})$

Mean bias

Figure 4.3 provides the ratios of the two-step estimates $\hat{\theta}_i(\hat{f})$ to the true values of the parameters θ_{0i} averaged over replications (the exact values of the ratios are given in Table B.1 in Appendix), whereas B.1 sheds light on their distributions.

We observe that, as T and N rise, the ratio for all elements of $\hat{\theta}(\hat{f})$ approaches one indicating consistency of the estimates. Interestingly, the convergence is achieved faster for the estimates of the parameters corresponding to the switch, i.e. β_0, β_1, p_0 and p_1, the deviation of the estimates of transition probabilities being no more than 3% even for very small N and T. The estimates of φ and σ^2 require greater N and T to approach their true values (the deviation is around 2%-4% for and φ and is much higher for σ^2).

As expected, the rate of convergence is lower for the two-step estimates in comparison to the estimates computed with the observed factor (see Table B.4 in the Appendix), the convergence of $\hat{\theta}(f)$ is achieved at $T = 150$ already.

In case of small T and N, the estimated values of the parameters deviate from their true values. The bias is generally a decreasing function of the sample size and the number of series, and for most design points is substantially different from zero. Consistent with Psaradakis and Sola (1998), the estimates of φ, p_0, p_1 and β_1[15] are always downward biased, whereas the estimates of β_0 and σ^2 are upward biased.

Standard error bias

The accuracy of the estimated small-sample standard errors as approximations of the sampling standard deviation of the two-step estimates also provides additional information on the convergence. Figure 4.4 reports the ratio between the estimated standard errors averaged over replications and sampling standard deviation of $\hat{\theta}_i(\hat{f})$.

The ratio approaches 1 as T and N rise, which advocates for consistency of the two-step estimates and for the accuracy of the numerical estimates of the standard errors. The only exception is the standard error of $\hat{\sigma}^2$, which tends to have an overestimated standard error when N is small, however, we observe convergence to one for $N > 300$. For smaller samples and number of observations, the standard errors appear to be overestimated for β_0, β_1, φ and p_0 and underestimated for p_1.

[15] In our setting, its true value is $\beta_1 = -2$; the values above 1 in Table B.1 indicate that the average value of $\hat{\beta}_1$ is more negative, i.e. downward biased, in small samples.

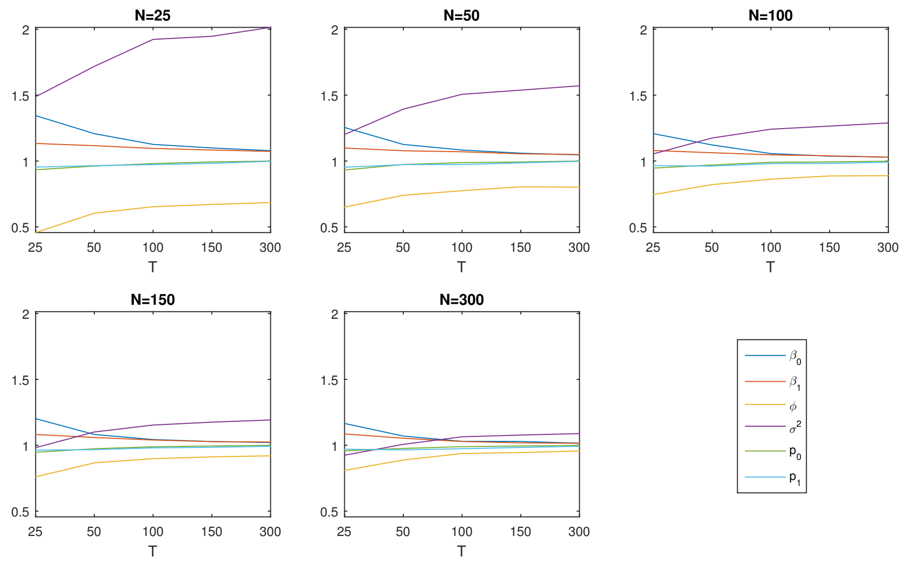


FIGURE 4.3: Mean ratio between the two-step estimate of the parameter $\hat{\theta}_i(\hat{f})$ and its true value θ_{0i}

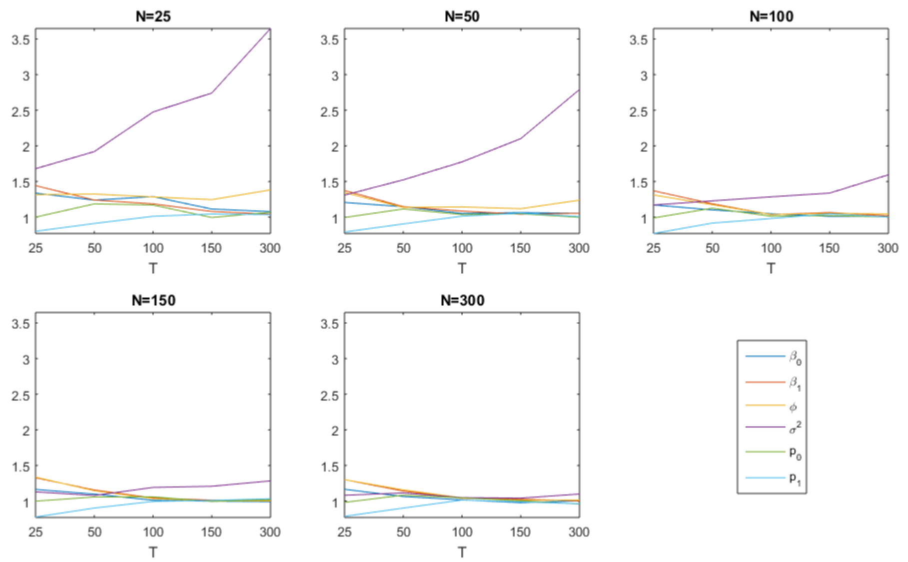


FIGURE 4.4: Ratio between mean estimated standard deviation $\hat{\sigma}_{\hat{\theta}_i(\hat{f})}$ and sample standard deviation of $\hat{\theta}_i(\hat{f})$

4.4.3 Identification of states

Figures 4.5 and 4.6 demonstrate the ability of the model to identify recession states.

As expected, the quality of state identification increases with the precision with which the factor is estimated, and thus with the number of series N. In the same time, the factor reveals more information about existing states when T is higher.

By comparing the sequence of filtered probabilities with the sequence of realized states we assess the quality of nowcasts of the current state of the cycle. Figure 4.5 shows that with $T > 50$ and $N > 100$, the model erroneously assigns a high probability of recession in at most 10% of cases ($FPS < 0.10$). In real empirical applications, the number of series that would produce the same quality should be lower, as the data are usually much less noisy (the results of scenario 4.1 confirm this hypothesis). The correlation of the filtered probability of recession with the true states r_1 is getting closer to r_2, its counterpart obtained with the observed factor, as N rises (up to almost coinciding when $N = 300$) and is very high under values of N and T close to those usually used in practice (above 0.8 for all T and $N > 150$).

The recession identification performance of the model is even higher for the retrospective analysis of the cycles, i.e. for the smoothed probabilities. With $T > 100$ and $N > 50$, the FPS is below 0.10 and attains 0.03 with 300 observations on 300 series.

To conclude, it is worth noting that notwithstanding the bias in the two-step estimates when the number of series and observations are small, the two-step estimates seem to be precise enough to insure high performance of the MS-DFM in terms of identification of states, especially a posteriori. This finding seems particularly encouraging to us, since dating of the moments of changes of states is the major output of the MS-DFM.

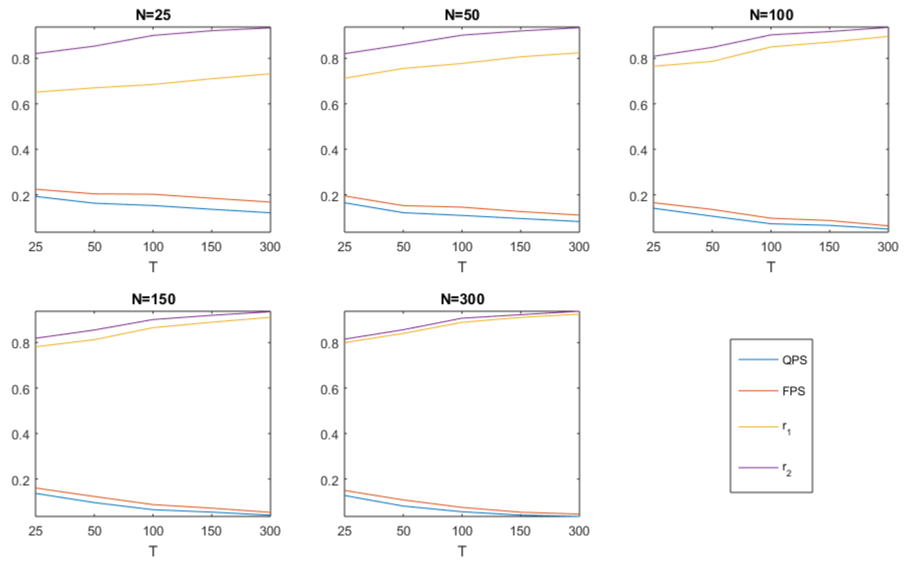


FIGURE 4.5: Quality of state identification: filtered probability of recession

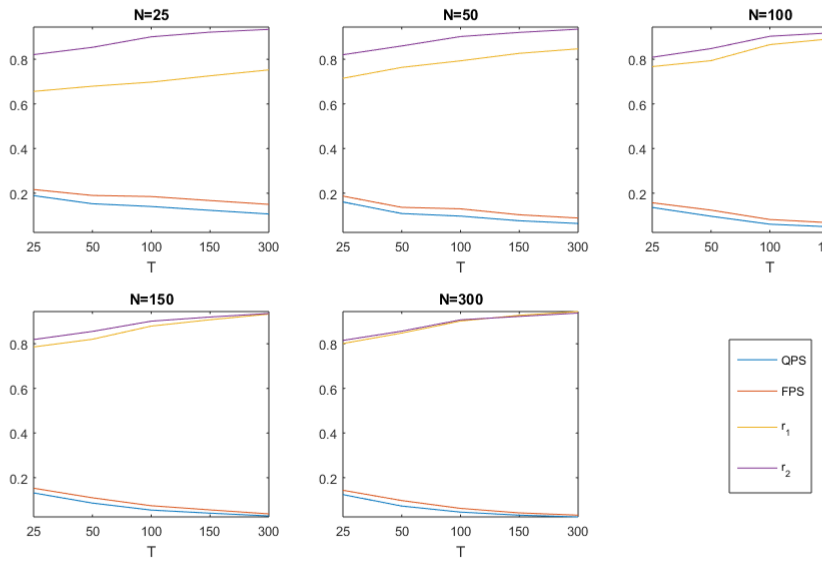


FIGURE 4.6: Quality of state identification: smoothed probability of recession

4.4.4 Analysis of t-statistics

Table B.5, Figure B.3 and Table B.6 in the Appendix show the mean, skewness and excess kurtosis of the t-statistics. As Table B.1, Table B.5 reports the bias of the estimates, but measured in standard deviations of the estimates. Once the standard deviations are accounted for, we observe that the bias of the estimates of p_0 and p_1 is very close to zero. The direction of bias of the t-statistics of the other estimates is in accordance with the results of Table B.1. One may notice that the values of the t-statistics in most design points is below 1.96 in absolute value, indicating that if the two-step estimates were asymptotically normal, the null $H_0 : E(\hat{\theta}_i) = \theta_{0i}$ would not be rejected.

For all values of N, the distribution of the t-statistics of the estimates are skewed, and skewness often changes sign when passing from small T to higher T and diminishes as N rises. Following the sign of the sample mean of the t-statistics, the skewness is negative for $t_{\hat{\varphi}}$ and $t_{\hat{\beta}_1}$, and positive for the other parameters. In particular, the empirical distribution of the two-step estimates of β_0, β_1 and σ^2 have thick right tails, whereas those of φ, p_0 and p_1 have thick left tails. The skewness of the two-step estimates of σ^2, p_0 and p_1 is probably not very surprising since the estimation was implemented under the constraint that the variance should be positive and the transition probabilities should lie in the open unit interval. These restrictions are likely to lead to finite-sample distributions resembling truncated ones, since more probability mass would lie in the required interval than would in a model with no restriction on the parameters. This effect is clearly visible on the boxplots of the ratio of the two-step estimates to their true values (see Figure B.1

in the Appendix).

Like the estimates obtained on the observed factors in small samples (see Psaradakis and Sola (1998)), the two-step estimates and their t-statistics are skewed and leptokurtic, at least in the samples of size considered in this paper. We find nevertheless that in some cases the t-statistics are Gaussian (see Figure B.4): the Kolmogorov-Smirnov test shows that normality of the distribution of $t_{\hat{p}_1}$ is not rejected at 5% for all N and high T (and for small N and high T in case of $t_{\hat{p}_0}$), however it is rejected for all other parameters. To the contrary, Jarque-Bera test does not reject normality for other parameters (see Figure B.5), so no unambiguous conclusion on the normality of the two-step estimates can be derived.[16]

To get additional insight on the potential normality of the distribution of the two-step estimates, we analyze the empirical size of the tests of the null $H_0 : E(\hat{\theta}_i) = \theta_{0i}$ (see Tables B.7 and B.8). We observe that due to the distortions in the distributions, the empirical size of the tests is always above its the nominal size. As distortions attenuate, the empirical size gets closer to its nominal counterpart, which we observe in case of β_0, β_1 (when the number of series and observations is high) and, in particular, p_0 and p_1 (for $N > 100$). Consistent with Psaradakis and Sola (1998), the difficulties are the most serious for the t-statistics of $\hat{\sigma}^2$.

All the observations listed above lead us to the conclusion that the two-step estimates $\hat{\theta}(\hat{f})$[17] are usually not normal for $N < 300$ and $T < 300$. This implies that the tests using this assumption (such as t-tests on significance of the coefficients, Wald-type tests, and other) are likely to be invalid and should be used with caution.

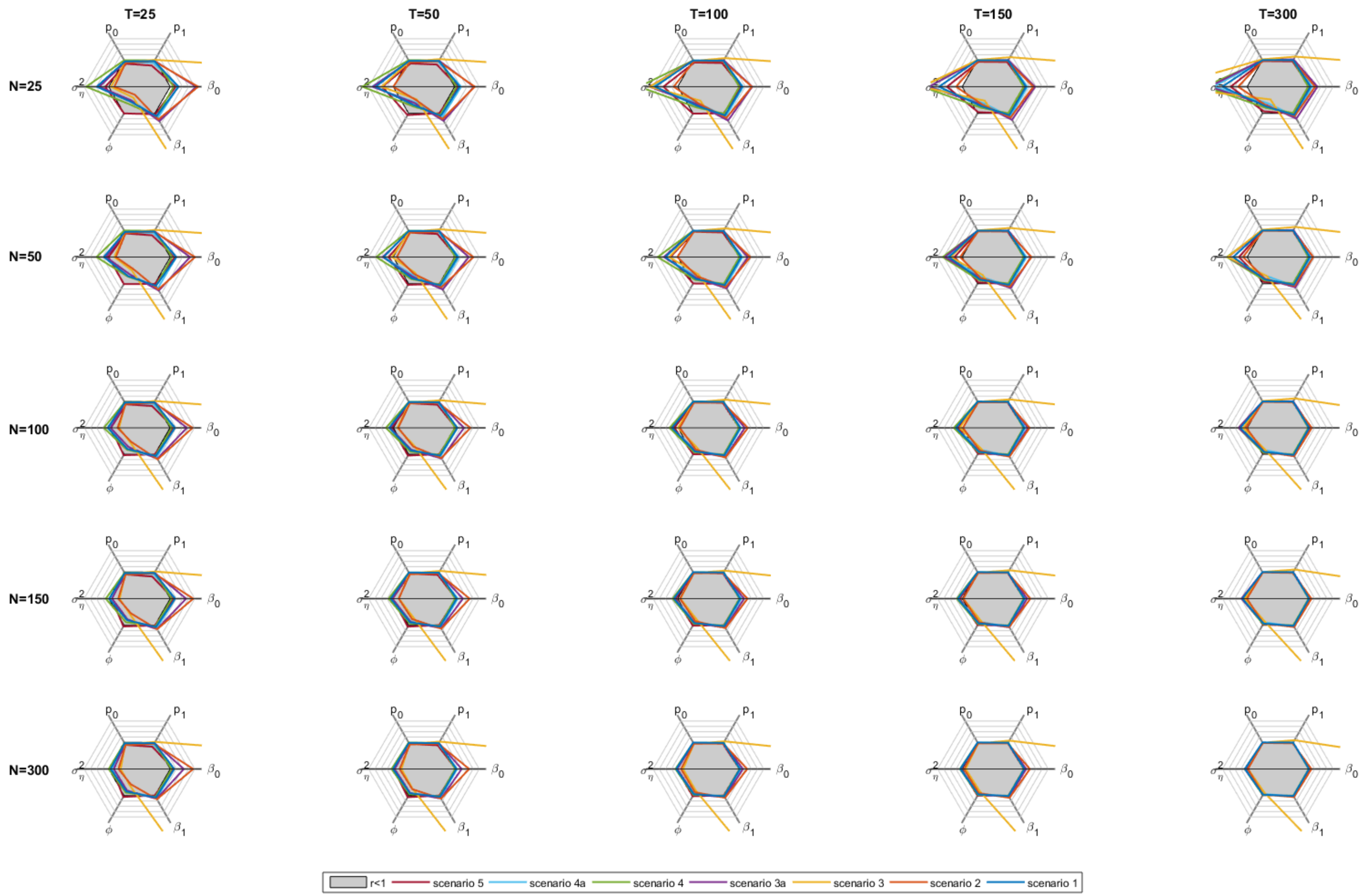
4.4.5 Other scenarios

To analyze the behavior of two-step estimates under different parameter sets, we discuss the results obtained with the other scenarios in terms of their deviation from the baseline scenario. Figures 4.7 and 4.8 show the ratio $\frac{\hat{\theta}_i(\hat{f})}{\theta_{0i}}$ and the indicators of quality of state identification, allowing us to compare the small-sample bias and the reliability of estimates of the current and past states.

[16] Different normality tests are known to have different power depending on the shape of the distribution. Jarque-Bera is considered to be the most powerful when the distribution is symmetrical (i.e. in case of $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\varphi}$), but it is overcome by Kolmogorov-Smirnov test in other cases (see Thadewald and Büning (2007) for details).

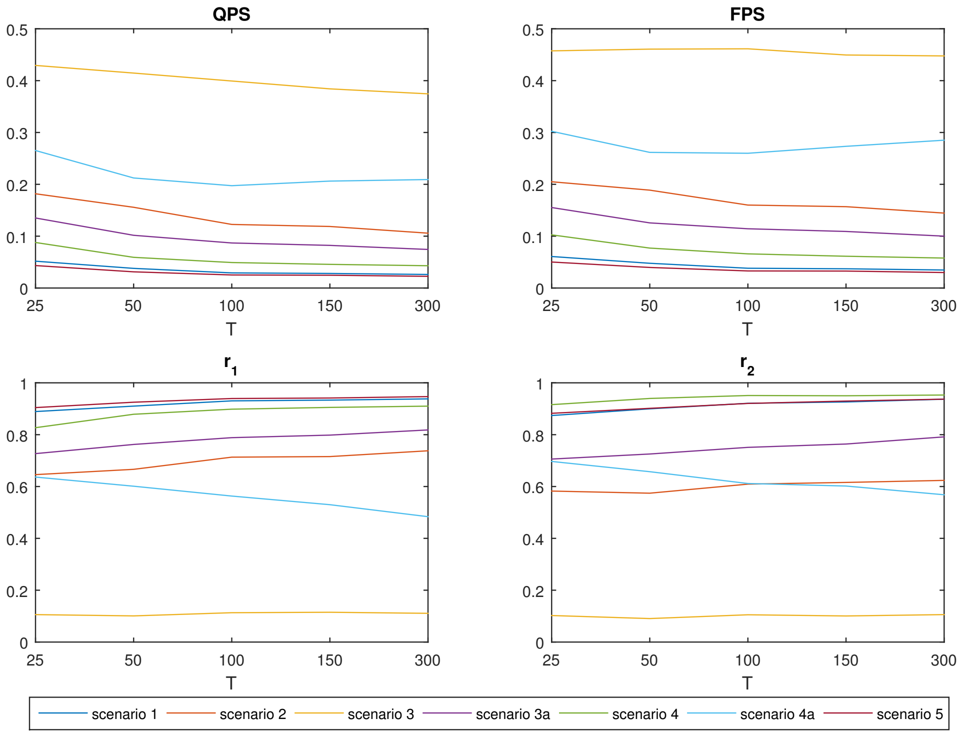
[17] as well as their counterpart $\hat{\theta}(f)$, as reported by Psaradakis and Sola (1998).

Figure 4.7: Mean values of two-step estimates for different scenarios



Note: The values on each axis varies from 0 to 2, the shaded area marking the values below or equal 1. The scenarios under consideration are: scenario 1 (baseline scenario): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.3$; scenario 2 (noisy factor): $c = 2$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.3$; scenario 3.1 (high autocorrelation): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.9$; scenario 3.2 (medium autocorrelation): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.6$; scenario 4.1 (impersistent states): $c = 5$, $p_0 = 0.5$, $p_1 = 0.5$, $u = 0.1$, $\varphi = 0.3$; scenario 4.2 (very persistent states): $c = 5$, $p_0 = 0.95$, $p_1 = 0.95$, $u = 0.1$, $\varphi = 0.3$; scenario 5 (homogeneous data): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.5$, $\varphi = 0.3$

FIGURE 4.8: Indicators of quality of state identification for various scenarios, smoothed probability, $N = 100$



Note: The scenarios under consideration are: scenario 1 (baseline scenario): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.3$; scenario 2 (noisy factor): $c = 2$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.3$; scenario 3.1 (high autocorrelation): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.9$; scenario 3.2 (medium autocorrelation): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.1$, $\varphi = 0.6$; scenario 4.1 (impersistent states): $c = 5$, $p_0 = 0.5$, $p_1 = 0.5$, $u = 0.1$, $\varphi = 0.3$; scenario 4.2 (very persistent states): $c = 5$, $p_0 = 0.95$, $p_1 = 0.95$, $u = 0.1$, $\varphi = 0.3$; scenario 5 (homogeneous data): $c = 5$, $p_0 = 0.9$, $p_1 = 0.8$, $u = 0.5$, $\varphi = 0.3$

In spite of the fact that scenarios relate to very different conditions, we can track several commonalities:

- for small N and T, φ tends to be underestimated, whereas β_0 and σ^2 are overestimated;
- σ^2 is estimated better when N rises; $\hat{\beta}_0$ and $\hat{\beta}_1$ are more accurate with higher T;
- for all scenarios except scenario 3.1, the two-step estimates are very close to their true values for $N > 150$ and $T > 150$;
- the two-step estimates of transition probabilities are the most accurate.

The differences between scenarios are clearly visible under small N and T. When the factor is noisy (scenario 2), the bias of the two-step estimates amplifies greatly for the autoregressive coefficient and β_0 and β_1. When the factor dynamics has high persistence (φ is high, scenario 3.1), the two-step method tends to confuse it with a large distance in mean, overestimating both constants to a large extent, and underestimating φ. The problem, however, disappears once the characteristic root is far enough from unity (scenario 3.2). The two-step method appears to be resistant to different degrees of persistence in states. For both low persistence and high persistence cases (scenario 4.1 and 4a, respectively), the distortions are comparable to the baseline scenario, being slightly higher in case of frequently changing regimes when N is low. The last scenario (homogeneous data) is, not surprisingly, the most favourable of all, bringing two-step estimates close to their true values at $N > 100$ and $T > 150$.

In terms of quality of state identification, with $N \geqslant 100$ the two-step estimates lead to performance usually considered as acceptable in all scenarios except the case of high autoregressive coefficient (scenario 3).[18] Not surprisingly, more favourable scenarios (homogeneous data, low persistence in states) lead to more accurate estimates of state, whereas high φ and very persistent states deteriorate the ability of the model to identify states.

Finally, Figures B.4 and B.5 show the results of the Kolmogorov-Smirnov and Jarque-Berra normality tests. The results appear to be contradictory, leaving the asymptotic distribution of the two-step estimates an open question.

4.5 Conclusion

In this paper we analyze the consistency and small-sample performance of a two-step estimator of the Markov-Switching Dynamic Factor model with the help of Monte Carlo simulations.

We observe that the empirical average of the estimates approaches the true value of the parameters when the number of observations and number of series rise. Together with convergence of the mean estimated standard errors to the sampling standard errors of the estimates, these two facts indicate consistency of the two-step method.

We find that, under values of parameters of the data generating process close to the ones usually observed in empirical applications, the estimates of the switching constants and transition probabilities are close to their true values when the the dataset contains more than 150 series and with at least 150 observations. The convergence of the estimates of the autoregressive coefficient and the variance of the error term requires higher N

[18] Interestingly, the estimates of the model on the observed factor also lead to poor estimates of current and past states. This is somehow in contradiction with Psaradakis and Sola (1998), where the consistency is achieved for all parameters of the model (including switching constants). This difference might be explained by the use of a different specification of the model, i.e. with a switch in mean instead of constant; this feature softens the effect of the switch).

and T. The results of the baseline scenario can be improved by the use of informative and homogeneous data (in terms of signal-to-noise), which brings the two-step estimates close to the true values at $N > 100$ and $T > 150$.

Consistent with previous results concerning the estimates of a simple autoregressive Markov Switching model (in the context of this paper this is equivalent to the hypothesis that the factor is observed), our findings indicate that the estimates are biased when T and N are small. In fact, the autoregressive coefficient tends to be underestimated, whereas the variance of the error terms and the constants (in absolute value) are overestimated. The precision of the variance of the error term increases when N rises, while the estimates of constants are more accurate with higher T. Importantly, the estimates of transition probabilities have almost no bias with N and T as small as $N = 50$ and $T > 50$. These observations are also valid for various deviations from the baseline DGP.

When the baseline DGP is modified, we observe that the bias of the two-step estimates increases a lot for the autoregressive coefficient and the constants when the factor is noisier. When the autoregressive coefficient is close to unity, the two-step method tends to confuse the effect of high persistence in the dynamics with a large distance in mean, overestimating both constants to a large extent, and underestimating the autoregressive component. The problem, however, disappears once the characteristic root is far enough from unity. The two-step method appears to be resistant to different degrees of persistence in states. For both low persistence and high persistence cases the distortions are comparable to the baseline scenario, being slightly higher in case of frequently changing regimes when N is low.

In spite of the bias in small samples, the two-step estimates still lead to plausible state-detection performance of the MS-DFM with a dataset of dimensions commonly used in the business cycle analysis ($T > 100$, $N > 100$), producing the filtered and smoothed probability of recession which are highly correlated with the true underlying sequence of states and giving a reasonable amount of false recession signals.

The empirical distributions of the t-statistics associated to the two-step estimates in finite samples tend to be skewed and leptokurtic and non-normal according to the Kolmogorov-Smirnov test. Therefore, some of the traditional tests using the normality assumption (such as t-student significance test or Wald-type test) are likely to be invalid. Similarly, the standard errors of the estimates should better be bootstrapped for small N and T. A positive exception are the parameters of transition probabilities, which were found to be normally distributed when $T > 300$.

This paper shows the general validity of the two-step estimation method for small-samples. It seems however likely that its performance can be improved by using more efficient estimates to estimate the factor on the first step. For example, the use of two-step method or Quasi-Maximum likelihood estimates proposed by Doz et al. (2011) and Doz et al. (2012) for the first step might probably lead to more precise estimates in the second step.

Chapter 5

Dynamical Interaction Between Financial and Business Cycles

Abstract

We adopt the Dynamical Influence model from computer science and transform it to study the interaction between business and financial cycles. For this purpose, we merge it with Markov-Switching Dynamic Factor Model (MS-DFM) which is frequently used in economic cycle analysis. The model suggested in this paper, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-FM), allows to reveal the pattern of interaction between business and financial cycles in addition to their individual characteristics. More specifically, this model allows to describe quantitatively the existing regimes of interaction in a given economy and to identify their timing, as well as to evaluate the effect of the government policy on the duration of each of the regimes. We are also able to determine the direction of causality between the two cycles for each of the regimes. The model estimated on the US data demonstrates reasonable results, identifying the periods of higher interaction between the cycles in the beginning of 1980s and during the Great Recession, while in-between the cycles evolve almost independently. The output of the model can be useful for policymakers since it provides a timely estimate of the current interaction regime, which allows to adjust the timing and the composition of the policy mix.

5.1 Introduction

Throughout the history, the financial sector has been given an increasing role with respect to the business cycle: from neutral intermediary in the theory of Modigliani-Miller to the early-warning indicator revealing the expectations of the economic agents about the business cycle in the framework of the efficient market hypothesis, then further to financial accelerator exacerbating the shocks in the real economy in models with financial frictions, and finally, to the independent source of shocks, on a par with technology and preference shocks in the New Keynesian DSGE models. Given the fast development and the increasing importance of the financial sector, the understanding of the interaction between the financial sector and the business cycle has become crucial for coordination of fiscal, monetary and macroprudential policies. For this purpose, the quantitative estimates of the role of the financial sector are essential.

The study of the financial sector and financial crises in particular gave rise to the notion of the financial cycle. For the moment, there is no single definition of the financial cycle. Instead, in most applied papers researchers refer to the fluctuations of credit, equity and house prices. In spite of the fact that these represent different parts of the financial sector, they possess similar cyclical features, which are therefore considered as the features of the financial cycle. Hubrich et al. (2013), Borio (2014), Stremmel (2015) find that the financial cycles are longer than the real business cycles and last about 12-15 years in US, France and Italy. Drehmann et al. (2012), Ciccarelli et al. (2016), Canova and Ciccarelli (2009), Canova and Ciccarelli (2012) find that the amplitude and duration of the financial cycle evolve. Borio (2006) states that the financial cycle depends on financial regime (liberalized market, controlled market), monetary policy (high and variable inflation causes financial instability) and the state of the business cycle (recession or expansion). In the same time, most of the studies agree that the business cycle, in turn, depends on the financial cycle, with the real shocks being more significant during the episodes of financial instability (see, for example, Bernanke and Gertler (1999), Kiyotaki and Moore (1997), Borio (2014), Hubrich et al. (2013), Claessens et al. (2012)).

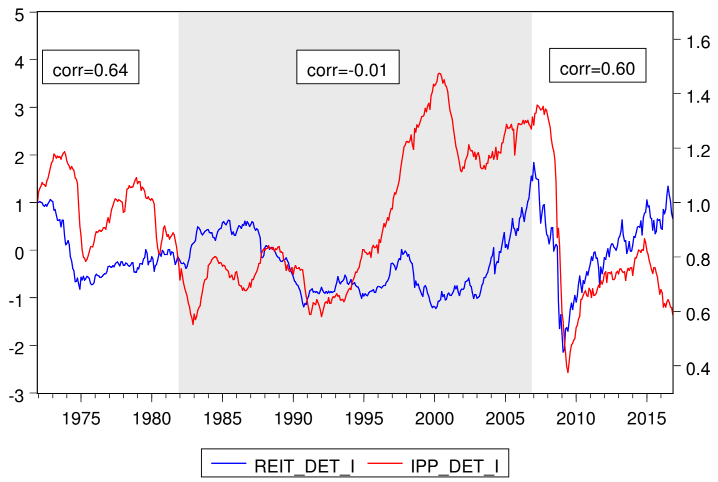
One particularly interesting feature of interdependence - the causality direction between the cycles - has been studied in many papers. In the same time, there is no consensus on whether the financial cycle leads the real cycle (Borio (2014), Adrian et al. (2010), Bandholz and Funke (2003), Chauvet (1999), Chauvet and Senyuz (2012)) or lags behind it (Runstler and Vlekke (2015)). This, however, is consistent with the fact that the financial cycles evolve over time and are longer than business cycles.

Given the changing character of the cycles, it is natural to expect that the interaction between them is also evolving. Indeed, a brief look on the dynamics of the business and financial cycle in the US (approximated by the index of industrial production and the index of house prices[1], respectively) shows that the degree of synchronization is different in different periods of time (Figure 5.1). The cycles are much more correlated in 1970s-beginning of 1980s and after the Global Financial Crisis (with correlation about 0.60),

[1] see ECB (2009) for discussion of the indicator characterizing the financial cycle

and much less in-between (the correlation is zero). The absolute value of the cross-correlations is even higher during these periods (see the dynamics of the absolute value of correlation and cross-correlation estimated on a moving window with width $w = 141$ on Figure 5.2).[2]

FIGURE 5.1: US industrial production index and US index of house prices



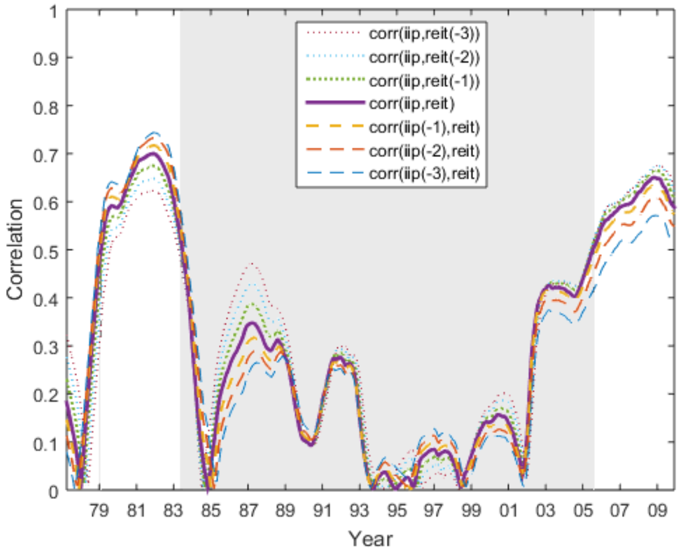
Note: US index of industrial production (red line, right axis, source: Federal Reserve Bank of St. Louis), US index of house prices (blue line, left axis, source: FTSE NAREIT US Real Estate Composite Index). Both series are detrended and seasonally-adjusted.

Taking into consideration the stylized facts mentioned above, an econometric framework that is used to study the joint dynamics of business and financial cycles should allow for the dynamical feedback between them. This idea was implemented in several different approaches. Among them are the time-varying VAR (in Hubrich et al. (2013)), Markov-Switching VAR model with time-varying transition probabilities (as, for example, in Billio et al. (2007)), versions of multivariate structural time series models (STSMs) (see Runstler and Vlekke (2015)), and time-varying Panel Bayesian VAR (see Ciccarelli et al. (2016)) for the analysis of the macro-financial linkages between countries.

In this paper we suggest an alternative model, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-FM) that provides rich statistical inference due to its three components: Dynamical Influence model by Pan et al. (2012), Markov-Switching model by Hamilton (1989) and Dynamic Factor model by Geweke (1977). Importantly, in contrast to the models mentioned above, the DI-MS-FM does not require an exogenous variable to drive the interaction but allows it to evolve intrinsically. More precisely, we assume that each of the cycles can be in several states (expansion and recession in case of the business cycle, boom and downturn in case of the financial cycle), and that

[2]The results are similar when the financial cycle is approximated with a time series of credit, as suggested by Drehmann et al. (2012).

Figure 5.2: Cross-correlations (in absolute value) between industrial production index and US index of house prices



Note: Cross-correlations between US index of industrial production and US index of house prices (FTSE NAREIT US Real Estate Composite Index) estimated on a moving window with width $w = 141$, i.e. a estimate for a date t is obtained using observations from $t - 70$ to $t + 70$.

there are several regimes of interaction which differ in degree of interdependence and leading/lagging relation. This assumption is formalized with the help of an hierarchical structure, where an exogenous unobservable Markov chain governs the mutual impact of the two other discrete processes characterizing the cycles. Besides average duration, qualitative characteristics, and filtered and smoothed probabilities of each state for each of the cycles, we get the same inference for the existing influence regimes. Additionally, for each of the influence regimes, we are able to identify the direction of causality between cycles and evaluate the relative importance of the past of each cycle on their present states. These estimates allow to perform a retrospective analysis of the cycles and their interaction as well as to make probabilistic inference on the current situation. Finally, they allow to provide forecasts of future states of each cycle given the current influence regime. Moreover, the estimate of the filtered probability of the influence regime corresponding to high interaction (as influence regime 2 in our empirical exercise below) can serve as an early-warning indicator of systemic risk (if one considers the notion of systemic risk in a broader sense, i.e. as a risk of a joint recession both in the financial and the business cycle simultaneously). These estimates can be useful for policymakers to design and adjust the policy mix.

The paper is organized as follows. In Section 2 we introduce the model, describe the underlying interaction mechanism and define Granger causality and suggest a possible extension of the model allowing to evaluate the effect of government policies. In Section 3 we discuss the estimation procedure, derive h-step ahead forecasts, examine in-sample and out-of-sample performance of the model. Section 4 contains the results of the application of the model to the US data. Section 5 concludes.

5.2 The DI-MS-FM

5.2.1 The general presentation

We adopt the Dynamical Influence model from computer science by Pan et al. (2012) and transform it to study the interaction between business and financial cycles. For this purpose, we merge it with Markov-Switching Dynamic Factor Model (MS-DFM) which is frequently used in economic cycle analysis. The resulting model, the Dynamical Influence Markov-Switching Dynamic Factor Model (DI-MS-FM), is presented below.

At date t, $t = 1, ..., T$, economic agents observe (or infer) the business cycle RF_t and the financial cycle FF_t[3] which have the following dynamics

$$RF_t = \mu(S_t^1) + \varphi(L)RF_t + \sigma(S_t^1)\varepsilon_t, \tag{5.1}$$

$$FF_t = \beta(S_t^2) + \psi(L)FF_t + \theta(S_t^2)\xi_t, \tag{5.2}$$

where S_t^1 and S_t^2 are unobservable discrete processes which are associated with a finite number of states and which govern the dynamics of the business cycle and the financial cycle, correspondingly, $\varphi(L) = \varphi_1 L + ... + \varphi_{p_1} L^{p_1}$ and $\psi(L) = \psi_1 L + ... + \psi_{p_2} L^{p_2}$ are lag polynomials of finite order p_1 and p_2 correspondingly, $\{\varepsilon_t\}$ and $\{\xi_t\}$ are independent standard Gaussian white noises. The functions $\mu(\cdot)$,$\sigma(\cdot)$, $\beta(\cdot)$, $\theta(\cdot)$ are known functions of the specified arguments with unknown parameters.

We assume that the interaction between the cycles happens at the level of unobservable processes S_t^1 and S_t^2, but not observations, which means that (5.1)-(5.2) is a restricted VAR.[4]

The current values of S_t^1 and S_t^2 are each dependent on the past of both processes and a variable r_t governing the interaction between S_t^1 and S_t^2, which is the crucial feature of the model:

$$P(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t) = A(S_{t-1}^1, S_{t-1}^2, r_t), \tag{5.3}$$

$$P(S_t^2|S_{t-1}^1, S_{t-1}^2, r_t) = B(S_{t-1}^1, S_{t-1}^2, r_t), \tag{5.4}$$

[3]The construction of RF_t and FF_t will be described later on in section 5.2.2.

[4]The lags of FF_t do not enter the equation for RF_t (equation (5.1)) and vice versa. When the interaction on the level of observation is also allowed for, the identification of each channel can be an issue.

$$P(r_t|r_{t-1}) = Q, \tag{5.5}$$

where $A(\cdot)$, $B(\cdot)$ are known functions with unknown parameters. We assume that the initial r_0, S_0^1, S_0^2, RF_0, FF_0 are not random. The process r_t, which we call the interaction regime process, is a Markov chain of first order[5] with a finite number of regimes and a transition probability matrix Q.

For the sake of simplicity, we suppose here that the variables r_t, S_t^1 and S_t^2 can take only two values (states) each ($S_t^1 = 1$ in case of expansion and $S_t^1 = 2$ in case of recession; $S_t^2 = 1$ in case of financial boom and $S_t^2 = 2$ in case of financial downturn; the interpretation of the states of $r_t \in \{1, 2\}$ is determined by the degree of mutual influence between the two chains in each regime estimated within the model[6].). Nevertheless, the analysis can be easily extended to incorporate chains of a higher (and different) order and with more states. Similarly, it is also feasible to allow the past of RF_t, FF_t or some observable covariate cause S_t^1 and S_t^2.

Unlike classic Markov-switching models used in business cycle analysis, strictly speaking, the processes S_t^1 and S_t^2 are not Markov chains since the current state of each of them depends on the past of the other chain, too. Moreover, the process (S_t^1, S_t^2) is not Markov either as it depends on all its lags. Nevertheless, for the ease of exposition, we address to S_t^1 and S_t^2 as "chains".

To understand the dynamics of the model, we present the conditional distributions of $\underline{RF_t}$, $\underline{FF_t}$, $\underline{S_t^1}$, $\underline{S_t^2}$, $\underline{r_t}$ using a generic notation $\underline{x_t} = (x_t, x_{t-1}, ..., x_0)$:

$$\mathfrak{L}(r_t|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{r_{t-1}}) = \mathfrak{L}(r_t|r_{t-1}), \tag{5.6}$$

$$\mathfrak{L}(S_t^1|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{r_t}) = \mathfrak{L}(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t), \tag{5.7}$$

$$\mathfrak{L}(S_t^2|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_t^1}, \underline{S_{t-1}^2}, \underline{r_t}) = \mathfrak{L}(S_t^2|S_{t-1}^1, S_{t-1}^2, r_t), \tag{5.8}$$

$$\mathfrak{L}(RF_t|\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_t^1}, \underline{S_t^2}, \underline{r_t}) = N(\mu(S_t^1) + \varphi(L)RF_t, \sigma^2(S_t^1)), \tag{5.9}$$

$$\mathfrak{L}(FF_t|\underline{FF_{t-1}}, \underline{RF_t}, \underline{S_t^1}, \underline{S_t^2}, \underline{r_t}) = N(\beta(S_t^2) + \psi(L)FF_t, \theta^2(S_t^2)). \tag{5.10}$$

The fundamental assumptions of the model are:

1. r_t is autonomous, i.e. S_t^1, S_t^2, RF_t and FF_t do not cause r_t in the Granger sense since $\underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{RF_t}, \underline{FF_t}$ do not appear in its conditional distribution.

2. RF_t and FF_t do not Granger cause S_t^1, S_t^2 and r_t.

[5]This assumption is not restrictive.

[6]The model can be easily extended for the case when the number of states of S_t^1 and S_t^2 is not equal

3. S_t^1 and S_t^2 are conditionally independent given $\underline{RF_{t-1}}, \underline{FF_{t-1}}, \underline{S_{t-1}^1}, \underline{S_{t-1}^2}, \underline{r_t}$.
4. The process (S_t^1, S_t^2, r_t) is an autonomous Markov chain.
5. RF_t and FF_t are conditionally independent given $\underline{r_t}$, $\underline{S_t^1}$ and $\underline{S_t^2}$.

To summarize, the dynamics of the model can be represented in the following way (with $\omega_t = (r_t, S_t^1, S_t^2, RF_t, FF_t)$):

$$r_t|\omega_{t-1} = r_t|r_{t-1} \tag{5.11}$$
$$S_t^1|r_t, \omega_{t-1} = S_t^1|r_t, S_{t-1}^1, S_{t-1}^2, \tag{5.12}$$
$$S_t^2|r_t, S_t^1, \omega_{t-1} = S_t^2|r_t, S_{t-1}^1, S_{t-1}^2, \tag{5.13}$$
$$RF_t|r_t, S_t^1, S_t^2, \omega_{t-1} = RF_t|S_t^1, \tag{5.14}$$
$$FF_t|RF_t, r_t, S_t^1, S_t^2, \omega_{t-1} = FF_t|S_t^2. \tag{5.15}$$

5.2.2 Construction of RF_t and FF_t

To construct the proxies for business and financial cycles RF_t and FF_t, we adopt the Dynamic Factor Model approach by Stock and Watson (1989). Following the concept of the business cycle by Burns and Mitchell (1946) as comovement of economic series, they assume that each of the indicators of the real sector of an economy (industrial production, consumption, stock, consumer and business surveys, etc.) can be decomposed into two parts. The first one refers to the comovement of series of the real sector (the business cycle) while the second part corresponds to the idiosyncratic dynamics:

$$x_t = \lambda RF_t + y_t, \tag{5.16}$$

where x_t is a $N \times 1$ vector of stationarized and deseasonalized economic indicators, RF_t is a $r \times 1$ vector of common factors of x_t, y_t is a $N \times 1$ vector of idiosyncratic components uncorrelated with RF_t at all leads and lags, λ is a $N \times r$ vector of factor loadings.

Bai (2003), Stock and Watson (2002) showed that $\hat{RF_t}$ can be consistently estimated with PCA when N and T are large. The use of PCA for factor extraction in the two-step procedures is very convenient since it is robust to some types of misspecifications, as was shown by Stock and Watson (2002). For example, under the number of series and observations sufficiently large, PCA provides consistent estimates of factors when the series of the database are weakly cross-sectionally correlated or autocorelated. Also, PCA does not require normality of the series. In the business cycle analysis, the first principal component usually explains most of the variance of x_t, so RF_t is actually one-dimensional.

Therefore, the first principal component of a rich database of macroeconomic variables is commonly accepted as a proxy to the business cycle. The proxy of the financial cycle $\hat{FF_t}$ is obtained similarly from the database of financial indicators.[7] These two proxies are then used to estimate (5.1)-(5.5).

To keep the notations simple, in what follows RF_t and FF_t (but not $\hat{RF_t}$ and $\hat{FF_t}$) refer to the proxies of business and financial cycles estimated with PCA.

5.2.3 The interaction mechanism

In order to describe the interaction between the chains, let us consider their joint dynamics. As we have mentioned above, the process (S_t^1, S_t^2, r_t) is a Markov chain. Each its component taking two values, the joint Markov chain has 8 states and thus a 8×8 transition matrix with 56 free parameters. By imposing a particular interaction mechanism, we parametrize this transition probability matrix with only 14 parameters, thus rendering the model more parsimonious. The interaction mechanism is organized as follows.

Consider two auxiliary variables, E_t^1 and E_t^2 (E standing for "effect"). Each of these variables is a binary variable and determines the current driving force for each of the corresponding chains, i.e.:

$$E_t^1 = \begin{cases} d, & \text{if } S_t^1 \text{ is impacted by } S_{t-1}^1, \text{ direct effect} \\ c, & \text{if } S_t^1 \text{ is impacted by } S_{t-1}^2, \text{ cross effect} \end{cases}, \tag{5.17}$$

$$E_t^2 = \begin{cases} d, & \text{if } S_t^2 \text{ is impacted by } S_{t-1}^2, \text{ direct effect} \\ c, & \text{if } S_t^2 \text{ is impacted by } S_{t-1}^1, \text{ cross effect} \end{cases}. \tag{5.18}$$

The chances of being under cross or direct effect for each of the chains depend on the interaction regime variable r_t. The exogenous process r_t is an ergodic first-order Markov Chain with 2 states, i.e.

$$P(r_t = j | r_{t-1} = i, r_{t-2} = k, ...) = P(r_t = j | r_{t-1} = i) = q_{ij}, \quad j, i, k \in \{1, 2\},$$

so r_t switches states according to the transition probabilities matrix

$$Q = \begin{bmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{bmatrix}. \tag{5.19}$$

[7] One can use single series to approximate business and financial cycles (industrial production index and housing prices index, for example). However, in practice factors are commonly used as they reflect a larger information set on each of the sectors.

The dynamic causality structure is the following:

1. the values of r_t are generated from a two-state Markov chain with the transition probability matrix Q;

2. for each value of r_t, E_t^1 is drawn in $\{d, c\}$ from the Bernoulli distribution $\mathcal{B}(R_{11}^{r_t})$, where $R_{11}^{r_t}$ is the probability of drawing d and $1 - R_{11}^{r_t} = R_{21}^{r_t}$ is the probability of drawing c;

3. for each value of r_t, E_t^2 is drawn in $\{d, c\}$ from the Bernoulli distribution $\mathcal{B}(R_{22}^{r_t})$, where $R_{22}^{r_t}$ is the probability of drawing d and $1 - R_{22}^{r_t} = R_{12}^{r_t}$ is the probability of drawing c;

4. for $E_t^1 = d$, $S_{t-1}^1 = i$, $S_{t-1}^2 = j$ $(i, j \in \{1, 2\})$, S_t^1 is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(D_{i1}^1)$, where D_{i1}^1 is the probability of drawing 1 and $1 - D_{i1}^1 = D_{i2}^1$ is the probability of drawing 2;
   for $E_t^1 = c$, $S_{t-1}^1 = i$, $S_{t-1}^2 = j$ $(i, j \in \{1, 2\})$, S_t^1 is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(C_{j1}^{21})$, where C_{j1}^{21} is the probability of drawing 1 and $1 - C_{j1}^{21} = C_{j2}^{21}$ is the probability of drawing 2;

5. for $E_t^2 = d$, $S_{t-1}^1 = i$, $S_{t-1}^2 = j$ $(i, j \in \{1, 2\})$, S_t^2 is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(D_{j1}^2)$, where D_{j1}^2 is the probability of drawing 1 and $1 - D_{j1}^2 = D_{j2}^2$ is the probability of drawing 2;
   for $E_t^2 = c$, $S_{t-1}^1 = i$, $S_{t-1}^2 = j$ $(i, j \in \{1, 2\})$, S_t^2 is drawn in $\{1, 2\}$ from the Bernoulli distribution $\mathcal{B}(C_{i1}^{12})$, where C_{i1}^{12} is the probability of drawing 1 and $1 - C_{i1}^{12} = C_{i2}^{12}$ is the probability of drawing 2.

Therefore, the interaction between the chains is fully described by a set of 14 parameters (q_{11}, q_{22}, R_{11}^1, R_{22}^1, R_{11}^2, R_{22}^2, D_{11}^1, D_{22}^1, C_{11}^{12}, C_{22}^{12}, D_{11}^2, D_{22}^2, C_{11}^{21}, C_{22}^{21}), which we organize in matrices Q defined above,

$$R^1 = \begin{bmatrix} R_{11}^1 & 1 - R_{22}^1 \\ 1 - R_{11}^1 & R_{22}^1 \end{bmatrix}, \; R^2 = \begin{bmatrix} R_{11}^2 & 1 - R_{22}^2 \\ 1 - R_{11}^2 & R_{22}^2 \end{bmatrix},$$

$$D^1 = \begin{bmatrix} D_{11}^1 & 1 - D_{11}^1 \\ 1 - D_{22}^1 & D_{22}^1 \end{bmatrix}, \; D^2 = \begin{bmatrix} D_{11}^2 & 1 - D_{11}^2 \\ 1 - D_{22}^2 & D_{22}^2 \end{bmatrix},$$

$$C^{12} = \begin{bmatrix} C_{11}^{12} & 1 - C_{11}^{12} \\ 1 - C_{22}^{12} & C_{22}^{12} \end{bmatrix}, \; C^{21} = \begin{bmatrix} C_{11}^1 & 1 - C_{11}^1 \\ 1 - C_{22}^1 & C_{22}^{21} \end{bmatrix}.$$

At period t the probability of being in a particular state of the business cycle S_t^1 depends on its own past S_{t-1}^1 and also on the previous state of the financial cycle S_{t-1}^2. The relative importance of each chain is determined by the matrix R^{r_t} with $r_t \in \{1, 2\}$, which assigns weights to S^1_{t-1} and S^2_{t-1}, thus determining their self effect and the effect of the other chain given the current interaction regime r_t. Therefore, the probability that the business cycle is in state S^1_t, given the states S^1_{t-1}, S^2_{t-1} and r_t, is a weighted average of probabilities to switch from $S^1_{t-1} = i$ to $S^1_t = k$ and from $S^2_{t-1} = j$ to $S^1_t = k$, where $i, j, k \in \{1, 2\}$, with weights determined by r_t. Formally, the probability that a chain S^1_t is in state k given the past of both chains and the current values of r_t is:

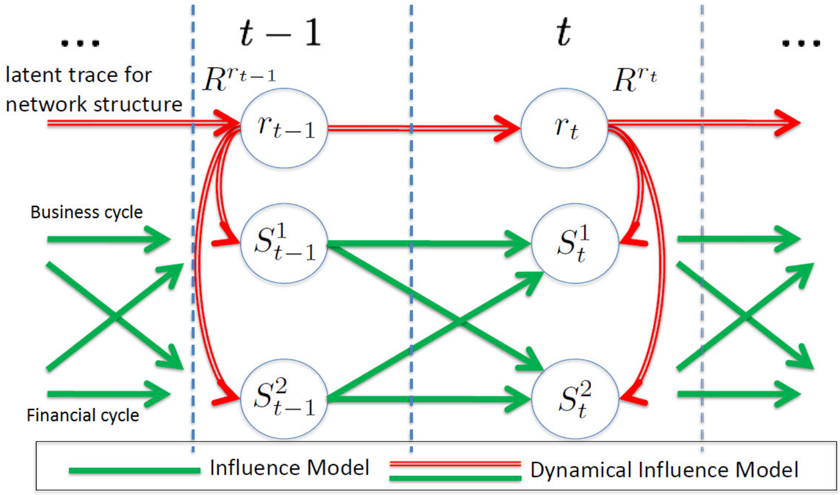
$$
\begin{aligned}
P(S^1_t = k|S^1_{t-1} = i, S^2_{t-1} = j, r_t) = \sum_{l=d,c} P(S^1_t = k, E^1_t = l|S^1_{t-1} = i, S^2_{t-1} = j, r_t) \\
= P(S^1_t = k|E^1_t = d, S^1_{t-1} = i, S^2_{t-1} = j, r_t)P(E^1_t = d|r_t) \\
+ P(S^1_t = k|E^1_t = c, S^1_{t-1} = i, S^2_{t-1} = j, r_t)P(E^1_t = c|r_t) \\
= D^1_{ik}R^{r_t}_{11} + C^{21}_{jk}(1 - R^{r_t}_{11}) \\
= D^1_{ik}R^{r_t}_{11} + C^{21}_{jk}R^{r_t}_{21}.
\end{aligned}
\tag{5.20}
$$

with $i, j, k \in \{1, 2\}$ and where X_{ij} denotes the element of the i-th row and j-th column of the matrix X. Similar logic applies to the financial cycle giving

$$
P(S^2_t = k|S^1_{t-1} = i, S^2_{t-1} = j, r_t) = D^2_{jk}R^{r_t}_{22} + C^{12}_{ik}R^{r_t}_{12}. \tag{5.21}
$$

Here D^i, $i \in \{1, 2\}$, is a matrix of parameters capturing the transition due to the direct effect, so that, for example, the element D^1_{11} shows the probability that the first chain stays in the regime 1 “expansion”. Similarly, the matrix C^{ki}, $i, k \in \{1, 2\}$, $i \neq k$, is a matrix of parameters that capture cross effect transitions, so that, for instance, the element C^{12}_{11} shows the probability that an expansion in the business cycle induces a boom in the financial cycle. Importantly, direct effect transitions and cross effect transitions do not depend on r_t. The value $R^{r_t}_{ki}$ shows the relative importance (the weight) of the past of chain k on the present of the chain i given the current interaction regime $r_t \in \{1, 2\}$. Therefore, the larger are the diagonal elements of this matrix, the higher is the self-impact, and more independent are the chains. The most important feature of this framework arises from the fact that the weights vary over time with r_t, thus rendering the interaction between the two chains dynamical. We illustrate schematically the Dynamical Influence model in Figure 1.

FIGURE 5.3: A graphical representation of the Dynamical Influence Model



Note: This is a modified version of Figure 2 from the paper by Pan et al. (2012)

This type of interaction is new in the economic literature. The existing methods based on the modeling of the joint process (S^1_t, S^2_t) allow either for a fixed relation between the chains (in case of static transition probability matrix) or exogenously driven relation (in case of transition probability matrix depending on some covariates). On the contrary, in this model the interaction is designed to be intrinsically dynamical, whether dependent on the covariates or not.

As we show in the next section, after introduction of a new state variable the model boils down to the classic Hamilton (1989) Markov-switching model. Therefore, once the estimates of the coefficients of (5.1) and (5.2), D^1, D^2, C^{12}, C^{21}, R^1, R^2 are obtained, the standard filtered and smoothed probabilities of each state of each chain can be calculated, including the smoothed probability $P(r_t = j|I_T)$, $j \in \{1, 2\}$ of being in a particular interaction regime j, where $I_\tau = (\underline{RF_\tau}, \underline{FF_\tau})$ is the information available up to time τ. On top of that, it would be possible to calculate the joint filtered and smoothed probabilities $P(S^1_t = i, S^2_t = j|I_t)$ and $P(S^1_t = i, S^2_t = j|I_T)$, $i, j \in \{1, 2\}$, which is useful for the purpose of analysis of joint crises in real and financial sectors.

5.2.4 Granger causality

As we said above, in this framework the two cycles RF_t and FF_t interact on the level of chains. Importantly, the estimated matrices of coefficients R^1, and R^2 can give us an idea about the causality relation between the two chains S^1_t and S^2_t.

Consider a process $\tilde{S}_t = (S_t^1, S_t^2, r_t)$ which is a Markov process with 8 states. We can decompose the transition probabilities as follows:

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | S_t^2, r_t, S_{t-1}^1, S_{t-1}^2, r_{t-1}) \times P(S_t^2 | r_t, S_{t-1}^1, S_{t-1}^2, r_{t-1}) P(r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}). \tag{5.22}$$

Using the assumptions (2) and (3) and equation (5.6), this expression can be simplified:

$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t) P(r_t | r_{t-1}). \tag{5.23}$$

Now, like Billio and Sanzo (2015), we can define Granger non-causality between S_t^1 and S_t^2 in *strong* sense, since it is specified by imposing restrictions on the parameters characterizing conditional distributions:

1. S_{t-1}^2 does not strongly cause S_t^1 one-step ahead given S_{t-1}^1 and r_t if
$$P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t) = P(S_t^1 | S_{t-1}^1, r_t) \quad \forall t. \tag{5.24}$$

2. S_{t-1}^1 does not strongly cause S_t^2 one-step ahead given S_{t-1}^2 and r_t if
$$P(S_t^2 | S_{t-1}^2, S_{t-1}^1, r_t) = P(S_t^2 | S_{t-1}^2, r_t) \quad \forall t. \tag{5.25}$$

We can also define the independence of two chains as follows:

3. S_t^1 and S_t^2 are independent given r_t if
$$P(S_t^1, S_t^2, r_t | S_{t-1}^1, S_{t-1}^2, r_{t-1}) = P(S_t^1 | r_t, S_{t-1}^1) P(S_t^2 | r_t, S_{t-1}^2) P(r_t | r_{t-1}). \tag{5.26}$$

Following the approach of Billio and Sanzo (2015), for a given parametrization (5.20), the conditions of the strong one-step ahead non-causality and independence can be derived as restrictions on the parameter space.

The restriction $H_{1 \not\Rightarrow 2}$ of the strong non-causality from S_t^1 to S_t^2 given r_t implies that the parameter related to S_{t-1}^1 is equal to zero. So, since

$$P(S_t^2 = k | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = R_{22}^{r_t} \times D_{jk}^2 + R_{12}^{r_t} \times C_{ik}^{12}, \tag{5.27}$$

the strong non-causality is implied by

$$H_{1\nRightarrow 2}: \quad R_{12}^{r_t} = 0 \tag{5.28}$$

Under $H_{1\nRightarrow 2}$ S_{t-1}^1 does not cause one-step ahead S_t^2 given S_{t-1}^2 and r_t. Since the terms related to S_{t-1}^1 are excluded from (5.27), therefore $P(S_t^2|S_{t-1}^2, S_{t-1}^1, r_t) = P(S_t^2|S_{t-1}^2, r_t)$.

On the other hand, the strong one-step ahead non-causality from S_t^2 to S_t^1 given S_{t-1}^1 and r_t, given the parametrization

$$P(S_t^1 = k|S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = R_{11}^{r_t} \times D_{ik}^1 + R_{21}^{r_t} \times C_{jk}^{21}, \tag{5.29}$$

is implied by

$$H_{2\nRightarrow 1}: \quad R_{21}^{r_t} = 0 \tag{5.30}$$

The term related to S_{t-1}^2 is excluded from (5.29), so $P(S_t^1|S_{t-1}^1, S_{t-1}^2, r_t) = P(S_t^1|S_{t-1}^1, r_t)$.

Finally, the restriction of the independence of S_t^1 and S_t^2 given r_t is implied by both restrictions (5.28) and (5.30) simultaneously:

$$H_{2\perp 1}: \quad R_{21}^{r_t} = R_{12}^{r_t} = 0 \tag{5.31}$$

Therefore, the value and significance of the off-diagonal coefficients of the matrices R^1 and R^2 allow to make inference on the causality between the two chains within each regime j. Moreover, since the elements in R^1 and R^2 are not necessarily 0 and 1, we can quantify the relative importance of each of the affecting chains.

Note that the values of the elements C^{ij}, $i, j \in \{1, 2\}$, $i \neq j$ give the idea of the global character of Granger causality between the two cycles, defining the channels of interaction irrespective of the current interaction regime. At the same time, the conditions on $R_{ij}^{r_t}$ refer to local changes in Granger causality, and can modify the intensity of the channel if it exists (the relevant element of C^{ij} is non-zero).

For the parametrization of the interaction described above, it is also possible to test for global non-causality, i.e. irrespective of the current interaction regime. However, in contrast to the local Granger non-causality, the null for the global non-causality can not be formulated in terms of restrictions on the elements of matrices R^{r_t}, for example, H_0 : $R_{12}^1 = R_{12}^2 = 0$ in case of testing for global non-causality of business cycle with respect to the financial cycle. Indeed, in this case, the parameters of the matrix C^{12} are not identified, and tests based on this null are not standard.[8] Instead, one can reformulate the null in terms of restrictions on the elements of C^{12} and C^{21}, thus avoiding the non-identification problem. Thus, S_t^1 does not strongly cause one-step ahead S_t^2 globally given S_{t-1}^2 and r_t if:

$$H_{1 \not\Rightarrow g 2} : C_{11}^{12} = C_{21}^{12}, \tag{5.32}$$

i.e. the impact of the recession is the same as the one of expansion, so the state of the business cycle is irrelevant for the future state of the financial cycle. Note that the null also implies $C_{22}^{12} = C_{12}^{12}$ since $C_{11}^{12} = 1 - C_{12}^{12}$, so under the null the matrix C^{12} has the form $\begin{bmatrix} C_{11}^{12} & 1 - C_{11}^{12} \\ C_{11}^{12} & 1 - C_{11}^{12} \end{bmatrix}$.

The null hypotheses for the strong global one-step ahead non-causality of S_t^2 with respect to S_t^1 given S_{t-1}^1 and r_t and strong global one-step ahead independence can be formulated in a similar way.

5.2.5 Extension: policy analysis

It is natural to assume that government policies may affect the cycles themselves as well as their interaction. One of possible ways to take this impact into account is through imposing dependence of the parameters describing state transitions on the policy variable vector z_t. Possible candidates for z_t series are the Federal Funds rate, the term premium as well as the series of tax shocks (see, for example, Mertens and Ravn (2013) and Romer and Romer (2010)).

Depending on assumptions on the impact of a particular policy measure, the dependence on policy variables can be introduced in different ways. While the rest of the framework stays unmodified, the changes may concern the matrices Q (impact on the duration of each of the interacting regimes), D^1 (impact on the business cycle), D^2 (impact on the financial cycle), C^{12} and C^{21} (impact on the mechanisms of transmission of states between the cycles). In the first case, for example, the transition probability matrix Q for the interaction regime variable r_t becomes dynamic, i.e. Q_t:

$$Q_t = \begin{bmatrix} q_{11}(z_{t-1}) & 1 - q_{11}(z_{t-1}) \\ 1 - q_{22}(z_{t-1}) & q_{22}(z_{t-1}) \end{bmatrix}. \tag{5.33}$$

Different functional forms of the transition probabilities mapping z_t into the unit interval can be considered (for example, the logistic function, probit function, Cauchy integral and other). The logistic function is a common case, therefore:

[8] A possible solution for this task would be to simulate the distribution of the test statistics under the null. Hansen (1996) also suggests a transformation of the test statistics based on a conditional probability measure which yields an asymptotic distribution free of the unidentifiable parameters.

$$q_{ii}(z_{t-1}) = \frac{exp(\delta_{i0} + \sum_{j=1}^{J} \delta_{ij} z_{t-j})}{1 + exp(\delta_{i0} + \sum_{j=1}^{J} \delta_{ij} z_{t-j})}, \tag{5.34}$$

where $\delta_{i0}, ..., \delta_{iJ}$, $i \in \{1, 2\}$ are parameters to estimate. The matrices D^1, D^2, C^{12} and C^{21} can be modified in a similar way.

5.3 Estimation and Forecasting

5.3.1 Maximum Likelihood Estimation

On the basis of observable data, we need to infer the distributions of the underlying latent variables and system parameters for the DI-MS-FM. If the interaction regime were constant, a standard approach to estimate (5.1)-(5.5) would be to construct an auxiliary state variable (S_t^1, S_t^2) with 2^2 states:

$$P(S_t^1 = k, S_t^2 = l | S_{t-1}^1 = i, S_{t-1}^2 = j) = (R_{11} \times D_{ik}^1 + R_{21} \times C_{jk}^{21})(R_{22} \times D_{jl}^2 + R_{12} \times C_{il}^{12}), \tag{5.35}$$

where $i, j, k, l \in \{1, 2\}$. However, when different interaction regimes come into play, the coefficients of matrices R_1 and R_2 are dependent on r_t, and the transition probability matrix of (S_t^1, S_t^2) becomes Markov-switching itself:

$$P(S_t^1 = k, S_t^2 = l | S_{t-1}^1 = i, S_{t-1}^2 = j, r_t) = (R_{11}^{r_t} \times D_{ik}^1 + R_{21}^{r_t} \times C_{jk}^{21})(R_{22}^{r_t} \times D_{jl}^2 + R_{12}^{r_t} \times C_{il}^{12}), \tag{5.36}$$

so the standard estimation procedures can not be applied. This problem is easily overcome by using the joint state variable $\tilde{S}_t = (S_t^1, S_t^2, r_t)$ with $2^3 = 8$ states instead of (S_t^1, S_t^2). In this case, the transition probability matrix Π is constant and is computed as follows:

$$\begin{aligned} \Pi = P(\tilde{S}_t | \tilde{S}_{t-1}) &= P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(r_t = j | r_{t-1} = k) \quad (5.37) \\ &= P(S_t^1 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times P(S_t^2 | S_{t-1}^1, S_{t-1}^2, r_t = j) \times Q_{kj}, \end{aligned}$$

$j, k \in \{1, 2\}$. Note that, as we have mentioned above, due to the hierarchical structure that we impose on the chains (S_t^1, S_t^2, r_t), the matrix Π has a more parsimonious representation than a transition matrix of a Markov chain with 8 states would usually have. Indeed, matrix Π contains only 14 parameters instead of 56, which certainly facilitates the numerical optimization of the likelihood. For notational use, we arrange the eight states of $\tilde{S}_t$ in the following order: $(S_t^1, S_t^2, r_t) = \{(0,0,0)\,(1,0,0)\,(0,1,0)\,(1,1,0)\,(0,0,1)\,(1,0,1)\,(0,1,1)\,(1,1,1)$

The classical Hamilton (1989) filter can then be applied. At each step, it updates the filtered probability $P(\tilde{S}_{t-1} = j|I_{t-1})$ to the next period $P(\tilde{S}_t = j|I_t)$, giving the likelihood $f(y_t|I_{t-1})$ as a by-product. Once the starting filtered probability $P(\tilde{S}_0 = j|I_0)$ is initiated (we suppose that the probability of starting in any of eight states of $\tilde{S}_0$ is equal, $P(\tilde{S}_0 = j|I_0) = 1/8$, $\forall j = 1, ..., 8$), the filtered probability for steps $t = 1, ..., T$ are calculated by iterating the following:

$$P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma) = P(\tilde{S}_t = j|\tilde{S}_{t-1} = i, \gamma)P(\tilde{S}_{t-1} = i|I_{t-1}, \gamma), \tag{5.38}$$

$$f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma) = f(y_t|\tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma)P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma) \tag{5.39}$$

$$f(y_t|I_{t-1}, \gamma) = \sum_{j=1}^{8}\sum_{i=1}^{8} f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma). \tag{5.40}$$

$$\begin{aligned} P(\tilde{S}_t = j, \tilde{S} = i|I_t, \gamma) &= \frac{f(y_t, \tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma)}{f(y_t|I_{t-1}, \gamma)} \\ &= \frac{f(y_t|\tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma) \times P(\tilde{S}_t = j, \tilde{S}_{t-1} = i|I_{t-1}, \gamma)}{f(y_t|I_{t-1}, \gamma)}, \end{aligned} \tag{5.41}$$

$$f(y_t|\tilde{S}_t = j, \tilde{S}_{t-1} = i, I_{t-1}, \gamma) = (2\pi)^{-1}(\sigma^2_{S_t^1}\theta^2_{S_t^2})^{-1/2} exp\{-\frac{1}{2}\frac{(\tilde{RF}_t)^2}{\sigma^2_{S_t^1}} - \frac{1}{2}\frac{(\tilde{FF}_t)^2}{\theta^2_{S_t^2}}\}, \tag{5.42}$$

$$P(\tilde{S}_t = j|I_t) = \sum_{i=1}^{2} P(\tilde{S}_t = j, \tilde{S}_t = i|I_t, \gamma), \tag{5.43}$$

where

$y_t = (RF_t, FF_t)$,

$\gamma = (D^1, D^2, C^{12}, C^{21}, R^1, R^2, \mu_1, \mu_2, \beta_1, \beta_2, \sigma_1^2, \sigma_2^2, \theta_1^2, \theta_2^2, \varphi_1 ... \varphi_{p_1}, \psi_1 ... \psi_{p_2})$,

$\mu_{S_t^1} = \mu_2(S_t^1 - 1) - \mu_1(S_t^1 - 2)$,

$$\sigma^2_{S^1_t} = \sigma^2_2(S^1_t - 1) - \sigma^2_1(S^1_t - 2),$$

$$\beta_{S^2_t} = \beta_2(S^2_t - 1) - \beta_1(S^2_t - 2),$$

$$\theta_{S^2_t} = \theta_2(S^2_t - 1) - \theta_1(S^2_t - 2),$$

$$\tilde{RF}_t = RF_t - \mu_{S^1_t} - \varphi(L)RF_t,$$

$$\tilde{FF}_t = FF_t - \beta_{S^2_t} - \psi(L)FF_t.$$

As a by-product of the Hamilton filter above, we obtain the log-likelihood function for the whole sample for any given value of γ:

$$\mathscr{L}(y,\gamma) = ln(f(y_T, y_{T-1}, ..., y_0|I_T,\gamma) = \sum_{t=1}^{T} ln(f(y_t|I_{t-1},\gamma)), \tag{5.44}$$

where $f(y_t|I_{t-1},\gamma)$ can be computed using formulas (5.38) to (5.43).

Once the filtered probability $P(\tilde{S}_t = j|I_t)$ is obtained for all $t = 1, ..., T$, it is possible to compute the smoothed probability $P(\tilde{S}_t = j|I_T)$ (we refer the reader to Hamilton (1989) for details). The filtered and smoothed probabilities for each chain can be obtained by integrating out the other chains in $\tilde{S}_t$, i.e.:

$$P(S^i_t = k|I_t) = \Sigma^2_{k=1}\Sigma^2_{j=1} P(S^i_t = i, S^{3-i}_t = k, r_t = j|I_t), \tag{5.45}$$

$$P(S^i_t = k|I_T) = \Sigma^2_{k=1}\Sigma^2_{j=1} P(S^i_t = i, S^{3-i}_t = k, r_t = j|I_T), \tag{5.46}$$

$$P(r_t = j|I_t) = \Sigma^2_{i=1}\Sigma^2_{k=1} P(S^i_t = i, S^{3-i}_t = k, r_t = j|I_t), \tag{5.47}$$

$$P(r_t = j|I_T) = \Sigma^2_{i=1}\Sigma^2_{k=1} P(S^i_t = i, S^{3-i}_t = k, r_t = j|I_T), \tag{5.48}$$

where $i \in \{1, 2\}$. Since the maximum likelihood is obtained with numerical algorithms, this estimation method can be applied only when the number of parameters is not too big. When more interacting chains with more states are involved, or when more interaction regimes are allowed for, the optimization algorithms may have difficulties to converge. In this case, the Forward-Backward algorithm and variational EM suggested by Pan et al. (2012) can be used. Pan et al. (2012) have successfully applied this approach to model the interaction between 50 states with 6 latent states each and 3 regimes of influence in order to evaluate flu epidemics.

The extended version of the model (see Section 5.2.5) can also be estimated with Maximum Likelihood after corresponding modifications in the transition probability matrix of $\tilde{S}_t$. Once any (or all) of the matrices Q, D^1, D^2, C^{12} and C^{21} becomes time-dependent, the matrix Π becomes dynamic as well, i.e. $\Pi_t = P(\tilde{S}_t|\tilde{S}_{t-1}, z_{t-1})$. Note that, since z_t is observable, the general form of the Hamilton filter (5.38)-(5.43) does not change and it can still be applied for the calculation of the likelihood function.

5.3.2 Forecasting

The in-sample analysis tools, such as filtered and smoothed probabilities discussed above, give a posteriori insight into the dating of both financial and business cycles and the types and timing of different interaction regimes. The out-of-sample analysis is a valuable complement, providing a probabilistic draft of future periods.

H-step ahead forecast of ergodic probability of the future state. Since the chain $\tilde{S}_t$ is the Markov chain of order one, it is straightforward that

$$P(\tilde{S}_{t+h}|\tilde{S}_t) = \Pi^h. \tag{5.49}$$

Then, the h-step ahead forecast for each individual chain can be computed by integrating the other two chains entering $\tilde{S}_t$ out. For example, the h-step ahead forecast for S^1_{t+h} is:

$$P(S^1_{t+h} = k|\tilde{S}_t) = \Sigma^2_{i=1}\Sigma^2_{j=1}P(S^1_{t+h} = k, S_{t+h} = i, r_{t+h} = j|\tilde{S}_t) = \Pi^h v, \tag{5.50}$$

where $i, j, k \in \{1, 2\}$, the vector v selects the columns of Π^h to be summed. For example, for $P(S^1_{t+h} = 1|\tilde{S}_t)$ the vector v is $v = (1\,0\,1\,0\,1\,0\,1\,0)'$.

H-step ahead forecast of the future state. It is also possible to compute an h-step ahead forecast of the state variable $\tilde{S}_t$

$$P(\tilde{S}_{t+h}|I_t) = \Sigma^8_{i=1}P(\tilde{S}_{t+h}|\tilde{S}_t = i)P(\tilde{S}_t = i|I_t) = P(\tilde{S}_t|I_t)'\Pi^h, \tag{5.51}$$

where $P(\tilde{S}_t|I_t)$ is the vector of filtered probabilities of being in state $\tilde{S}_t = i$, $i = \{1, ..., 8\}$. As in the previous case, the h-step ahead forecast for each chain separately can be calculated by integrating the other chains out. For example, for S^1_{t+h} we obtain:

$$P(S^1_{t+h} = k|I_t) = \Sigma^2_{i=1}\Sigma^2_{j=1}P(S^1_{t+h} = k, S^2_{t+h} = i, r_{t+h} = j|I_t) = P(\tilde{S}_t|I_t)'\Pi^h v, \tag{5.52}$$

where, as before, the vector v selects the columns to sum over. For example, for $P(S^1_{t+h} = 2|\tilde{S}_t)$ the vector v is $v = (0\,1\,0\,1\,0\,1\,0\,1)'$.

H-step ahead forecast of factors. Given equations (5.1)-(5.5), the h-step ahead forecasts of the factors are obtained recursively as for regular $AR(p)$ forecasts. For example, if

$$RF_{t+h} = \mu(S^1_{t+h}) + \varphi(L)RF_{t+h-1} + \sigma(S^1_{t+h})\varepsilon_{t+h}, \tag{5.53}$$

then

$$\hat{RF}_{t+h} = E(RF_{t+h}|I_t) = E(\mu(S^1_{t+h})|I_t) + \varphi(L)E(RF_{t+h}|I_t), \tag{5.54}$$

where the $E(\mu(S^1_{t+h})|I_t)$ is a known function of $P(S^1_{t+h}|I_t)$ defined in (5.52), and $\varphi(L)E(RF_{t+h}|I_t)$ can be calculated using the forecasts obtained in the previous iterations, i.e. for $h-1$, $h-2$, etc. The h-step ahead forecast of the financial factor $\hat{FF}_{t+h|t}$ can be obtained in a similar way.

It is important to notice, however, that the DI-MS-FM is designed for the identification of the latent interaction regime and performs poorly in the forecasts of factors. For this reason in the following sections we focus solely on the in-sample and out-of-sample performance for the forecasts of states.

5.3.3 In-sample and out-of-sample performance

In this section we evaluate and compare the quality of in-sample and out-of-sample forecasts. We also verify whether the dynamical influence feature, which is obviously a complication to a regular two-factor Markov-switching Dynamic Factor model, actually helps to obtain better forecasts, both in-sample and out-of-sample.

In order to evaluate the performance of the model in terms of identification of the current state of each of the chains, it is difficult to use empirical data since we have no reference dating for the financial cycle and the interaction regimes. For this reason, we run a Monte Carlo experiment on the simulated data. We use the data generating process described in equations (5.1)-(5.5) with the parameters set to their estimated values that we obtained using data described in the following section (see Table **5.3**).[9] For simplicity we assume that there is no external intervention into the system, so z_t is omitted. The generated sample has $T = 500$ observations and is simulated 1000 times.

[9]The simulations show that model is very sensitive to the difference between the interaction regimes. For this reason, to generate our data, we use the estimates with a large difference between $\hat{R}_1$ and $\hat{R}_2$. However, according to our observations, in case the regimes are close, this does not deteriorate the accuracy of the identification of the states of the financial and business cycles.

For the analysis of the accuracy of identification of states, we use the following indicators (we use a generic notation X_t for any of the chains S_t^1, S_t^2 or r_t and X_t^* for the corresponding sequence of true states; T is the total number of observations, T_1 is the out-of-sample period, indices *is* and *oos* correspond to in-sample and out-of-sample cases, respectively):

1. QPS, the quadratic probability score. This indicator is conceptually similar to the mean squared error and is calculated in the following way:

$$QPS_{is}(X) = \frac{\Sigma_{t=1}^{T}(P(X_t = 2|I_T, \hat{\gamma}) - (X_t^* - 1))^2}{T}, \tag{5.55}$$

$$QPS_{oos}(X) = \frac{\Sigma_{t=0}^{T_1-1}(P(X_{T+t+1} = 2|I_{T+t}, \hat{\gamma}) - (X_{T+t+1}^* - 1))^2}{T_1}, \tag{5.56}$$

where $P(X_t = 2|I_T, \hat{\gamma})$ is the smoothed probability of state 2 given in equation (5.46), $P(X_{T+t+1} = 2|I_{T+t}, \hat{\gamma})$ is the one-step-ahead forecast of probability of state 2 given in equation (5.52).

2. FPS, the false positive score. This indicator gives the proportion of misidentified states and is calculated as

$$FPS_{is}(X) = \frac{\Sigma_{t=1}^{T}(I_{P(X_{T+t+1}=2|I_{T+t},\hat{\gamma})>\alpha} - (X_t^* - 1))^2}{T}, \tag{5.57}$$

$$FPS_{oos}(X) = \frac{\Sigma_{t=0}^{T_1-1}(I_{P(X_{T+t+1}=2|I_{T+t},\hat{\gamma})>\alpha} - (X_{T+t+1}^* - 1))^2}{T_1}, \tag{5.58}$$

where $I_{\hat{P}>\alpha}$ is the indicator function taking value one when $\hat{P}$ is higher than a threshold α, conventionally set at 0.5.

3. $AUROC$, the area under the Receiver Operating Characteristic (ROC) curve. ROC curve gives the information on the accuracy of identification of each state as the threshold varies. In other words, it provides pairs of ratios - a fraction of correctly identified recession (financial downturn) periods and a fraction of missed expansion (financial boom) periods - for each arbitrary chosen level of α.[10]

[10] According to its definition, ROC curve is a graphical plot which juxtaposes the false positive rate (FPR, on horizontal axis) and the true positive rate (TPR, on vertical axis) as the threshold of the classifier (in this case, the cut-off smoothed probability for a state to be identified as recession state) varies. TPR and FPR are defined as

A better identification performance would imply a higher ratio of correct guesses and a lower percentage of mistakes for a given α. Then, the Area Under the ROC curve calculated as an integral over α measures discrimination, i.e. the general ability of the model to distinguish the states of a process (independently of α). $AUROC$ takes the value in $[0; 1]$, $AUROC = 1$ meaning that the state identification performance is perfect.

4. J, Youden's J statistic. J shows the identification performance at each given level of α and is calculated as a sum of fractions of correct guesses for each of the states, i.e.:[11]

$$J = \frac{TP}{TP + FN} + \frac{TN}{TN + FP} - 1.$$

J takes values in $[-1; 1]$, with $J = 1$ meaning that the states are identified correctly in all periods, so the state identification performance is perfect. $J = 0$ means that the discrimination ability of the model is the same as of a regular coin, so the model is useless.

For comparability, we set the threshold level α equal to 0.5 for all chains.

QPS can be considered as a mean squared error computed for the forecasts (nowcasts) of states, and is informative only in comparison of several models. The other three measures can be used independently, as the absolute values of FPS, $AUROC$ and J-statistic are informative b themselves.

5.3.3.1 In-sample performance

We present below the indicators of the in-sample performance of the DI-MS-FM. These results are opposed to the ones estimated with the help of a one-regime Markov-Switching VAR (see Billio and Sanzo (2015) for more details) with factors used as observable variables, i.e. discarding r_t from the framework, so that the interaction between S_t^1 and S_t^2 is described by an unrestricted transition probability matrix with four states (see Table 5.1). In this way, we intend to measure the potential losses of quality due to time-invariance of the interaction between the cycles.

$TPR = \frac{TP}{TP+FN}$, $FPR = \frac{FP}{TN+FP}$,

where where TP is the number of true positives (correctly identified recessions or financial downturns), FN is the number of false negatives (incorrectly identified expansions or financial booms), TN is the number of true negatives (correctly identified expansions or financial booms) and FP is the number of false positives (incorrectly identified recessions or financial downturns).

[11] See the previous footnote.

We observe that the DI-MS-FM performs very well in the identification of the individual cycles - the error rate measured with QPS_{is} and FPS_{is} is low, whereas the classification quality measured with $AUROC_{is}$ and J_{is} is high. The interaction regime is more difficult to identify (both QPS_{is} and FPS_{is} are higher, whereas $AUROC_{is}$ and J_{is} are lower), however values of QPS_{is} and FPS_{is} do not exceed the ones usually obtained in the empirical papers for the business cycle.

When comparing the performance of DI-MS-FM to one-regime MS-VAR-DFM (see Table 5.1), one may notice that, all four measures of quality pointing in the same direction, neglecting the dynamics of the interaction deteriorates the accuracy of the identification of states of the individual cycles.

TABLE 5.1: In-sample performance: smoothed probabilities of the second state

	DI-MS-FM			
	QPS_{is}	FPS_{is}	$AUROC_{is}$	J_{is}
S^1- business cycle	0.0182	0.0238	0.9754	0.9222
S^2- financial cycle	0.0444	0.0567	0.9527	0.6819
r - interaction regimes	0.2034	0.2537	0.7343	0.3951

	One interaction regime MS-DFM			
	QPS_{is}	FPS_{is}	$AUROC_{is}$	J_{is}
S^1- business cycle	0.0765	0.0888	0.9340	0.8163
S^2- financial cycle	0.2454	0.2618	0.8353	0.6259
r - interaction regimes	-	-	-	-

Note: The table describes the ability of the models to identify state two of each of the chains: "recession" for S_t^1, "high volatility" for S_t^2 and "Interdependent chains" for r_t.

5.3.3.2 Out-of-sample performance

One-step ahead forecasts of states. For the out-of-sample analysis on simulated data, the sample is split into in-sample period with $T_1 = 1, ..., T - 60$ observations and out-of-sample period with $T_2 = 60$ observations, so that the number of observations in the in-sample period corresponds to the one we used in the real sample (395 observations)). The out-of-sample forecasts $P(\tilde{S}_{T_1+t+1}|I_{T_1+t})$, for $t = 1, ..., T - T_1 - 1$ are then constructed using the equations (5.51). As in case of in-sample analysis, we fit both DI-MS-FM and MS-VAR-DFM ('One interaction regime MS-DFM') to the generated data.

The results of the simulation are given in Table 5.2.

As expected, the out-of-sample behavior is inferior compared to in-sample performance. However, the quality is still satisfactory, the values of QPS_{oos} and FPS_{oos} for the business cycle corresponding to the ones usually obtained in the empirical exercises (see, for example, Matas-Mir et al. (2008)). Similarly to the in-sample performance, the introduction of switches in the interaction regime improves the quality of the out-of-sample identification of the individual cycles.

TABLE 5.2: Out-of-sample performance: one-step ahead forecast of the future state

	DI-MS-FM			
	QPS_{oos}	FPS_{oos}	$AUROC_{oos}$	J_{oos}
S^1- business cycle	0.0615	0.0712	0.8952	0.6574
S^2- financial cycle	0.1309	0.1279	0.6683	0.2435
r - interaction regimes	0.2857	0.3639	0.6195	0.1628

	One interaction regime MS-DFM			
	QPS_{oos}	FPS_{oos}	$AUROC_{oos}$	J_{oos}
S^1- business cycle	0.1128	0.1583	0.8721	0.6576
S^2- financial cycle	0.3412	0.4076	0.8004	0.4367
r - interaction regimes	-	-	-	-

5.4 Interaction between financial and business cycles in the US

In this empirical exercise we apply the DI-MS-FM to the US data in order to identify the existing interaction regimes between the financial cycle and the business cycle and to determine when each of them was activated. We leave the analysis of the impact of particular government policies on this interaction for further research.

We set $\varphi_{p_1} = \psi_{p_2} = 0$, $\forall p_1, p_2$.[12] We also impose several technical constraints in order to increase the convergence to the correct local maximum. More specifically, we set $diag(Q) > 0.5e$ (where e is a vector of ones) to avoid the situations when the influence

[12]The number of lags has been chosen according to the information criteria. Inclusion of lags of the dependent variables in equations (5.1) and (5.2) does not change the estimates significantly.

regimes are not persistent. The initial values of β_0, β_1 and σ_0^2, σ_1^2 are set to the mean and the variance for the business cycle observations above and below 0 (the initial values of μ_0, μ_1 and θ_0^2, θ_1^2 are set similarly for the financial factor). The initial values of the matrices R^1 and R^2 are set to their potential values, for example, $diag(R^1) = [0.9, 0.9]'$, $diag(R^2) = [0.9, 0]'$.[13]

5.4.1 Data description

We perform our analysis for the business and the financial cycle of the United States. To construct the business cycle indicator we use the Stock-Watson database of indicators from CITIBASE and available in the databank of the Federal Reserve Bank of Saint Louis.[14] The first principal component explains just 18% of the total variance, however it is highly correlated with the GDP growth, contrary to the other components. In practice, the first component is usually enough to describe the business cycle, the other inclusion of the other components giving only marginal improvement (see Doz and Petronevich (2015), for example). The full list of variables and the corresponding factor loadings can be found in Table C.2 and Figure C.1.1 in the Appendix.

We approximate the financial cycle with the first principal component extracted from the database containing 31 indicators of different segments of the financial sector most used in the empirical papers on financial cycles. In particular, we extended the list of indicators used by Guidolin et al. (2013) with the information on deposits, monetary aggregates, loans, reserve balances and other. The complete list is given in Table C.1 in the Appendix, while the factor loadings can be found in Figure C.1.2.[15]

All data are seasonally adjusted, stationarized (by taking first differences of logarithms) and standardized. The time-span covers the period 1976m06-2014m12. The dynamics of the factors and the correlation between them is presented in Appendix C.2.

[13]Setting the initial values of these parameters to random leads to instability in the results. To solve this problem, we try different plausible values: 1) $diag(R^1) = [0, 0]'$, $diag(R^2) = [0.9, 0.9]'$, 2) $diag(R^1) = [0, 0.9]'$, $diag(R^2) = [0.9, 0.9]'$, 3) $diag(R^1) = [0.9, 0]'$, $diag(R^2) = [0.9, 0.9]'$, 4) $diag(R^1) = [0.9, 0.9]'$, $diag(R^2) = [0.9, 0.9]'$, 5) $diag(R^1) = [0, 0]'$, $diag(R^2) = [0, 0]'$, 6) $diag(R^1) = [0.5, 0]'$, $diag(R^2) = [0.5, 0.5]'$, 7) $diag(R^1) = [0, 0.5]'$, $diag(R^2) = [0.5, 0.5]'$. The output obtained with different these sets of initial values are equivalent qualitatively and very similar quantitatively.

[14]see Stock and Watson (2005)

[15]Other datasets were also tested, see section 4.4.

5.4.2 Characteristics of cycles and identified interaction regimes

The estimation results are given in Table 5.3. According to the estimates, switches in the regime of the business cycle happen mostly in mean, whereas the variance stays relatively stable. On the contrary, the financial factor switches primarily in variance. We also find that expansions in both cycles, as well as recessions of the business cycle, are very persistent ($\hat{D}^1_{11}$, $\hat{D}^2_{11}$, $\hat{D}^1_{22}$, are close to one). Recessions in the financial cycle are less persistent ($\hat{D}^2_{22}$ is below 0.9). These estimates match the findings in the previous literature.

Now consider the parameters characterizing the influence. The business cycle is capable of transmitting both expansion and recession to the financial cycle (the coefficients $\hat{C}^{12}_{11}$ and $\hat{C}^{12}_{22}$ are above 0.9). The transmitting ability is reciprocal, although the financial cycle less likely to transmit expansion to the business cycle ($\hat{C}^{21}_{11}$ is only 0.74). A similar asymmetry of influence between the business cycle and the financial cycle (measured as industrial production growth rate and excess returns, correspondingly) was also detected by Billio and Sanzo (2015).

Table 5.3: Estimation results

	Business cycle			Financial cycle	
	$\hat{\gamma}$	$\hat{\sigma}_{\hat{\gamma}}$		$\hat{\gamma}$	$\hat{\sigma}_{\hat{\gamma}}$
$\hat{\mu}_1$	0.5718	(0.0374)	$\hat{\beta}_1$	0.1584	(0.0343)
$\hat{\mu}_2$	-0.8512	(0.0712)	$\hat{\beta}_2$	-0.8587	(0.2806)
$\hat{\sigma}^2_1$	0.3103	(0.0276)	$\hat{\theta}^2_1$	0.2742	(0.0311)
$\hat{\sigma}^2_2$	0.8107	(0.0851)	$\hat{\theta}^2_2$	4.0480	(0.9117)
$\hat{D}^1_{11}$	0.9897	(0.0156)	$\hat{D}^2_{11}$	0.9888	(0.0192)
$\hat{D}^1_{22}$	0.9809	(0.0121)	$\hat{D}^2_{22}$	0.8799	(0.2985)
$\hat{C}^{12}_{11}$	0.9059	(0.1311)	$\hat{C}^{21}_{11}$	0.7481	(0.0121)
$\hat{C}^{12}_{22}$	0.9899	(0.7864)	$\hat{C}^{21}_{22}$	0.9889	(0.0029)

Influence regimes			
	"Independent chains"		"Interdependent chains"
$\hat{R}^1_{11}$	0.9815	$\hat{R}^2_{11}$	0.8562
$\hat{R}^1_{22}$	0.9426	$\hat{R}^2_{22}$	0.1853
$\hat{q}_{11}$	0.9900	$\hat{q}_{22}$	0.9677

Note: The estimated specification is $RF_t = \mu_{s^1_t} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2_{S^1_t})$, $FF_t = \beta_{s^2_t} + \xi_t$, $\xi_t \sim N(0, \theta^2_{S^2_t})$.

The detected abilities of state transmission are clearly necessary for understanding of the relation between the chains. In our framework these should be considered together with the parameters responsible for the influence regimes. The model identified two distinct and very persistent influence regimes ($\hat{q}_{11}$ and $\hat{q}_{22}$ are above 0.96). The values $\hat{R}^1_{11}$, $\hat{R}^1_{22}$, $\hat{R}^2_{11}$, $\hat{R}^2_{22}$ suggest that the first and the second regimes can be interpreted as "Independent cycles" and "Interdependent cycles", correspondingly. According to information criteria, the two regimes are not redundant: in case of a single influence regime the values of the information criteria are $AIC = 2047.6$, $BIC = 2135.3$, $HQ = 2080.2$, which is above $AIC = 2285.8$, $BIC = 2368.5$, $HQ = 2318.3$ for the DI-MS-FM.

We perform a Likelihood-ratio test in order to find out the direction of causality in each of the regimes. More precisely, we test if the high values of $\hat{R}^1_{11}$, $\hat{R}^1_{22}$ can be interpreted as the absence of causality in the first regime, and if the high value of $\hat{R}^2_{11}$ and the low value of $\hat{R}^2_{22}$ actually implies that in the second regime the business cycle leads the financial cycle. We test a joint hypothesis H_0 versus the alternative H_1, where

$$H_0 : \begin{cases} \hat{R}^1_{11} = 1 \\ \hat{R}^1_{22} = 1 \\ \hat{R}^2_{11} = 1 \\ \hat{R}^2_{22} = 0 \end{cases}, \qquad H_1 : \begin{cases} \hat{R}^1_{11} \neq 1 \\ \hat{R}^1_{22} \neq 1 \\ \hat{R}^2_{11} \neq 1 \\ \hat{R}^2_{22} \neq 0 \end{cases}. \tag{5.59}$$

The value of the test statistics is $LR = 81.91$ and largely overcomes the critical value at 5% of confidence probability ($\chi^2_{0.95,4} = 9.49$), so H_0 is not rejected.

5.4.3 Identifying the periods of recession, financial downturn and high interdependence between the cycles

The estimated smoothed probabilities of recession $P(S^1_t = 2|I_T)$, financial downturn $P(S^2_t = 2|I_T)$ and second influence regime $P(r_t = 2|I_T)$ are presented in Figures 5.4-5.6. Shaded areas correspond to NBER business cycle recessions and are given to verify the validity of the obtained estimates. On Figure 5.4 one can see that the model captures all business cycle recessions well. The smoothed probability of recession spikes exactly with the beginning of the NBER recession, without either false signals or missed recessions. Whereas the double-dip crisis of 1980 and 1981-1982 is identified very accurately, the duration of the other three recessions observed in the time-span - the early 1990s recession, the dot-com bubble and the Great Recession - appears to be overestimated by the model. This imprecision might be due to the fact that the US business cycle is reported to have at least three states (recession, expansion and slow growth), one of which we have omitted in this simple specification of the model.[16]

The adequacy of the estimated smoothed probabilities of financial downturns is difficult to evaluate since there is no benchmark dating of financial cycles. To provide at least some reference, we use the dates of the beginning of banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009) to pinpoint the gravest events in the US banking sector (September 1988 and July 2007) which certainly correspond to financial crises, even though it is possible that they do not cover all financial crises but only those in the banking sector. Comparing the graphs of the smoothed probability with these reference dates on Figure 5.5, we can see that the model captures the banking crisis of 2008 with much precision, but foreruns the crisis of 1988 by about 10 months. In general, smoothed probability of financial downturn detects all the major events in the last 40 years: the savings and loans crisis and bank crisis during the double-dip recession of 1980 and 1981-1982, Black Monday of 1987, early 1990s recession, the Russian crisis of 1998, bursting of dot-com bubble in 2001, the global financial crisis of 2008.

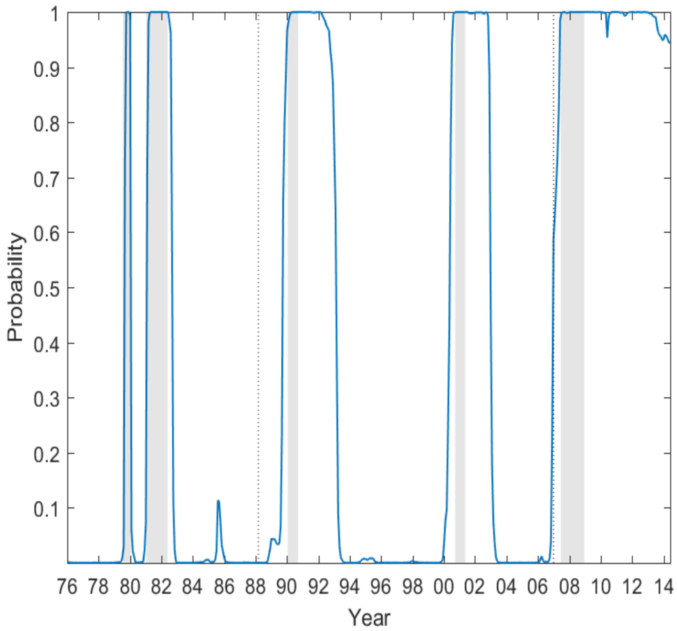
The ongoing influence regime at each point of time is clearly visible from Figure 5.6. The "Interdependent cycles" regime was active during the double-dip recession and the Great Recession. Both cases (and not during the other two observed recessions during the period under consideration) were marked with increased panic on the stock exchange, which can probably be an explanation of the higher interaction between the financial and the business cycles during these periods. This idea is consistent with the theory of sunspot equilibria: the exogeneous random Markov-Switching process r_t can be viewed as an extrinsic variable, influencing the economy through expectations but not affecting the fundamentals. In other words, if the agents' beliefs are such that the current shock (either financial or economic) is likely to be devastating, they act accordingly on the stock exchange, launching a self-reinforcing mechanism of transition of the shock from the financial sector to the real and back - the economy enters the "Interdependent cycles regime". Otherwise, if the agents are sure that the shock is temporary (as the Black Monday of 1987, for example), the interaction is just not activated ("Independent cycles" influence regime is on), and the shock does not propagate.

The estimates of the periods of high interaction seem reasonable. However, one may argue that the direction of causality between the two cycles (identified as business cycle leading the financial cycle) might not be the same in 1980-1982 and 2008. This misidentification of causality in the second case might arise from the fact that in this empirical exercise we allow for just two influence regimes. Given the relatively long period of low correlation between the two cycles in the middle of the sample, the model identified the regime of independent cycles and attributed any sort of other relation to the other

[16] Indeed, the GDP growth rate was recovering much slower during the last three recessions comparing to the preceding ones.

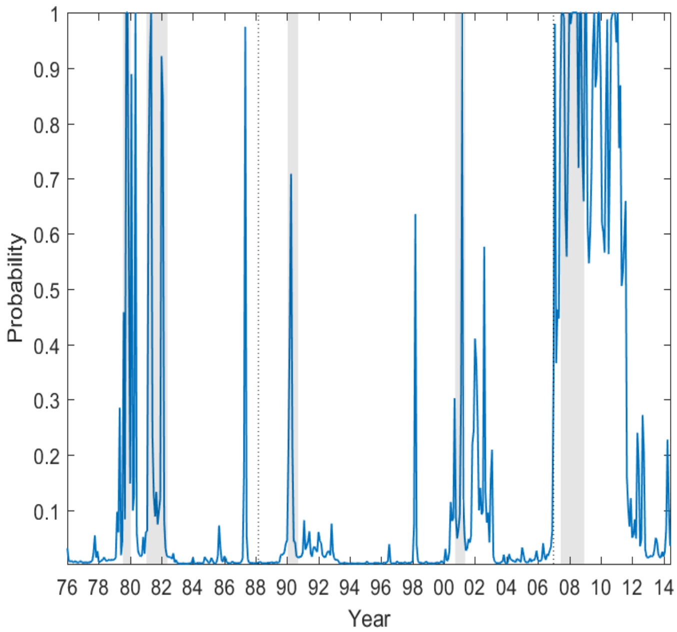
regime. Therefore, once more influence regimes are allowed for, the model might be able to distinguish different types of interdependence. A certain evidence for this hypothesis is shown in the robustness check exercise below, where the causality direction in the second regime is shown to be different in different subsamples.

Figure 5.4: Smoothed probability of recession in the business cycle



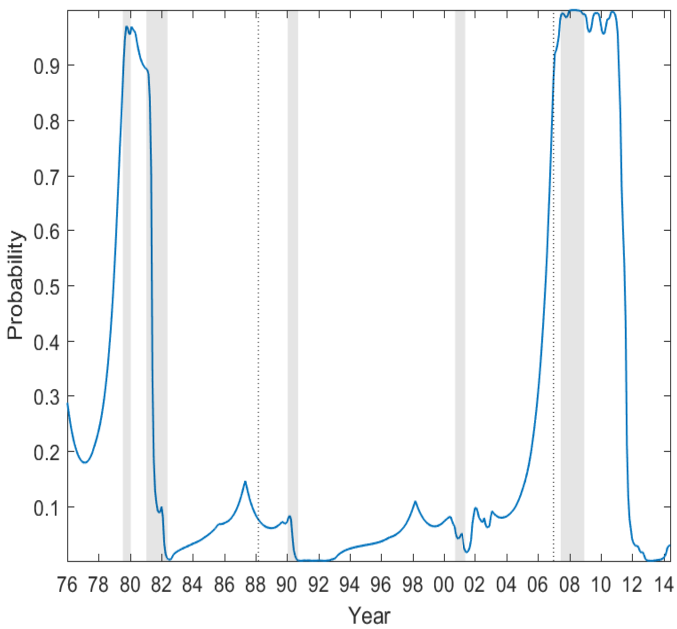
Note: Grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009).

FIGURE 5.5: Smoothed probability of financial downturn



Note: Grey shaded areas correspond to NBER recessions, while dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009).

FIGURE 5.6: Smoothed probability of the "Interdependent cycles" regime



Note: Grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2013)) and Reinhart (2009).

5.4.4 Robustness check

The dynamics of interaction suggested by the estimates of DI-MS-DFM, generally coherent with observations on the comovement of financial and business cycles in the US in the beginning and in the end of the sample, contributes to the discussion on the degree of interaction between 1982 and 2008. Currently, there is no consensus on it in the literature. For example, Gilchrist et al. (2009) find that "credit market shocks have contributed significantly to U.S. economic fluctuations during the 1990-2008 period". Gertler and Lown (1999) and Mody and Taylor (2004) suggest that yield spreads based on indexes of high yield corporate bonds perform well in forecasting of output during 1980s-1990s. Meeks (2012) find that "adverse credit shocks have contributed to declining output in every post-1982 recession". On the other hand, Stock and Watson (2003), find mixed evidence for the high-yield spread as a leading indicator of the business cycle. Rachdi and Ben Mbark (2013) find that the link between the cycles is bi-directional. Our own findings are in line with those of Rousseau and Watchel (2011), Valickova et al. (2015) who show that the link between financial sector and output growth has weakened worldwide and especially in the developed countries.

Given the ambiguity of findings on the interaction between the cycles, we check the robustness of our results by performing two auxiliary exercises: use of other indicators for the financial and business cycles and estimation of the model on subsamples. In this section we briefly present the main results of these exercises. More details can be found in Appendix C.3.1 and Appendix C.3.2.

We verify the validity of use of business and financial cycle indicators RF_t and FF_t by replacing them by two other proxies commonly used in the literature. According to Leamer (2015), the number of housing starts (New Privately Owned Housing Units Started) is a "critical part of the U.S. business cycle" and is therefore a good proxy for the business cycle[17] used by Conrad and Loch (2015), Ferrara and Vigna (2010), Luciani (2015) and others. In the same time, Claessens et al. (2012), Runstler and Vlekke (2015) and Drehmann et al. (2012) suggest that house prices, on a par with credit and equity markets, characterize the financial cycle.

To evaluate the impact of each of the indicators, for our robustness check we consider three alternative datasets: (RC1) RF_t and house price index;[18] (RC2) number of house

[17]Certainly, other series could have been used to approximate the business cycle, either univariate (such as index of industrial production, for example) or composite indexes (such as Conference Board business cycle indicators), as well as the enhanced versions of the factors (with time-varying weights, for instance). We prefer to perform the robustness check on a single (but not composite) indicator in order to eliminate a possible additional impact of the method used for the construction of the aggregate indicator. The choice has been made in favor of housing starts since the industrial production index appeared to be not informative enough to capture all the business cycle recessions.

[18]We use NAREIT Composite Index as a measure of house price.

starts[19] and FF_t; (RC3) number of house starts and house price index. The three cases are compared to the results obtained with the baseline scenario (BL).

Table C.3 and Figure C.3.1 show that the estimates obtained with four datasets are very similar. Importantly, the recessions and financial downturns identified with alternative proxies match the ones previously obtained very closely, with two exceptions. The number of house starts completely misses out the dot-com bubble crisis; so does the house price index, as it ignores the stress evoked on the equity market. These observations allow us to conclude that RF_t and FF_t approximate the business and financial cycles at least as good as single series indicators, and, moreover, provide a more comprehensive view on each of the sectors.

Even more importantly, in all cases the two identified regimes of interaction correspond to independence and interdependence, as in the baseline case. While RC1 and RC3 confirm the independence between 1982 and 2008 crises, thus bringing another evidence on the weakening of the finance-growth nexus after 1980s. Also, the use of house price for financial cycle tends to exacerbate the degree of dependence of the financial cycle on the business cycle. The result of case RC2 is more ambiguous: the dependence is weaker and present between 1982 and 2008 as well, indicating that the results on this period should be considered with caution.

Table C.4 and Figure C.3.3 demonstrate the results obtained on the right and left subsamples, i.e. omitting the first and the last 100 observations (the double dip recession and the Great recession). The results indicate the interaction regimes is robustly identified as "Independent cycles" and "Interdependent cycles". However, the type of interdependence in terms of causality appears to be dependent on the subsample: while in the beginning the financial cycle seems to lead the business cycle, which is in line with the literature, later the causality inverses the direction. This finding suggests that the hypothesis of just two regimes of interaction is somewhat restrictive. Figure C.3.3 demonstrates that the level of systemic risk during the period of Great recession is comparable only to the double-dip recession, as the other two critical periods - the early 1990s recessions and the dot-com bubble - are classified as periods of "Independent cycle" regimes when sample spans the Great recession.

5.4.5 Transition probabilities and smoothed probabilities of future states

Table (5.4) contains the estimated one-step ahead transition probabilites for the business cycle and the financial cycle ($P(S_t^i|S_{t-1}^i, S_{t-1}^k, r_{t-1})$, $i, k \in \{1, 2\}$, $i \neq k$) calculated

[19]Although the Conference Board considers this indicator as leading with respect to the cycle, the correlation with the RF_t and index of industrial production is the highest when the series are considered simultaneously, i.e. with zero lag. This observation has been also considered by Kydland et al. (2016).

using equation (5.49). These estimates are important since they provide a description of the individual characteristics of each of the cycles. So save space, we report only the probability to switch to expansion (financial boom) $P(S_t^i = 1|S_{t-1}^i, S_{t-1}^k, r_{t-1})$, the probability of recession (financial downturn) being $(P(S_t^i = 2|S_{t-1}^i, S_{t-1}^k, r_{t-1}) = 1 - P(S_t^i = 1|S_{t-1}^i, S_{t-1}^k, r_{t-1})$. Table (5.4) contains the forecasts for all possible combinations of the past values of the chains S_{t-1}^1, S_{t-1}^2, r_{t-1} known at $t-1$.

TABLE 5.4: Estimated one-step ahead probability of expansion and financial boom $(P(S_t^i = 1|S_{t-1}^i, S_{t-1}^k, r_{t-1}))$

	Probability of expansion in the business cycle $P(S_t^1 = 1\|S_{t-1}^1, S_{t-1}^2, r_{t-1})$	
	"Independent chains"	"Interdependent chains"
$S_{t-1}^1 = 1$, $S_{t-1}^2 = 1$	0.99	0.95
$S_{t-1}^1 = 1$, $S_{t-1}^2 = 2$	0.97	0.85
$S_{t-1}^1 = 2$, $S_{t-1}^2 = 1$	0.02	0.11
$S_{t-1}^1 = 2$, $S_{t-1}^2 = 2$	0.01	0.01

	Probability of financial boom $P(S_t^2 = 1\|S_{t-1}^1, S_{t-1}^2, r_{t-1})$	
	"Independent chains"	"Interdependent chains"
$S_{t-1}^2 = 1$, $S_{t-1}^1 = 1$	0.98	0.92
$S_{t-1}^2 = 1$, $S_{t-1}^1 = 2$	0.93	0.21
$S_{t-1}^2 = 2$, $S_{t-1}^1 = 1$	0.75	0.87
$S_{t-1}^2 = 2$, $S_{t-1}^1 = 2$	0.69	0.16

For the business cycle, the probability to switch to expansion depends on the previous state of the business cycle to a large extent. Both expansion and recession states are very persistent (the probability to stay in expansion for any past conditions is above 0.85; similarly, the probability to stay in recession is above 0.89). However, when the "interdependent cycles regime" is active, the impact of the financial cycle is not negligible: financial downturn decreases the probability that the business cycle switches from recession to expansion (from 0.11 to 0.01) confirming the findings of Claessens et al. (2012) who found that downturns in financial sector tend to make recessions longer. In the same manner, financial downturn reduces chances to stay in expansion in the business cycle (the probability decreases from 0.95 to 0.85).

The probability of financial boom depends both on its past and on the past influence regime. In the "Independent cycles" regime the boom state is very persistent contrary to the downturn state (with the probability to stay in the state above 0.93 and 0.25 (under any past conditions) correspondingly). In the "Interdependent cycles" regime, the past state of the business cycle plays a decisive role. When the business cycle is in expansion, the probability to stay in financial boom is high and is close to the corresponding one in the "Independent cycles" regime. However, a recession in the business cycle decreases this probability dramatically: from 0.92 to 0.21 (for the probability of staying in financial boom), and from 0.87 down to 0.16 (for the probability to switch from financial downturn to boom).

The findings above indicate that the downturns in the financial cycles are temporary by their nature, as the financial market in the developed economies is flexible enough to absorb the shocks relatively quickly. For this reason, on Figure 5.5 the episodes of financial instability are presented just as spikes in the smoothed probability during the "Independent cycles" regime. To the contrary, when the financial cycle enters into interaction with the business cycle, the downturn state becomes much more persistent.

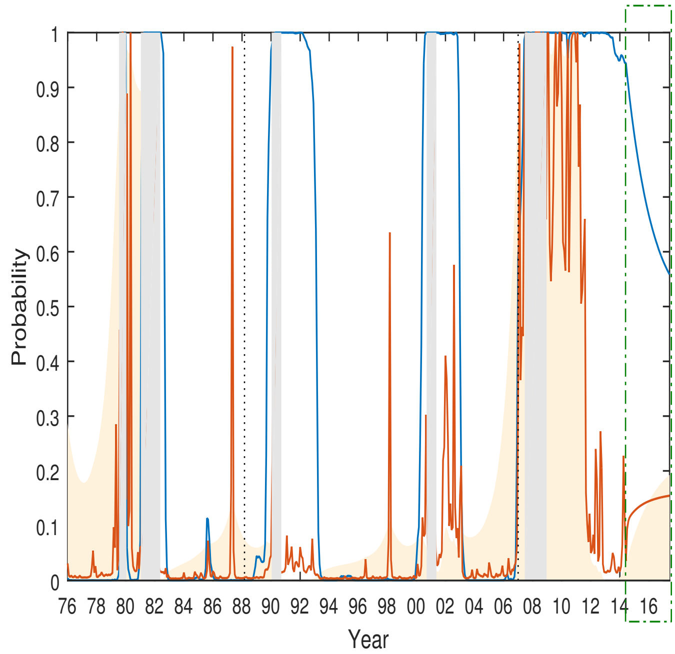
What are the projections of the model for the future? Figure 5.7 gathers the 36 months ahead forecasts of the smoothed probability of recession (blue line), financial downturn (red line) and "Interdependent cycle" regime (yellow area). The model thus predicts that by 2018 the period of low growth rates in the real sector will be over, financial sector will be stable and the "Independent cycles" regime will dominate.

What sort of implication can this have for policy-makers? Even though at this moment theoretical models do not have an unequivocal answer to the question on linkages between financial and business cycles, the impact of certain instruments of monetary, macro- and microprudential policy, and so do not provide an optimal policy rule, the knowledge of the current state of both cycles as well as the level of their interaction can be helpful for policy adjustments. For example, when the cycles are independent, the spillover effects documented by Zdzienicka et al. (2015), such as the impact of monetary policy on the stability of the financial sector, can be quite limited, which may allow to run more aggressive policies to stimulate either of the cycles. Similarly, the trade-off between financial stability and economic prosperity in the environment of the low interest rates discussed by Coimbra and Rey (2017) and Heider and Schepens (2017) can be less pronounced. On the contrary, when the cycles are interdependent, the regulator should be prepared to implement large interventions since the the recessions appear to be longer and more severe (Claessens et al. (2012)), and the financial sector needs increased support to stabilize.

Given the aggravated character of recessions during the periods of high interaction between the cycles, the set of monetary, fiscal and macroprudential measures should be directed towards the reduction of the procyclicality of the financial sector. Cerutti et al. (2015) find that macroprudential policy is an effective instument for this purpose and works better during the bust phase of the financial cycle and are more efficient in emergent economies rather than advanced ones. Blanchard et al. (2010) suggest that the monetary policy should take into account the assets price movements, too, however, by now is it not clear how to operationalize this. Another solution for mitigating credit cycles and dramatically reducing the level of government and public debt, proposed by Fischer (1936) and recently rediscovered by Kumhof and Benes (2014), is the radical idea of separation of monetary and credit functions of the banking system, also known as Chicago plan.

Whatever the relevant policy is, given the usual lag between the moment when a problem in an economy is recognized and the moment when the undertaken policy starts giving the first effects, timing is very important. In this concern, the probabilities of the influence regimes and states of individual cycles are of a great use since they provide an operative measure of the current state of the economy and future tendencies, and can be updated as soon as new information arrives. Moreover, once the causality direction is identified for each of the influence regimes, the leading cycle can serve as an early-warning indicator.

FIGURE 5.7: 36 months ahead forecast of smoothed probability



Note: Blue line and red line correspond to the smoothed probability of recession in the business cycle and the downturn state in the financial cycle, respectively. Yellow area marks the smoothed probability of being in the "Interdependent cycle" regime. Grey shaded areas correspond to NBER recessions.

5.5 Conclusion

Previous findings in the literature on business and financial cycles have shown that the cycles evolve, and so does the interaction between them. In this paper we suggest a flexible econometric framework, the Dynamical Influence Markov Switching Dynamic Factor model (DI-MS-FM), which allows to capture the changes in this interaction. Contrary to the existing models of the joint dynamics of business and financial cycles, we allow the interaction to be intrinsically dynamical, which implies that there is no need to search for an exogenous variable which could serve as as a proxy for the process governing the interaction. Based on the mix of the Dynamical influence model from computer science and the classical Markov-Switching model, the DI-MS-FM produces a wide range of statistical tools which can be very useful to design a relevant policy mix for mitigating the effects of downturns in both cycles as well as for reduction of the procyclicality of the financial cycle. More precisely, besides the individual characteristics of the cycles, the model allows to characterize the existing influence regimes in terms of leading-lagging relation between them as well as the degree of their interdependence, and to provide a probabilistic indicator of being in a particular regime of interaction at each point of time. Forecasts of the future states and future influence regimes can also be calculated.

We applied the model to the macroeconomic and financial series of the US for the period 1976m06 to 2014m12. The obtained estimates complement the findings in the previous literature. The model clearly identifies two distinct influence regimes, "Independent cycles" and "Interdependent cycles", the second being active during the double-dip recession in July 1979-November 1981 and the Great Recession in January 2007-January 2012. The periods of higher interaction are well detected, although the results may be even more telling if one allows for three influence regimes.

As any other model, the DI-MS-FM has several limitations. First, it requires the time span to be long enough in order to make sure that all the regimes of all chains are observed at least once. This implies that the more flexibility one introduces into the model (by increasing the number of chains, individual states, influence regimes), the more data is needed, which can obviously be a problem especially for the analysis of developing countries. Secondly, the simulations show that the influence regimes are well identified only when they are different enough, however, this does not deteriorate the quality of the estimates the individual behavior of each cycle. Nevertheless, this issue can be solved by a more accurate selection of initial values for the optimization process.

The model can be extended in several ways. The one mentioned in this paper concerns the introduction of policy-dependence into the parameters responsible for state transitions in order to evaluate the effect of government policy on the duration of recessions, financial downturns and regime of high interaction between business and financial cycles. The other straightforward direction is the generalization of the model for a larger number of influence regimes and states of each of the cycles. Secondly, it seems appealing to engage more chains into the dynamical interaction, for example, by letting the credit and equity part of the financial market each follow their individual chain. Another interesting application of this kind concerns the interaction of business and financial cycles of several countries (for example, the core countries of the Euro area) which would allow to asses the contribution of each country to the cross-country systemic risk, identify the clusters of interdependence, and construct an indicator of systemic risk in the region. Third, we can let the cycles to interact not only on the level of underlying latent finite-state processes, but also on the level of observations by allowing for cross-correlation in the error terms of the DGPs of the cycles and/or by introducing a VAR structure in equations (5.1) and (5.2), which might improve the forecasting ability of the model. In this case the identification issues concerning the distinction between the observation-level and chain-level interaction should be resolved, as well as the causality definition is to be reconsidered.

The Dynamic Influence Markov-Switching Dynamic Factor Model, to our knowledge, is the first instrument for objective and reproducible empirical identification of the regimes of interaction between the real and the financial sectors. Even in its basic form, it appears to produce meaningful inference on individual features of cycles as well as the dynamics of their interaction. All this information can be useful for policy-makers as it enables to adjust the fiscal, monetary and macroprudential policy according to the current influence regime.

Appendix A

Appendix to "Dating business cycle turning points for the French economy: an MS-DFM approach"

A.1 Datasets

Table A.1: Series used for the two-step estimation

Series full name	Source	SA	Lag
Industrial production by industry			
General			
France, OECD MEI, Production Of Total Industry, SA, Change P/P	Macrobond	SA	2
France, OECD MEI, Production Of Total Industry, SA, Index	Macrobond	SA	2
France, OECD MEI, Production Of Total Manufactured Intermediate Goods, SA, Index	Macrobond	SA	2
France, OECD MEI, Production In Total Manufacturing, SA, Index	Macrobond	SA	2
France, OECD MEI, Production Of Total Manufactured Investment Goods, SA, Index	Macrobond	SA	2
France, Industrial Production, Total Industry Excluding Construction, Calendar Adjusted, SA, Index	Macrobond	SA	1
France, Capacity Utilization, Total Industry, SA	Macrobond	SA	0
Mining			
France, Eurostat, Industry Production Index, Extraction of Crude Petroleum & Natural Gas, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, Other Mining & Quarrying, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, Mining & Quarrying, Calendar Adjusted, Change Y/Y	Macrobond		1
Nondurables			
France, Eurostat, Industry Production Index, Manufacture of Food Products, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, Manufacture of Beverages, Calendar Adjusted, Change Y/Y	Macrobond		1

France, Eurostat, Industry Production Index, Manufacture of Tobacco Products, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Textiles, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Wearing Apparel, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Leather & Related Products, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Paper & Paper Products, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Printing & Service Activities Related to Printing, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Coke & Refined Petroleum Products, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Chemicals & Chemical Products, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Rubber Products, Calendar Adjusted, Change Y/Y	Macrobond	1
Durables		
France, Eurostat, Industry Production Index, Manufacture of Computer, Electronic & Optical Products, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Electric Motors, Generators, Transformers & Electricity Distribution & Control Apparatus, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Electrical Equipment, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Machinery & Equipment N.E.C., Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Motor Vehicles, Trailers, Semi-Trailers & of Other Transport Equipment, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Building of Ships & Boats, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacture of Furniture, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Industry Production Index, Manufacturing, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Construction, Building & Civil Engineering, Construction & Production Index, Buildings, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Construction, Building & Civil Engineering, Construction & Production Index, Civil Engineering Works, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Eurostat, Construction, Building & Civil Engineering, Construction & Production Index, Construction, Calendar Adjusted, Change Y/Y	Macrobond	1
France, Metropolitan, Construction by Status, Number, Permits, Residential Buildings, Total	Macrobond	1
France, Metropolitan, Construction by Status, Number, Housing Starts, Residential Buildings, Total	Macrobond	1
France, Construction by Status, Number, Permits, Residential Buildings, Total	Macrobond	1
France, Construction by Status, Number, Housing Starts, Residential Buildings, Total	Macrobond	1
Utilities		
France, Eurostat, Industry Production Index, Electricity, Gas, Steam & Air Conditioning Supply, Total, Calendar Adjusted, Change Y/Y	Macrobond	1
Industrial production by market		
Durables		

France, Eurostat, Industry Production Index, MIG - Capital Goods, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Consumer Goods (Except Food, Beverages & Tobacco), Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Durable Consumer Goods, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Intermediate & Capital Goods, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Intermediate Goods, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Consumer Goods, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Expenditure Approach, Household Consumption Expenditure, Automobiles, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Housing Equipment, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Durable Personal Equipment, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
Nondurables			
France, Eurostat, Industry Production Index, MIG - Non-Durable Consumer Goods, Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Energy (Except D & E), Calendar Adjusted, Change Y/Y	Macrobond		1
France, Eurostat, Industry Production Index, MIG - Energy (Except Section E), Calendar Adjusted, Change Y/Y	Macrobond		1
France, Energy Production, Transmission & Distribution, Electric Power Generation, Transmission & Distribution, Calendar Adjusted, SA, Index	Macrobond	SA	1
France, Eurostat, Industry Production Index, Manufacture of Products of Wood, Cork, Straw & Plaiting Materials, Calendar Adjusted, Index	Macrobond		1
France, Eurostat, Industry Production Index, Manufacture of Basic Metals & Fabricated Metal Products, Except Machinery & Equipment, Index	Macrobond		1
France, Expenditure Approach, Household Consumption Expenditure, Textiles & Leather, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Other Manufactured Goods, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Energy, Water & Waste Treatment, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Petroleum Products, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Food, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
France, Expenditure Approach, Household Consumption Expenditure, Goods, Calendar Adjusted, Constant Prices, SA, EUR	Macrobond	SA	1
Equipment			
France, Manufacturing, Computers & Peripheral Equipment, Calendar Adjusted, SA, Index	Macrobond	SA	1
France, Manufacturing, Optical Instruments & Photographic Equipment, Calendar Adjusted, SA, Index	Macrobond	SA	1
France, Manufacturing, Electric Lighting Equipment, Calendar Adjusted, SA, Index	Macrobond	SA	1
France, Manufacturing, Other Electrical Equipment, Calendar Adjusted, SA, Index	Macrobond	SA	1

France, Manufacturing, Repair of Fabricated Metal Products, Machinery & Equipment, Calendar Adjusted, SA, Index	Macrobond	SA	1
France, Manufacturing, Electrical Equipment, Calendar Adjusted, SA, Index	Macrobond	SA	1
Materials			
France, Manufacturing, Clay Building Materials, Calendar Adjusted, SA, Index	Macrobond	SA	1
Employment by skill and gender			
France, Metropolitan, Unemployment, Job Seekers, Men, Total, Categories A, B & C, Calendar Adjusted, SA	Macrobond	SA	1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Under 25 Years, Categories A, B & C, Calendar Adjusted, SA	Macrobond	SA	1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Aged 25-49 Years, Categories A, B & C, Calendar Adjusted, SA	Macrobond	SA	1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Aged 50 & More, Categories A, B & C, Calendar Adjusted, SA	Macrobond	SA	1
France, Unemployment, Job Seekers, Women & Men, Total, Categories A, B & C, Calendar Adjusted, SA	Macrobond	SA	1
France, Metropolitan, Unemployment, Job Seekers, Men, Under 25 Years, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Aged 25-49 Years, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Aged 50 & More, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Total, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Under 25 Years, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Aged 25-49 Years, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Aged 50 & More, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Total, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Under 25 Years, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Aged 25-49 Years, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Aged 50 & More, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Total, Categories A, B & C	Macrobond		1
France, Unemployment, Job Seekers, Women & Men, Total, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Labourers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Labourers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Labourers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Professional Workers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Professional Workers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Professional Workers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Skilled Manual Workers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Skilled Manual Workers, Categories A, B & C	Macrobond		1

France, Metropolitan, Unemployment, Job Seekers, Women & Men, Skilled Manual Workers, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Non-Qualified Employed Persons, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women, Non-Qualified Employed Persons, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Women & Men, Non-Qualified Employed Persons, Categories A, B & C	Macrobond		1
France, Metropolitan, Unemployment, Job Seekers, Men, Qualified Employed Persons, Categories A, B & C	Macrobond		1
Trade			
Credit			
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Households & NPISH, Loans for House Purchasing Adjusted for Sales & Securitisation, Total, Flows, EUR	Macrobond		1
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Households & NPISH, Loans for Other Purposes Adjusted for Sales & Securitisation, Total, Flows, EUR	Macrobond		1
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Households & NPISH, Loans Adjusted for Sales & Securitisation, Total, Flows, EUR	Macrobond		1
Durables			
France, OECD MEI, CLI New Car Registrations, SA	Macrobond	SA	1
France, OECD MEI, Total Car Registrations, SA	Macrobond	SA	1
France, OECD MEI, Passenger Car Registrations, SA, Index	Macrobond	SA	1
Retail			
France, OECD MEI, Total Retail Trade (Volume), SA, Index	Macrobond	SA	1
France, OECD MEI, Total Retail Trade (Value), SA, Index	Macrobond	SA	1
France, Domestic Trade, Vehicle Sales & Registrations, New, Passenger Cars, Total, Calendar Adjusted, SA	Macrobond	SA	0
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale of Automotive Fuel in Specialised Stores, Calendar Adjusted, Index	Macrobond		2
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale via Mail Order Houses or via Internet, Index	Macrobond		2
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale of Food, Beverages & Tobacco, Trend Adjusted, Index	Macrobond		1
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale of Textiles, Clothing & Leather Goods in Specialised Stores, Index	Macrobond		2
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale of Textiles, Clothing, Footware & Leather Goods in Specialised Stores, Calendar Adjusted, Index	Macrobond		2
France, Eurostat, Retail Trade & Services, Total Market, Dispensing Chemist, Retail Sale of Medical & Orthopaedic Goods, Cosmetic & Toilet Articles in Specialised Stores, Calendar Adjusted, Index	Macrobond		2
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale of Non-Food Products (Incl. Fuel), Index	Macrobond		1
France, Eurostat, Retail Trade & Services, Total Market, Retail Sale of Non-Food Products (Excl. Fuel), Index	Macrobond		1
Foreign trade			
France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR	Macrobond	SA	1
France, Foreign Trade, Export, Calendar Adjusted, SA, EUR	Macrobond	SA	1
France, Foreign Trade, Import, Calendar Adjusted, SA, EUR	Macrobond		1
France, OECD MEI, BOP Capital Account Credit, EUR	Macrobond		1

France, OECD MEI, BOP Capital Account Debit, EUR	Macrobond		1
Surveys			
Retail			
France, OECD MEI, Manufacturing Business Situation Future, SA	Macrobond	SA	0
France, OECD MEI, Manufacturing Finished Goods Stocks Level, SA	Macrobond	SA	0
France, OECD MEI, Manufacturing Production Future Tendency, SA	Macrobond	SA	0
France, OECD MEI, Manufacturing Production Tendency, SA	Macrobond	SA	0
France, OECD MEI, Manufacturing Selling Prices Future Tendency, SA	Macrobond	SA	0
France, OECD MEI, Manufacturing Industrial Confidence Indicator, SA	Macrobond	SA	0
France, OECD MEI, Manufacturing Export Order Books Level, SA	Macrobond	SA	0
Consumers			
France, Consumer Surveys, INSEE, Consumer Confidence Indicator, General Economic Situation, Past 12 Months, Balance of Replies, SA	Macrobond	SA	0
France, Consumer Surveys, INSEE, Consumer Confidence Indicator, General Economic Situation, Next 12 Months, Balance of Replies, SA	Macrobond	SA	0
France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Major Purchases Intentions, Next 12 Months, Balance of Replies, SA	Macrobond	SA	0
France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Financial Situation, Last 12 Months, Balance of Replies, SA	Macrobond	SA	0
France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Financial Situation, Next 12 Months, Balance of Replies, SA	Macrobond	SA	0
Industry			
France, Business Surveys, INSEE, Building Industry, Global, Past Activity Tendency	Macrobond		0
France, Business Surveys, INSEE, Building Industry, Global, Expected Activity	Macrobond		0
France, Business Surveys, INSEE, Building Industry, Global, Order Books Level	Macrobond		0
France, Business Surveys, INSEE, Building Industry, Global, Past Workforce Size	Macrobond		0
France, Business Surveys, Bank of France, Industry, Inventories of Final Goods, Manufacturing Industry, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Manufacturing Industry, SA	Macrobond	SA	0
France, Business Surveys, INSEE, Industry, Manufacturing, Personal Production Expectations, Balance of Replies, SA	Macrobond	SA	0
France, Business Surveys, INSEE, Industry, Manufacturing, Demand & Export Order Books, Balance of Replies, SA	Macrobond	SA	0
France, Business Surveys, INSEE, Industry, Manufacturing, General Production Expectations, Balance of Replies, SA	Macrobond	SA	0
France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA	Macrobond	SA	0
France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Manufacture of Food Products, Beverages & Tobacco Products, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Manufacture of Electrical, Computer & Electronic Equipment, Manufacture of Machinery, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Computer, Electronic & Optical Products, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Machinery & Equipment, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Transport Equipment, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Automotive Industry, SA	Macrobond	SA	0

France, Business Surveys, Bank of France, Industry, Current Order Books, Other Transport Equipment, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Other Manufacturing, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Metal & Metal Products Manufacturing, SA	Macrobond	SA	0
France, Business Surveys, Bank of France, Industry, Current Order Books, Other Manufacturing Industries (Including Repair & Installation of Machinery), SA	Macrobond	SA	0
Services			
France, Service Surveys, DG ECFIN, Services Confidence Indicator, Balance, SA	Macrobond	SA	0
France, Service Surveys, INSEE, Services, Past Trend of Employment, All Non-Temporary Services, Including Transportation, Balance of Replies, SA	Macrobond	SA	0
France, Service Surveys, INSEE, Services, Expected Trend of Activity, All Non-Temporary Services, Including Transportation, Balance of Replies, SA	Macrobond	SA	0
France, Service Surveys, INSEE, Services, Past Trend of Activity, All Non-Temporary Services, Including Transportation, Balance of Replies, SA	Macrobond	SA	0
Retail trade			
France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Business Activity (Sales) Development over the Past 3 Months, Balance, SA	Macrobond	SA	0
France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Business Activity Expectations over the Next 3 Months, Balance, SA	Macrobond	SA	0
France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Employment Expectations over the Next 3 Months, Balance, SA	Macrobond	SA	0
France, OECD MEI, Retail Trade Orders Intentions, SA	Macrobond	SA	0
Prices			
France, Consumer Price Index, Total, Index	Macrobond		0
France, Consumer Price Index, Housing, Water, Electricity, Gas & Other Fuels, Rent of Primary Residence, Index	Macrobond		0
France, Eurostat, Producer Prices Index, Domestic Market, Manufacture of Plastics Products, Change P/P	Macrobond		1
Germany, Bundesbank, Price of Gold in London, Afternoon Fixing *, 1 Ounce of Fine Gold = USD ..., USD	Macrobond		0
World, IMF IFS, International Transactions, Export Prices, Linseed Oil (Any Origin)	Macrobond		6
Commodity Indices, UNCTAD, Price Index, End of Period, USD	Macrobond		0
Financial sector			
Indexes			
NYSE Euronext Paris, cac40 (^FCHI), price index, beginning of period, EUR	Macrobond		0
United Kingdom, Equity Indices, FTSE, All-Share, Index, Price Return, End of Period, GBP	Macrobond		0
Germany, Bundesbank, Capital Market Statistics, General Survey, Key Figures from the Capital Market Statistics 2, DAX Performance Index, End 1987 = 1000, End of Month, Index	Macrobond		0
Japan, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, Nikkei 225, Balance	Macrobond		0
United States, Equity Indices, S&P, 500, Index (Shiller), Cyclically Adjusted P/E Ratio (CAPE)	Macrobond		0
Exchange rates			
France, FX Indices, BIS, Real Effective Exchange Rate Index, CPI Based, Broad	Macrobond		
France, FX Indices, BIS, Nominal Effective Exchange Rate Index, Broad	Macrobond		0
REER Euro/Chinese yuan, CPI deflated	BCE		0
REER Euro/UK pound, CPI deflated	BCE		0
REER Euro/Japanese yen, CPI deflated	BCE		0

REER Euro/US dollar, CPI deflated	BCE		0
Interest rates			
France, 3 months treasury bills, reference interest rate - monthly average	BDF		0
France, 12 months treasury bills, reference interest rate - monthly average	BDF		0
France, Government Benchmarks, Eurostat, Government Bond, 10 Year, Yield	Macrobond		0
Loans			
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Domestic Non-Financial Corporations, Loans Adjusted for Sales & Securitisation, Total, EUR	Macrobond		1
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Domestic Non-Financial Corporations, Investment Loans Adjusted for Sales & Securitisation, Total, EUR	Macrobond		1
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Domestic Non-Financial Corporations, Short-Term Loans Adjusted for Sales & Securitisation, Total, EUR	Macrobond		1
France, Deposits & Loans, Credit Institutions, Loans, By Entity, to Domestic Non-Financial Corporations, Other Loans Adjusted for Sales & Securitisation, Total, EUR	Macrobond		1
Monetary aggregates			
France, Monetary Aggregates, M1, Total, EUR	Macrobond		1
France, Monetary Aggregates, M2, Total, EUR	Macrobond		2
France, Monetary Aggregates, M3, Total, EUR	Macrobond		2
International			
Germany, Economic Sentiment Surveys, ZEW, Financial Market Report, Current Economic Situation, Balance	Macrobond		0
Germany, OECD MEI, Manufacturing Business Situation Present, SA	Macrobond	SA	1
Germany, OECD MEI, Production Of Total Industry, SA, Index	Macrobond	SA	3
United States, Employment, CPS, 16 Years & Over, SA	Macrobond	SA	1
United States, Unemployment, CPS, 16 Years & Over, Rate, SA	Macrobond	SA	1
United States, Industrial Production, Total, SA, Index	Macrobond	SA	1
United States, Domestic Trade, Retail Trade, Retail Sales, Total, Calendar Adjusted, SA, USD	Macrobond	SA	1
United States, Industrial Production, Industry Group, Manufacturing, Total (SIC), SA, Index	Macrobond	SA	1
United States, Equity Indices, S&P, 500, Index, Price Return, End of Period, USD	Macrobond		1

Table A.2: List of series used for the one-step estimation

N	Series name	Publication Lag
1	France, Capacity Utilization, Total Industry, SA	1
2	France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Synthetic Index, SA	0
3	France, Domestic Trade, Vehicle Sales & Registrations, New, Passenger Cars, Total, Calendar Adjusted, SA	0
4	France, OECD MEI, Retail Trade Orders Intentions, SA	0
5	France, OECD MEI, CPI All Items, Change Y/Y	3
6	France, OECD MEI, Production Of Total Industry, SA, Index	3
7	France, OECD MEI, Total Retail Trade (Volume), SA, Change P/P	1
8	France, Metropolitan, Unemployment, Job Seekers, Men, Total, Categories A, B & C, Calendar Adjusted, SA	1
9	France, OECD MEI, Manufacturing Finished Goods Stocks Level, SA	0
10	France, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, CAC-40, Balance	1
11	France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR	1
12	France, Foreign Trade, Export, Calendar Adjusted, SA, EUR	2
13	France, Foreign Trade, Import, Calendar Adjusted, SA, EUR	2
14	France, 3 months treasury bills, reference interest rate - monthly average	3
15	France, 12 months treasury bills, reference interest rate - monthly average	3
16	United Kingdom, Equity Indices, FTSE, All-Share, Index, Price Return, End of Period, GBP	0
17	Japan, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, Nikkei 225, Balance	0
18	United States, Equity Indices, S&P, 500, Index (Shiller), Cyclically Adjusted P/E Ratio (CAPE)	0
19	France, OECD MEI, Manufacturing Business Situation Future, SA	0
20	France, OECD MEI, Manufacturing Industrial Confidence Indicator, SA	3
21	France, Business Surveys, INSEE, Building Industry, Global, Expected Activity	0
22	France, Business Surveys, INSEE, Building Industry, Global, Order Books Level	0
23	France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA	0
24	France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA	0
25	France, Service Surveys, DG ECFIN, Services Confidence Indicator, Balance, SA	0

A.2 One-step estimation results

TABLE A.3: Frequency of 25 French economic indicators in 72 selected combinations for one-step estimation

No	Freq.	Name of series
24	22	France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA (Business survey)
1	21	France, Capacity Utilization, Total Industry, SA
12	20	France, Foreign Trade, Export, Calendar Adjusted, SA, EUR
8	19	France, Metropolitan, Unemployment, Job Seekers, Men, Total, Categories A, B & C, Calendar Adjusted, SA
23	19	France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA (Business survey)
11	18	France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR
4	16	France, OECD MEI, Retail Trade Orders Intentions, SA (Business survey)
7	15	France, OECD MEI, Total Retail Trade (Volume), SA, Change P/P
9	15	France, OECD MEI, Manufacturing Finished Goods Stocks Level, SA
17	13	Japan, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, Nikkei 225, Balance
18	12	United States, Equity Indices, S&P, 500, Index (Shiller), Cyclically Adjusted P/E Ratio (CAPE)
13	11	France, Foreign Trade, Import, Calendar Adjusted, SA, EUR
6	10	France, OECD MEI, Production Of Total Industry, SA, Index
14	10	France, 3 months treasury bills, reference interest rate - monthly average
16	10	United Kingdom, Equity Indices, FTSE, All-Share, Index, Price Return, End of Period, GBP
22	10	France, Business Surveys, INSEE, Building Industry, Global, Order Books Level
3	8	France, Domestic Trade, Vehicle Sales & Registrations, New, Passenger Cars, Total, Calendar Adjusted, SA
25	8	France, Service Surveys, DG ECFIN, Services Confidence Indicator, Balance, SA
2	7	France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Synthetic Index, SA
15	7	France, 12 months treasury bills, reference interest rate - monthly average
19	6	France, OECD MEI, Manufacturing Business Situation Future, SA
20	5	France, OECD MEI, Manufacturing Industrial Confidence Indicator, SA
21	4	France, Business Surveys, INSEE, Building Industry, Global, Expected Activity
5	2	France, OECD MEI, CPI All Items, Change Y/Y
10	0	France, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, CAC-40, Balance

The numbers in the last column stand for the length of lag of data updates publication, in months

TABLE A.4: Economic indicators describing major crises

Crisis	Composition				FPS	QPS
March 1992-October 1993	7	8	11	22	0.0000	0.0142
	9	12	18	23	0.0000	0.0331
	9	14	17	23	0.0000	0.0580
April 1995-January 1997	3	4	11	24	0.1828	0.1141
	6	8	12	25	0.1828	0.1171
	7	8	11	22	0.1828	0.1216
January 2001-June 2003	3	9	12	25	0.0000	0.0108
	2	11	16	24	0.0000	0.0460
	4	7	17	24	0.0000	0.0477
January 2008-June 2009	3	9	12	25	0.0000	0.0108
	2	11	16	24	0.0000	0.0460
	4	7	17	24	0.0000	0.0477
October 2011-January 2013	7	8	11	22	0.0000	0.0205
	3	9	12	25	0.0625	0.0828
	7	8	11	23	0.0625	0.0870

Note: Here the QPS and FPS are calculated for each recession period only. See Table A.2 for the series corresponding to the numbers in the column "Combinations".

TABLE A.5: Top 25 combinations with the lowest QPS, FPS and the highest $Corr$

Combinations	Rating	Component series				FPS	QPS	$Corr$	Factor loadings			
Combination 1	1	**4**	7	17	**24**	35	0.1287	0.7155	**0.3665**	0.1006	-0.0026	**0.4463**
-	7	**8**	12	23	24	49	0.1493	0.7554	**-0.1454**	0.0247	0.1301	0.0005
Combination 2	2	2	11	**16**	**24**	39	0.1315	0.6899	0.1775	-0.0584	**0.6137**	**0.3779**
Combination 3	3	3	4	11	**24**	45	0.1254	0.7491	0.0001	0.1807	-0.0461	**0.8876**
Combination 4	4	4	9	**19**	24	46	0.1328	0.7006	0.3058	-0.0799	**0.6722**	0.3337
Combination 5	5	**4**	7	11	**24**	46	0.1412	0.7082	**0.3139**	0.0933	-0.0768	**0.3958**
Combination 6	6	8	18	23	**24**	47	0.1184	0.6815	0.0738	-0.0452	-0.0805	**-0.3292**
Combination 7	8	7	**8**	11	23	50	0.1492	0.5607	0.0216	**-0.1829**	-0.0371	-0.0013
-	9	8	9	15	23	54	0.1674	0.5506	-0.1125	-0.0945	0.0344	0.1510
-	10	1	8	16	22	58	0.1963	0.5929	0.4595	0.0025	0.7876	0.0015
-	11	1	9	13	18	58	0.2135	0.5346	0.0983	-0.1404	0.0905	-0.0010
-	12	4	9	12	24	59	0.1656	0.6488	0.2292	-0.0746	0.1633	0.6130
-	13	1	4	13	15	62	0.2091	0.5255	0.2255	0.1715	0.2888	-0.0014
-	14	4	8	11	23	64	0.1724	0.5033	0.0880	-0.2649	-0.0377	-0.0031
-	15	4	8	19	24	64	0.1795	0.6511	0.3272	-0.0022	0.5487	0.3608
-	16	15	16	17	24	64	0.2092	0.5597	-0.0010	1.0500	-0.0027	0.3222
-	17	9	12	18	23	65	0.1993	0.5321	-0.2665	0.0362	0.0384	0.1614
-	18	18	22	23	24	67	0.2147	0.4249	0.0219	0.0130	0.0738	0.0002
-	19	7	14	24	25	68	0.2233	0.4967	0.0016	-0.0004	0.5101	0.0428
-	20	7	8	11	24	69	0.1702	0.6538	-0.1141	-0.0003	-0.0066	-0.8400
-	21	14	15	18	24	70	0.1799	0.5312	0.0636	0.0063	0.0082	0.7306
-	22	11	16	21	22	71	0.1862	0.5639	-0.0006	0.9812	0.0170	-0.0007
-	23	1	12	13	17	71	0.1863	0.5439	0.8973	0.2825	0.3013	0.0495
-	24	6	8	18	22	71	0.2058	0.4326	0.0262	-0.2072	0.0283	0.0328
	25	12	14	20	23	72	0.1912	0.5221	0.1444	0.0687	0.6750	0.1026

Note: The series with the highest loadings are in bold. Retained combinations are the combinations retained for the one-step analysis. The second-ranked combination in not included as it produces extra signals. See Table A.2 for the series corresponding to the numbers in the column "Combinations". The first 8 entries belong to the best 10% by three indicators simultaneously

FIGURE A.1: One-step estimation: filtered probability of recession in a current period (blue line) vs OECD recession dating (shaded area)

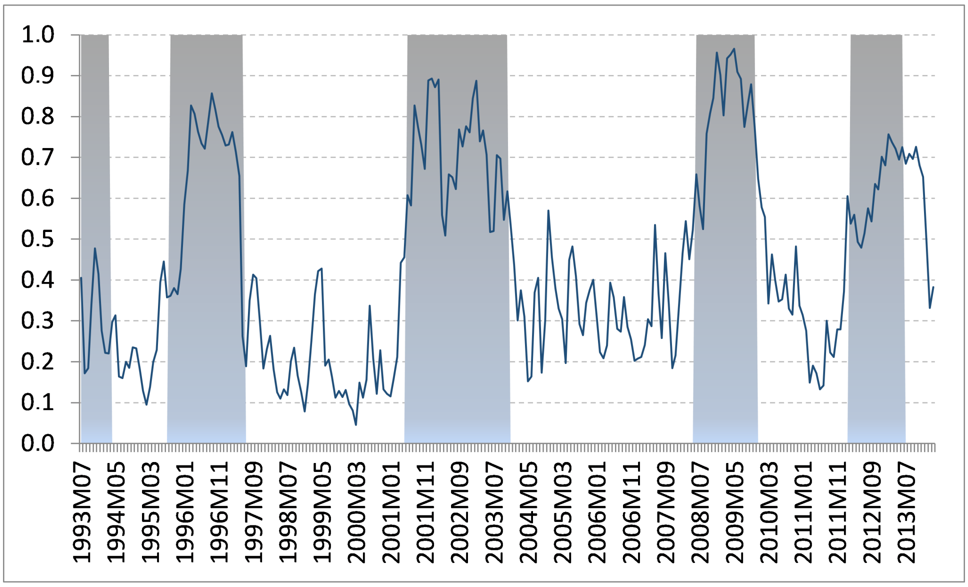
One-step combination 1

4 - France, OECD MEI, Retail Trade Orders Intentions, SA

7 - France, OECD MEI, Total Retail Trade (Volume), SA, Change P/P

17 - Japan, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, Nikkei 225, Balance

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA



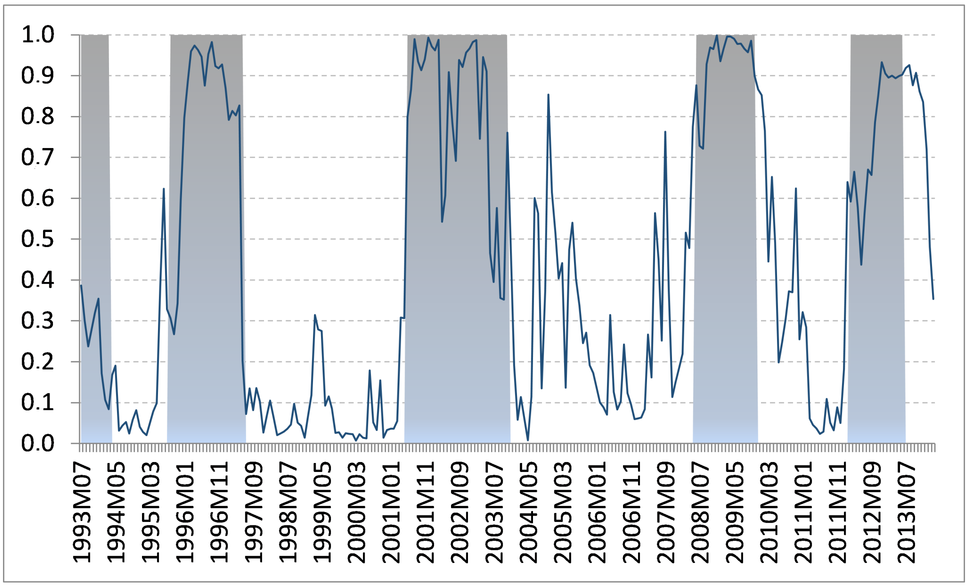
One-step combination 2

3 - France, Domestic Trade, Vehicle Sales & Registrations, New, Passenger Cars, Total, Calendar Adjusted, SA

4 - France, OECD MEI, Retail Trade Orders Intentions, SA

11 - France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA



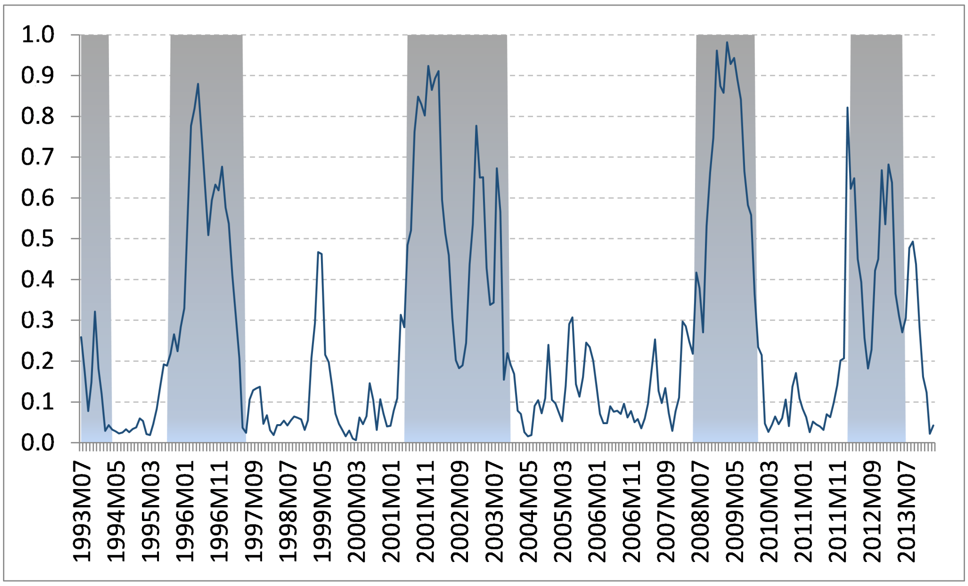
One-step combination 3

4 - France, OECD MEI, Retail Trade Orders Intentions, SA

9 - France, OECD MEI, Manufacturing Finished Goods Stocks Level, SA

19 - France, OECD MEI, Manufacturing Business Situation Future, SA

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA



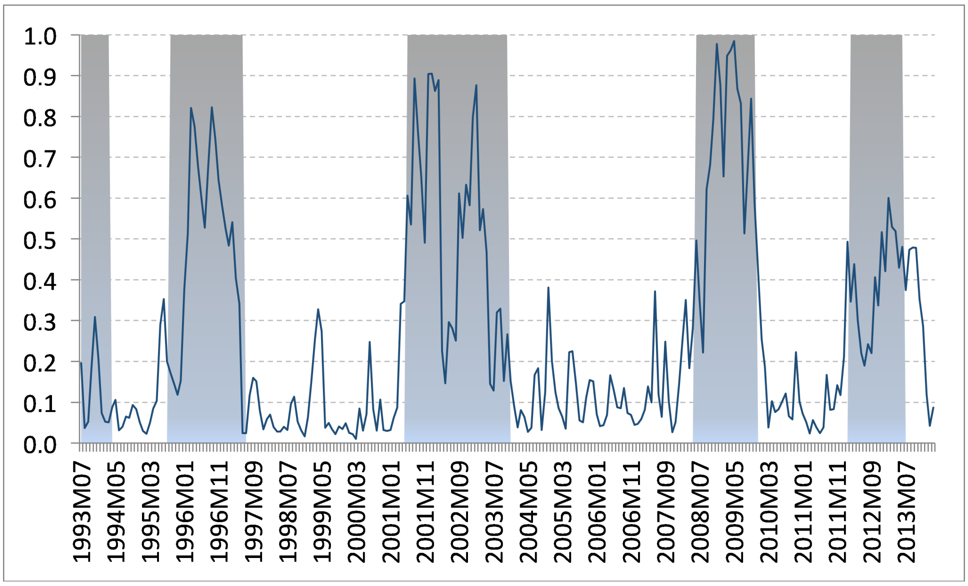
One-step combination 4

4 - France, OECD MEI, Retail Trade Orders Intentions, SA

7 - France, OECD MEI, Total Retail Trade (Volume), SA, Change P/P

11 - France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA



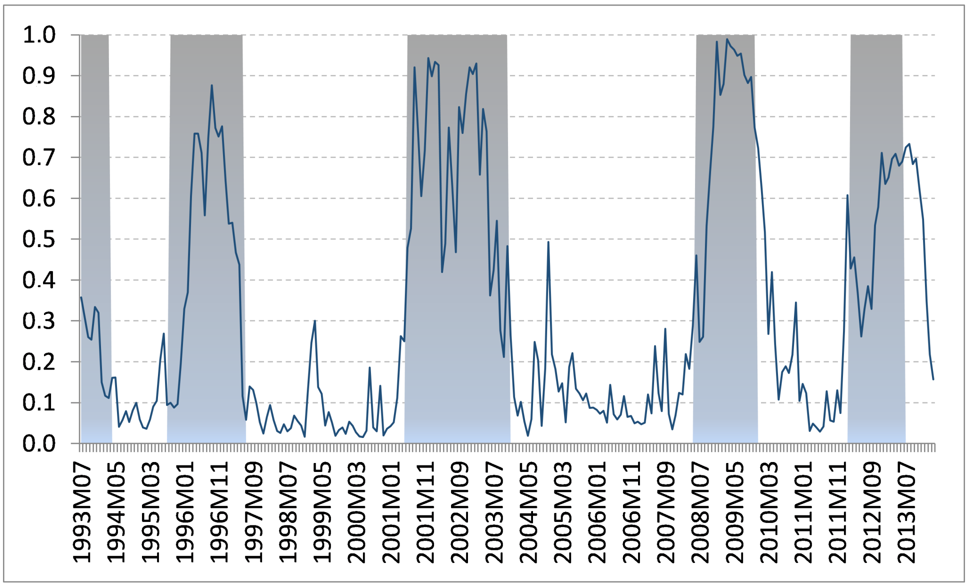
One-step combination 5

8 - Unemployed, total

18 - United States, Equity Indices, S&P, 500, Index (Shiller), Cyclically Adjusted P/E Ratio (CAPE)

23 - France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA



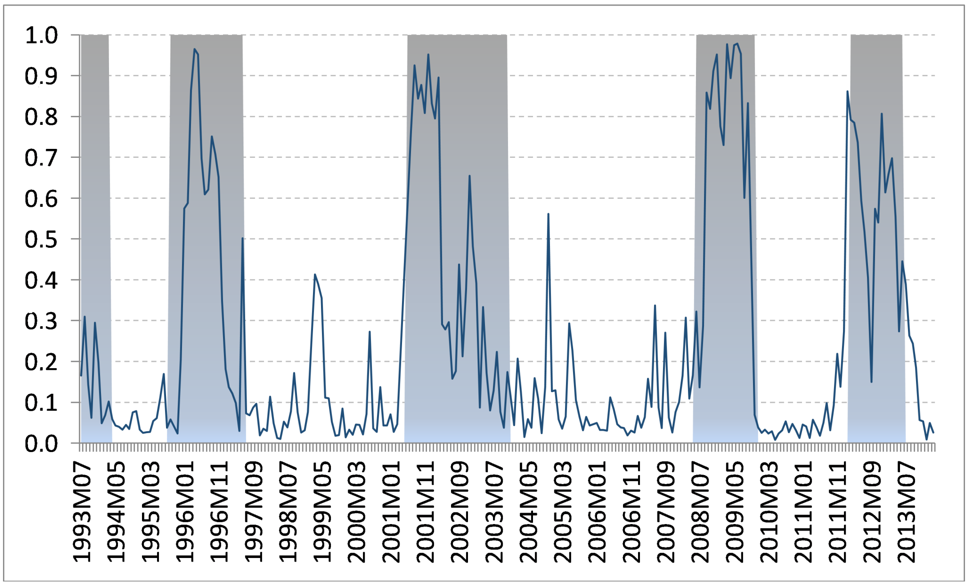
One-step combination 6

8 - Unemployed, total

12 - France, Foreign Trade, Export, Calendar Adjusted, SA, EUR

23 - France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA



One-step combination 7

7 - France, OECD MEI, Total Retail Trade (Volume), SA, Change P/P

8 - Unemployed, total

11 - France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR

23 - France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA

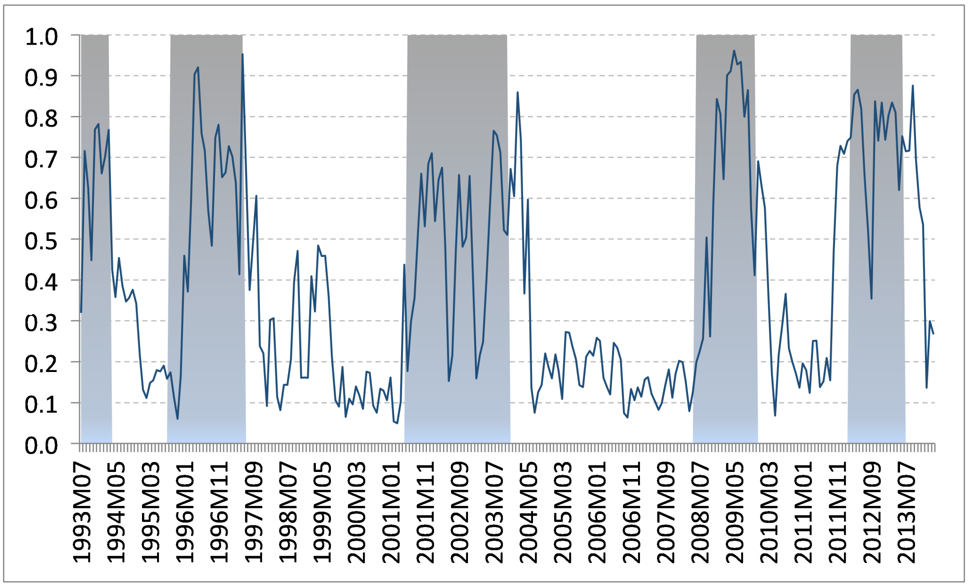
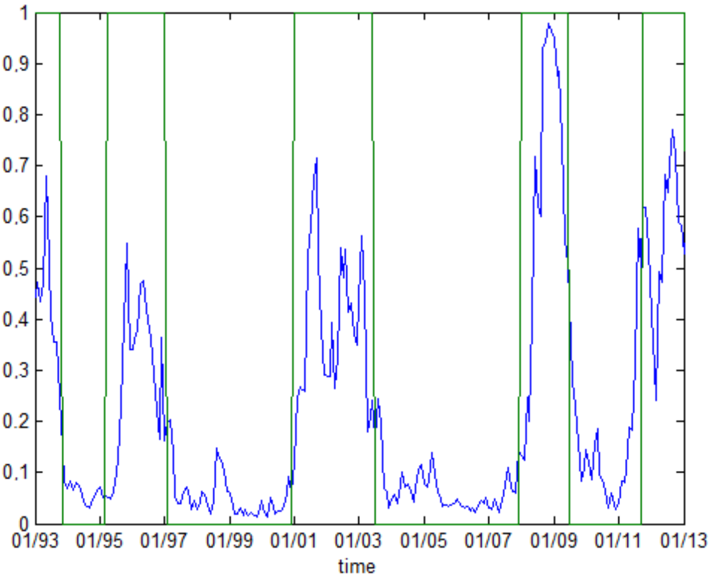


FIGURE A.2: The filtered probability of recession, estimated with one-step method on 13 series of the finally selected information set (blue line) vs OECD recession dating (shaded area



The blue line corresponds to the filtered probability of recession, the green line corresponds to the OECD turning points

Series:

4 - France, OECD MEI (Enquete de Conjoncture INSEE), Retail Trade Orders Intentions, SA;

8 - Unemployed, total

19 - France, OECD MEI (Enquete de Conjoncture INSEE), Manufacturing Business Situation Future, SA

24 - France, Business Surveys, DG ECFIN, Construction Confidence Indicator, Balance, SA

2 - France, Consumer Surveys, INSEE, Consumer Confidence Indicator, Synthetic Index, SA

3 - France, Domestic Trade, Vehicle Sales & Registrations, New, Passenger Cars, Total, Calendar Adjusted, SA

7 - France, OECD MEI, Total Retail Trade (Volume), SA, Change P/P

9 - France, OECD MEI, Manufacturing Finished Goods Stocks Level, SA

11 - France, Foreign Trade, Trade Balance, Calendar Adjusted, SA, EUR

12 - France, Foreign Trade, Export, Calendar Adjusted, SA, EUR

17 - Japan, Economic Sentiment Surveys, ZEW, Financial Market Report, Stock Market, Nikkei 225, Balance

18 - United States, Equity Indices, S&P, 500, Index (Shiller), Cyclically Adjusted P/E Ratio (CAPE)

23 - France, Business Surveys, DG ECFIN, Retail Trade Confidence Indicator, Balance, SA

A.3 Estimation results for one-step and two-step methods

Table A.6: Estimated parameters, one-step and two-step methods

	Two-step		One-step						
Parameters	(switch in μ)	(switch in μ and σ^2)	Comb 1	Comb 2	Comb 3	Comb 4	Comb 5	Comb 6	Comb 7
ϕ_1	0.0010	0.0012	0.0018*	-0.0142*	-0.0031*	-0.0033*	0.8348*	0.1864*	0.0753*
ϕ_2	0.8926*	0.8685*	0.0016	-0.0070	0.0021*	0.0049	-0.2047*	0.7788*	0.6955*
ψ_{11}		-	-0.4524*	-0.4922*	-0.3708*	-0.3659*	0.0027	0.0626*	-0.7269*
ψ_{12}		-	0.0046*	-0.2011*	-0.0063*	-0.0012	-0.0027*	0.1096*	-0.4214*
ψ_{21}		-	-0.7354*	-0.3727*	0.9060*	-0.7475*	0.9889*	-0.5624*	0.2608*
ψ_{22}		-	-0.4208*	0.0016	0.0119	-0.4271*	-0.0003*	-0.2637*	0.3266*
ψ_{31}		-	0.8955*	-0.6637*	0.1708*	-0.6240*	0.5139*	0.4689*	-0.6276*
ψ_{32}		-	-0.0046	-0.2873*	0.1148*	-0.2025*	0.3528*	0.2521*	-0.2046*
ψ_{41}		-	-0.0023	-0.0043*	-0.0014*	-0.0056	0.0007*	0.0040*	0.0007*
ψ_{42}		-	-0.0025*	-0.0075*	-0.0022*	-0.1115*	-0.0003	-0.0043*	-0.0031*
σ_1		-	0.6622	0.6343	0.6470	0.6589	0.6228	0.7256	0.6714
σ_2		-	0.6736	0.6331	0.8150	0.6823	0.9353	0.6539	0.7130
σ_3		-	0.7925	0.6590	0.7136	0.6589	0.7410	0.7500	0.6609
σ_4		-	0.7244	1.0006	0.6676	0.7260	0.6853	0.6905	0.7864
γ_1		-	0.3665*	0.0001*	0.3058*	0.3139*	0.0738*	-0.1454*	0.0216*
γ_2		-	0.1006*	0.1807*	-0.0799*	0.0933*	-0.0452*	0.0247*	-0.1829*
γ_3		-	-0.0026*	-0.0461*	0.6722*	-0.0768*	-0.0805*	0.1301*	-0.0371*
γ_4		-	0.4463*	0.8876*	0.3337*	0.3958*	-0.3292*	0.0005*	-0.0013*
μ_0	1.0452*	1.2251*	0.3089*	0.4710*	0.2107*	0.3075*	0.8431*	0.6056*	0.7906*
μ_1	-1.7789*	-1.5245*	-0.3162*	-0.5072*	-0.7062*	-0.8129*	-0.2725*	-1.6863*	-1.1919*
$\sigma_{\eta 0}$	0.5770*	0.4028*	-	-	-	-	-	-	-
$\sigma_{\eta 1}$	0.5770*	0.7524*	-	-	-	-	-	-	-
p_0	0.9532*	0.9432*	0.9549	0.9585	0.9728	0.9673	0.9636	0.9650	0.9352
p_1	0.9029*	0.9149*	0.9442	0.9525	0.9284	0.9120	0.9385	0.8949	0.9011

For the composition of Combination i see Table A.5 and Table A.2. Estimates marked with * are significant on 5% level of confidence probability. $\sigma_{\eta 0}$ and $\sigma_{\eta 1}$ stand for the standard error of η_t (the stochastic term in factor dynamics) in expansion and recession states, respectively.

Appendix B

Appendix to "On the consistency of the two-step estimates of the MS-DFM: A Monte-Carlo study"

B.1 Variance of the factor

The formula for the variance of the factor is obtained in the following way.

$$V(f_t) = V(\beta_{S_t}) + \varphi^2 V(f_{t-1}) + V(\eta_t) + 2\varphi Cov(\beta_{S_t}, f_{t-1}). \tag{B.1}$$

The process (f_t) is stationary, so

$$V(f_t) = \frac{1}{1-\varphi^2}\left[V(\beta_{S_t}) + V(\eta_t) + 2\varphi Cov(\beta_{S_t}, f_{t-1})\right]. \tag{B.2}$$

Let us consider each part of $V(f_t)$ separately. The variance of the switching constant is:

$$\begin{aligned} V(\beta_{S_t}) &= V(\beta_0 + (\beta_1 - \beta_0)S_t) \\ &= (\beta_1 - \beta_0)^2 V(S_t) \\ &= (\beta_1 - \beta_0)^2 (E(S_t^2) - E^2(S_t)) \\ &= (\beta_1 - \beta_0)^2 (\pi - \pi^2). \end{aligned} \tag{B.3}$$

where $E(S_t) = E(S_t^2) = \pi$.

The covariance term $Cov(\beta_{S_t}, f_{t-1})$ is

$$
\begin{aligned}
Cov(\beta_{S_t}, f_{t-1}) &= Cov(\beta_{S_t}, \sum_{i=0}^{\infty} \varphi^i \beta_{S_{t-1-i}} + \sum_{i=0}^{\infty} \varphi^i \eta_{S_{t-1-i}}) \\
&= Cov(\beta_{S_t}, \sum_{i=0}^{\infty} \varphi^i \beta_{S_{t-1-i}}) \\
&= \sum_{i=0}^{\infty} \varphi^i Cov(\beta_{S_t}, \beta_{S_{t-1-i}}) \\
&= (\beta_1 - \beta_0)^2 \sum_{i=0}^{\infty} \varphi^i Cov(S_t, S_{t-i-1}),
\end{aligned} \tag{B.4}
$$

where

$$
\begin{aligned}
Cov(S_t, S_{t-i-1}) &= E(S_t S_{t-i-1}) - E(S_t)E(S_{t-i-1}) \\
&= E(S_t S_{t-i-1}) - \pi^2 \\
&= \sum_{j=0}^{1}\sum_{k=0}^{1} jkP(S_t = j|S_{t-i-1} = k)P(S_{t-i-1} = k) - \pi^2 \\
&= P(S_t = 1|S_{t-i-1} = 1)\pi - \pi^2.
\end{aligned} \tag{B.5}
$$

If $P(S_t|S_{t-1})$ is the transition probability matrix for one time step:

$$
P(S_t|S_{t-1}) = \begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}, \tag{B.6}
$$

and $P(S_t|S_{t-i-1})$ is the transition probability matrix for i time steps, then, according to Chapman-Kolmogorov theorem,

$$
P(S_t|S_{t-i-1}) = (P(S_t|S_{t-1}))^{i+i}. \tag{B.7}
$$

Using Cayley-Hamilton theorem or a diagonalization of this matrix, it can be shown that:

$$
\begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}^n = \frac{\lambda_2\lambda_1^n - \lambda_1\lambda_2^n}{\lambda_2 - \lambda_1} I_2 + \frac{\lambda_2^n - \lambda_1^n}{\lambda_2 - \lambda_1} \begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}, \tag{B.8}
$$

where λ_1 and λ_2 are the eigenvalues of the matrix $\begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}$, such that $\lambda_1 = 1$, $\lambda_2 = p_0 + p_1 - 1$. So,

$$
\begin{pmatrix} p_0 & 1-p_0 \\ 1-p_1 & p_1 \end{pmatrix}^n = A\frac{1}{p_0 + p_1 - 2}, \tag{B.9}
$$

where

$$A = \begin{bmatrix} (p_0-1)(p_0+p_1-1)^n + p_1 - 1 & (1-p_0)(p_0+p_1-1)^n + p_0 - 1 \\ (1-p_1)(p_0+p_1-1)^n + p_1 - 1 & (p_1-1)(p_0+p_1-1)^n + p_0 - 1 \end{bmatrix}.$$

Therefore,

$$\begin{aligned} P(S_t = 1|S_{t-i-1} = 1) &= (P(S_t|S_{t-1}))^{i+i}_{(2,2)} \\ &= \frac{1}{p_0+p_1-2}\big((p_1-1)(p_0+p_1-1)^{i+1} + p_0 - 1\big) \\ &= (1-\pi)(p_0+p_1-1)^{i+1} + \pi. \end{aligned} \tag{B.10}$$

Coming back to $Cov(S_t, S_{t-i-1})$:

$$\begin{aligned} Cov(S_t, S_{t-i-1}) &= (1-\pi)\pi(p_0+p_1-1)^{i+1} + \pi^2 - \pi^2 \\ &= (1-\pi)\pi(p_0+p_1-1)^{i+1}. \end{aligned} \tag{B.11}$$

Putting all terms together,

$$\begin{aligned} Cov(\beta_{S_t}, f_{t-1}) &= \sum_{i=0}^{\infty} \varphi^i(\beta_1-\beta_0)^2(1-\pi)\pi(p_0+p_1-1)^{i+1} \\ &= \frac{(\beta_1-\beta_0)^2(1-\pi)\pi(p_0+p_1-1)}{1-\varphi(p_0+p_1-1)}. \end{aligned} \tag{B.12}$$

Finally,

$$\begin{aligned} V(f_t) &= \frac{1}{1-\varphi^2}\left[(\beta_1-\beta_0)^2(\pi-\pi^2) + \sigma^2 + \frac{2\varphi(\beta_1-\beta_0)^2(\pi-\pi^2)(p_0+p_1-1)}{1-\varphi(p_0+p_1-1)}\right] \\ &= \frac{1}{1-\varphi^2}\left[\sigma^2 + (\beta_1-\beta_0)^2(\pi-\pi^2)\frac{1+\varphi(p_0+p_1-1)}{1-\varphi(p_0+p_1-1)}\right]. \end{aligned} \tag{B.13}$$

The nullity of the $E(f_t)$ implies that $\beta_1 = \beta_0(1-\frac{1}{\pi})$, so the final expression for $V(f_t)$ becomes:

$$V(f_t) = \frac{1}{1-\varphi^2}\left[\sigma^2 + \beta_0^2\left(\frac{1-p_1}{1-p_0}\right)\left(\frac{1+\varphi(p_0+p_1-1)}{1-\varphi(p_0+p_1-1)}\right)\right]. \tag{B.14}$$

B.2 Two-step estimates distribution

TABLE B.1: Mean ratio between the two-step estimate of the parameter $\hat{\theta}_i(\hat{f})$ and its true value θ_0

N	T	$\hat{\beta_0}/\beta_0$	$\hat{\beta_1}/\beta_1$	$\hat{\varphi}/\varphi$	$\hat{\sigma^2}/\sigma^2$	$\hat{p_0}/p_0$	$\hat{p_1}/p_1$
25	25	1.34	1.13	0.46	1.49	0.93	0.95
25	50	1.21	1.12	0.60	1.72	0.96	0.96
25	100	1.13	1.10	0.65	1.92	0.98	0.97
25	150	1.10	1.08	0.67	1.95	0.99	0.98
25	300	1.08	1.07	0.68	2.01	1.00	1.00
50	25	1.25	1.10	0.65	1.20	0.93	0.95
50	50	1.13	1.08	0.74	1.39	0.97	0.97
50	100	1.08	1.07	0.77	1.51	0.99	0.97
50	150	1.06	1.05	0.80	1.54	0.99	0.99
50	300	1.05	1.05	0.80	1.57	1.00	1.00
100	25	1.21	1.08	0.74	1.05	0.95	0.97
100	50	1.12	1.06	0.82	1.17	0.97	0.96
100	100	1.06	1.05	0.86	1.24	0.99	0.98
100	150	1.04	1.04	0.89	1.26	0.99	0.98
100	300	1.03	1.03	0.89	1.29	1.00	0.99
150	25	1.20	1.08	0.76	0.98	0.95	0.96
150	50	1.08	1.06	0.87	1.10	0.97	0.97
150	100	1.04	1.04	0.90	1.15	0.99	0.98
150	150	1.03	1.03	0.91	1.18	0.99	0.98
150	300	1.02	1.02	0.92	1.19	1.00	0.99
300	25	1.17	1.09	0.81	0.92	0.96	0.97
300	50	1.07	1.05	0.89	1.01	0.98	0.97
300	100	1.03	1.03	0.94	1.06	0.99	0.97
300	150	1.03	1.02	0.94	1.08	0.99	0.98
300	300	1.02	1.02	0.96	1.09	1.00	0.99

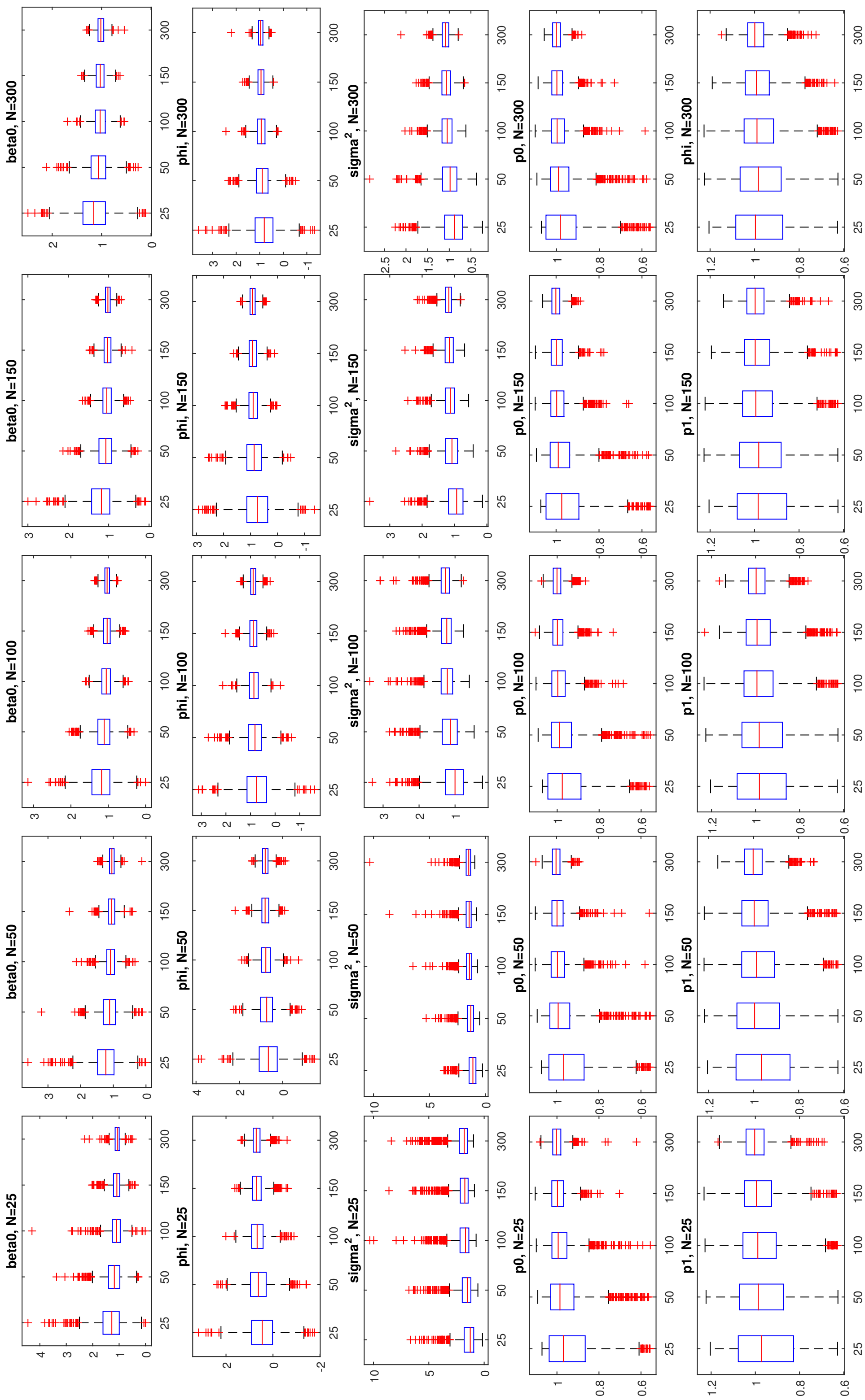


FIGURE B.1: Sample characteristics of the ratio of the two-step estimates to their true values

B.3 Results on unfiltered data

TABLE B.2: Test statistic of the Kolmogorov-Smirnov test

N	T	β_0	β_1	φ	σ^2	p_0	p_1
25	25	**0.04**	0.11	0.08	0.07	0.20	0.08
25	50	0.08	0.11	0.12	0.08	0.27	0.09
25	100	0.10	0.16	0.18	0.12	0.37	0.12
25	150	0.07	0.14	0.19	0.12	0.42	0.18
25	300	0.12	0.22	0.16	0.15	0.44	0.30
50	25	**0.04**	0.08	**0.03**	**0.03**	0.15	0.10
50	50	**0.04**	0.08	0.07	0.08	0.14	0.06
50	100	0.06	0.10	0.10	0.10	0.29	0.11
50	150	0.12	0.15	0.11	0.12	0.32	0.16
50	300	0.07	0.20	0.15	0.15	0.40	0.26
100	25	0.06	0.06	**0.03**	0.06	0.11	0.08
100	50	**0.04**	0.06	**0.04**	**0.04**	0.11	0.05
100	100	**0.03**	**0.04**	0.07	**0.03**	0.10	0.09
100	150	0.06	0.09	0.08	0.08	0.13	0.05
100	300	0.06	0.11	0.10	0.10	0.20	0.16
150	25	0.05	**0.03**	0.05	**0.02**	0.13	0.09
150	50	0.06	**0.04**	**0.04**	**0.02**	0.08	0.05
150	100	**0.03**	**0.03**	0.05	**0.04**	0.11	0.06
150	150	0.05	0.06	0.07	0.05	0.14	**0.04**
150	300	**0.04**	0.09	0.09	0.08	0.12	0.06
300	25	0.05	**0.03**	**0.04**	**0.02**	0.05	0.07
300	50	**0.04**	**0.03**	**0.04**	**0.04**	0.06	0.05
300	100	**0.03**	**0.04**	**0.04**	0.05	0.06	0.05
300	150	**0.04**	**0.02**	**0.02**	**0.04**	**0.02**	**0.02**
300	300	0.08	0.06	0.07	0.06	0.10	0.06

The null hypothesis of the test is that $\hat{\theta}_i(\hat{f})$ and $\hat{\theta}_i(f)$, with $\theta = (\beta_0, \beta_1, \varphi, \sigma^2, p_0, p_1)$, are from the same continuous distribution. The null is rejected when $KS > 0.043$. The cases when the null is not rejected are marked with bold font.

TABLE B.3: Mean ratio between the two-step estimate of the parameter $\hat{\theta}_i(\hat{f})$ and its true value

N	T	$\hat{\beta_0}/\beta_0$	$\hat{\beta_1}/\beta_1$	$\hat{\varphi}/\varphi$	$\hat{\sigma^2}/\sigma^2$	$\hat{p_0}/p_0$	$\hat{p_1}/p_1$
25	25	1.40	0.90	0.87	1.45	0.74	0.72
25	50	1.20	0.93	1.05	1.94	0.77	0.74
25	100	1.11	0.91	1.07	2.28	0.78	0.79
25	150	1.06	0.92	1.04	2.37	0.81	0.81
25	300	1.03	0.93	0.97	2.50	0.85	0.84
50	25	1.27	0.90	1.00	1.18	0.78	0.73
50	50	1.10	0.94	1.07	1.52	0.84	0.81
50	100	1.05	0.93	1.06	1.72	0.86	0.86
50	150	1.03	0.95	1.02	1.74	0.90	0.89
50	300	1.01	0.99	0.98	1.77	0.93	0.91
100	25	1.23	0.88	1.01	1.04	0.82	0.76
100	50	1.08	0.91	1.12	1.26	0.85	0.82
100	100	1.02	0.97	1.03	1.33	0.92	0.90
100	150	1.01	0.98	1.01	1.36	0.95	0.94
100	300	1.01	1.01	0.94	1.34	0.98	0.97
150	25	1.22	0.88	0.98	0.97	0.83	0.78
150	50	1.06	0.92	1.11	1.19	0.87	0.84
150	100	1.02	0.96	1.04	1.23	0.92	0.92
150	150	1.01	0.98	1.01	1.23	0.96	0.95
150	300	1.01	1.01	0.96	1.22	0.99	0.98
300	25	1.19	0.89	1.02	0.93	0.84	0.79
300	50	1.04	0.93	1.08	1.09	0.90	0.86
300	100	1.00	0.97	1.05	1.13	0.94	0.93
300	150	1.01	0.99	1.00	1.11	0.97	0.96
300	300	1.01	1.01	0.98	1.11	0.99	0.98

B.4 ML estimates obtained with the observable factor f_t

TABLE B.4: Mean ratio between $\hat{\theta}_i(f)$ and its true value θ_{0i}

T	$\hat{\beta}_0/\beta_0$	$\hat{\beta}_1/\beta_1$	$\hat{\varphi}/\varphi$	$\hat{\sigma^2}/\sigma^2$	$\hat{p_0}/p_0$	$\hat{p_1}/p_1$
25	1.13	0.84	1.04	0.82	0.84	0.80
50	1.03	0.92	1.08	0.96	0.91	0.87
100	1.00	0.97	1.04	1.00	0.96	0.94
150	1.00	0.98	1.03	1.00	0.98	0.96
300	1.00	1.00	1.00	1.00	1.00	0.99

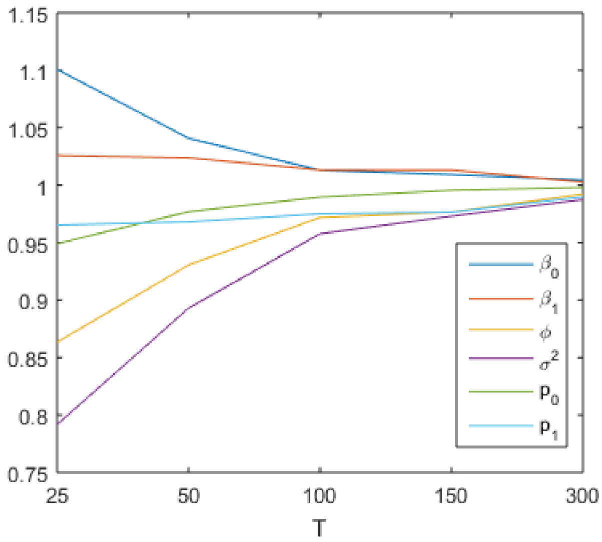


FIGURE B.2: Mean ratio between $\hat{\theta}_i(f)$ and its true value θ_{0i}

B.5 Properties of empirical distributions of t-statistics corresponding to the two-step estimates

TABLE B.5: Sample mean of the t-statistics corresponding to $\hat{\theta}(\hat{f})$

N	T	$t_{\hat{\beta_0}}$	$t_{\hat{\beta_1}}$	$t_{\hat{\varphi}}$	$t_{\hat{\sigma^2}}$	$t_{\hat{p_0}}$	$t_{\hat{p_1}}$
25	25	0.82	-0.35	-1.03	0.37	-0.24	0.06
25	50	0.66	-0.50	-0.96	1.27	-0.05	0.08
25	100	0.54	-0.57	-1.16	2.26	0.01	0.05
25	150	0.56	-0.63	-1.34	2.90	0.09	0.06
25	300	0.63	-0.75	-1.80	4.35	0.17	0.14
50	25	0.70	-0.29	-0.66	-0.01	-0.25	0.07
50	50	0.46	-0.37	-0.68	0.82	0.02	0.13
50	100	0.45	-0.48	-0.82	1.64	0.04	0.03
50	150	0.38	-0.47	-0.86	2.16	0.06	0.07
50	300	0.43	-0.59	-1.22	3.30	0.16	0.12
100	25	0.60	-0.25	-0.50	-0.35	-0.15	0.11
100	50	0.50	-0.32	-0.48	0.32	-0.06	0.04
100	100	0.34	-0.36	-0.54	0.93	0.07	0.06
100	150	0.28	-0.37	-0.55	1.32	0.04	0.01
100	300	0.31	-0.40	-0.75	2.12	0.10	0.00
150	25	0.63	-0.24	-0.47	-0.56	-0.15	0.10
150	50	0.34	-0.32	-0.37	0.11	-0.03	0.07
150	100	0.28	-0.31	-0.40	0.58	0.04	0.06
150	150	0.22	-0.28	-0.43	0.92	0.07	0.03
150	300	0.24	-0.35	-0.56	1.54	0.13	0.03
300	25	0.52	-0.31	-0.35	-0.75	-0.08	0.15
300	50	0.32	-0.29	-0.31	-0.27	0.03	0.06
300	100	0.19	-0.24	-0.25	0.18	0.02	-0.01
300	150	0.24	-0.19	-0.28	0.38	0.06	0.01
300	300	0.19	-0.23	-0.31	0.74	0.10	0.01

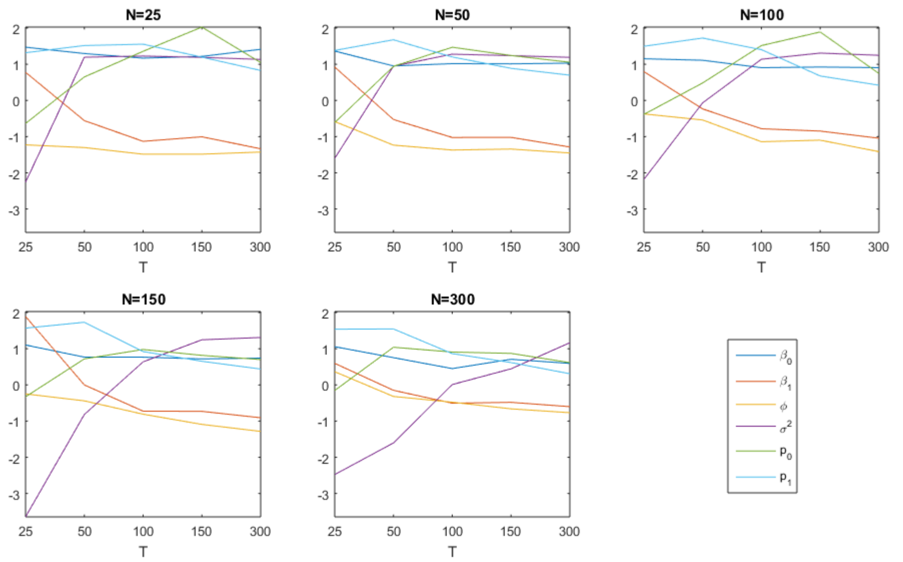


FIGURE B.3: Skewness of the t-statistics corresponding to the estimates $\hat{\theta}_i(\hat{f})$ at different N and T

TABLE B.6: Excess kurtosis of t-statistics of the estimates $\hat{\theta}_i(\hat{f})$

N	T	$t_{\hat{\beta}_0}$	$t_{\hat{\beta}_1}$	$t_{\hat{\varphi}}$	$t_{\hat{\sigma^2}}$	$t_{\hat{p_0}}$	$t_{\hat{p_1}}$
25	25	0.50	5.38	0.27	21.22	-0.64	1.79
25	50	0.29	0.51	0.37	-1.07	1.11	3.79
25	100	0.11	0.17	-0.11	-1.35	4.87	5.34
25	150	0.00	-0.11	-0.31	-1.45	13.38	3.62
25	300	0.13	-0.31	-0.62	-1.63	1.53*	0.82*
50	25	0.71	4.39	1.57	4.01	-0.75	1.79
50	50	0.20	0.63	0.05	-0.66	1.73	4.13
50	100	-0.09	0.15	-0.14	-1.14	6.55	3.49
50	150	-0.02	-0.01	-0.31	-1.31	4.57	2.46
50	300	0.17	0.13	-0.36	-1.44	2.36*	0.51*
100	25	-0.04	5.32	1.82	5.29	-0.75	2.30
100	50	0.13	1.06	1.04	0.35	1.01	6.28
100	100	0.20	0.18	0.16	-0.77	5.64	7.32
100	150	0.32	-0.12	-0.05	-0.97	18.36	1.77
100	300	0.16	0.02	-0.07	-1.27	0.94	0.53*
150	25	0.35	12.71	0.79	24.17	-0.84	2.50
150	50	0.23	2.57	1.07	1.86	1.29	5.65
150	100	0.29	0.10	0.52	-0.21	2.21	1.73
150	150	0.21	0.10	0.06	-0.70	1.05	0.70*
150	300	0.15	-0.04	0.03	-1.02	0.22	0.42*
300	25	0.18	4.12	3.41	6.04	-0.83	2.26
300	50	0.24	1.67	1.39	3.08	1.69	4.75
300	100	-0.02	0.19	0.45	0.25	2.92	2.82
300	150	0.01	0.05	-0.12	-0.08	1.67	0.67
300	300	0.11	0.01	0.62	-0.69	0.02	0.46*

Note: the symbol '*' marks the cases for which the null of normality of the Kolmogorov-Smirnov test is not rejected at 5% level of confidence probability.

Table B.7: Empirical size of two-tailed tests based on t-statistics, nominal size=5%

N	T	$t_{\hat{\beta_0}}$	$t_{\hat{\beta_1}}$	$t_{\hat{\varphi}}$	$t_{\hat{\sigma^2}}$	$t_{\hat{p_0}}$	$t_{\hat{p_1}}$
25	25	0.17	0.17	0.23	0.09	0.01	0.06
25	50	0.13	0.13	0.21	0.26	0.08	0.07
25	100	0.11	0.11	0.25	0.61	0.08	0.07
25	150	0.10	0.10	0.28	0.79	0.06	0.07
25	300	0.10	0.12	0.42	0.96	0.06	0.07
50	25	0.14	0.15	0.18	0.09	0.01	0.06
50	50	0.11	0.10	0.14	0.13	0.07	0.06
50	100	0.08	0.09	0.17	0.38	0.05	0.06
50	150	0.08	0.08	0.15	0.59	0.05	0.07
50	300	0.08	0.09	0.26	0.86	0.06	0.06
100	25	0.11	0.15	0.16	0.11	0.00	0.05
100	50	0.10	0.10	0.11	0.08	0.06	0.06
100	100	0.07	0.07	0.09	0.14	0.05	0.06
100	150	0.06	0.07	0.09	0.25	0.05	0.06
100	300	0.06	0.07	0.12	0.56	0.05	0.05
150	25	0.12	0.13	0.15	0.13	0.00	0.05
150	50	0.08	0.10	0.10	0.06	0.05	0.06
150	100	0.06	0.07	0.08	0.09	0.06	0.06
150	150	0.05	0.06	0.08	0.15	0.05	0.06
150	300	0.05	0.07	0.09	0.34	0.06	0.05
300	25	0.10	0.13	0.14	0.15	0.00	0.06
300	50	0.07	0.08	0.10	0.09	0.05	0.05
300	100	0.06	0.06	0.07	0.05	0.05	0.06
300	150	0.05	0.06	0.06	0.05	0.05	0.05
300	300	0.06	0.06	0.06	0.12	0.05	0.05

Table B.8: Empirical size of two-tailed tests based on t-statistics, nominal size=10%

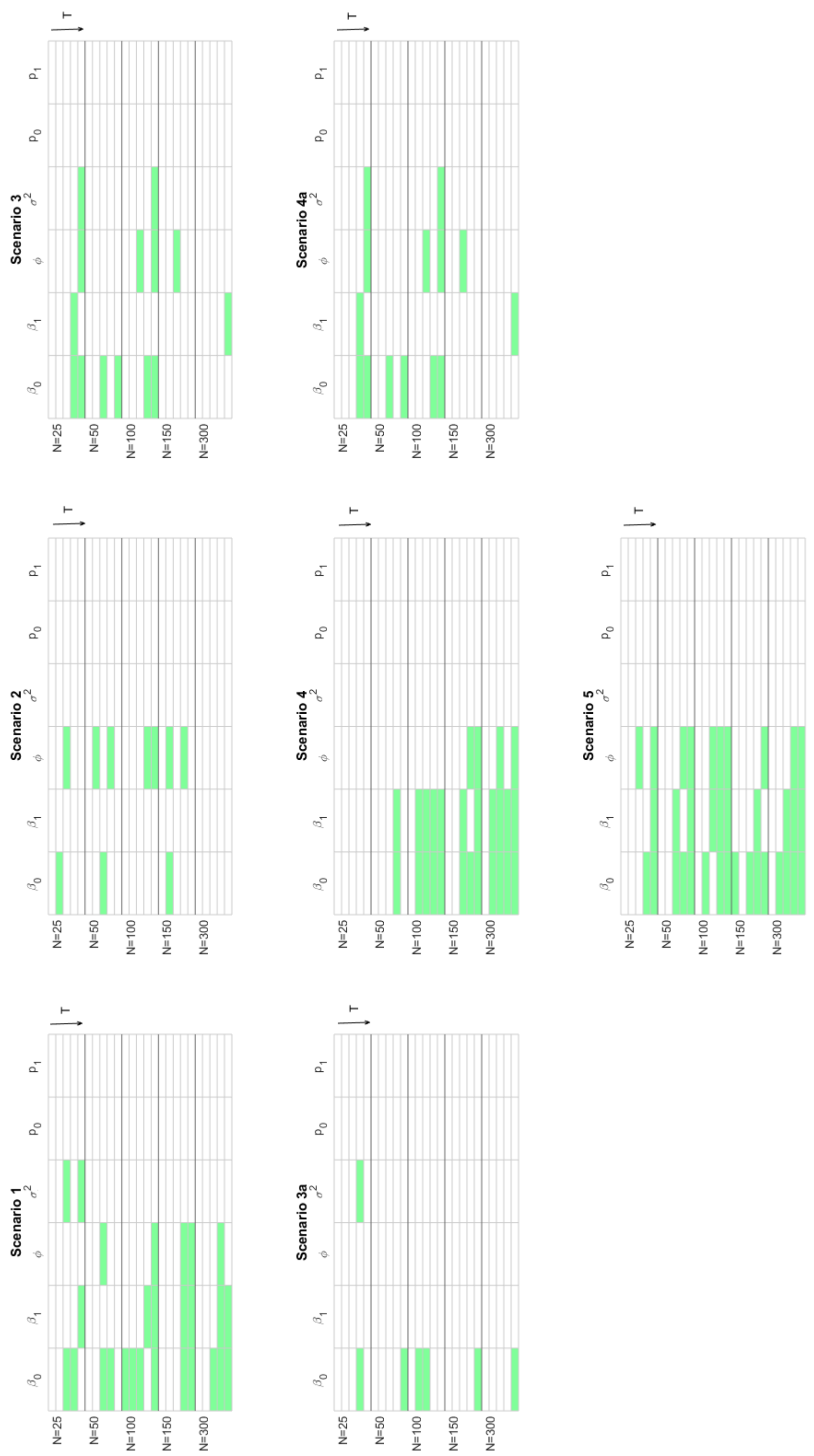
N	T	$t_{\hat{\beta_0}}$	$t_{\hat{\beta_1}}$	$t_{\hat{\varphi}}$	$t_{\hat{\sigma^2}}$	$t_{\hat{p_0}}$	$t_{\hat{p_1}}$
25	25	0.24	0.24	0.33	0.19	0.04	0.08
25	50	0.20	0.21	0.31	0.38	0.13	0.11
25	100	0.18	0.18	0.35	0.72	0.12	0.12
25	150	0.17	0.18	0.38	0.86	0.11	0.12
25	300	0.16	0.18	0.52	0.97	0.10	0.11
50	25	0.20	0.24	0.26	0.15	0.03	0.09
50	50	0.18	0.16	0.21	0.24	0.10	0.10
50	100	0.13	0.15	0.24	0.52	0.10	0.10
50	150	0.14	0.13	0.24	0.70	0.10	0.11
50	300	0.13	0.17	0.36	0.91	0.11	0.11
100	25	0.20	0.23	0.23	0.16	0.03	0.07
100	50	0.17	0.17	0.18	0.14	0.09	0.09
100	100	0.13	0.13	0.15	0.26	0.09	0.10
100	150	0.11	0.14	0.17	0.38	0.10	0.11
100	300	0.11	0.13	0.19	0.69	0.10	0.09
150	25	0.20	0.21	0.23	0.17	0.03	0.08
150	50	0.14	0.16	0.16	0.11	0.08	0.09
150	100	0.11	0.13	0.13	0.17	0.11	0.11
150	150	0.10	0.12	0.14	0.25	0.10	0.10
150	300	0.10	0.13	0.16	0.47	0.11	0.10
300	25	0.18	0.19	0.22	0.20	0.02	0.08
300	50	0.13	0.15	0.16	0.14	0.09	0.09
300	100	0.11	0.12	0.12	0.10	0.09	0.10
300	150	0.10	0.11	0.13	0.11	0.09	0.09
300	300	0.11	0.12	0.11	0.20	0.10	0.09

B.6 Simulations with various scenarios

Figure B.4: Results of the Kolmogorov-Smirnov normality test for the t-statistics of the two-step estimators, $\alpha = 5\%$

Note: cases when the hypothesis is not rejected are marked in green.

FIGURE B.5: Results of the Jarque-Berra normality test for the t-statistics of the two-step estimators, $\alpha = 5\%$



Note: cases when the hypothesis is not rejected are marked in green.

Appendix C

Appendix to “Dynamical Interaction between Financial and Business Cycles”

C.1 Composition of factors RF_t and FF_t

TABLE C.1: List of financial variables used for describing the financial cycle in the US

Series	**Source**
Series from Guidolin et al. (2013) database	
Monthly SP500 portfolio returns	FREDII
3mtb, monthly rate	FREDII
10-Year Treasury Constant Maturity Rate	FREDII
2-Year Treasury Constant Maturity Rate	FREDII
Moody’s Seasoned Baa Corporate Bond Yield (to change, see Shiller)	FREDII
Composite NAREIT	NAREIT
Equity REITs	NAREIT
Mortgage REITs	NAREIT
Excess return on a value-weighted market	FREDII
S&P 500 dividend yield — (12 month dividend per share)/price.	FREDII
Moody’s Seasoned Baa Corp. Bond Yield to Yield on 10-Year Treasury Const. Maturity	FREDII
10-Year Treasury Constant Maturity Minus 3-Month Treasury Constant Maturity	FREDII
10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity Rate	FREDII
Unexpected inflation rate	FREDII
Industrial production index	FREDII
Real personal consumption expenditures	FREDII
Other series	
3-month Tbill rate of return minus CPI	FREDII

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Table C.1 – continued from previous page

Series	**Source**
SP500 PE ratio	FREDII
Federal funds effective rate	FREDII
Monetary Base; Total	FREDII
Total Reserve Balances Maintained with Federal Reserve Banks	FREDII
M1 Money Stock	FREDII
M2 Money Stock	FREDII
Federal Debt: Total Public Debt as Percent of Gross Domestic Product	FREDII
Median Sales Price for New Houses Sold in the United States	FREDII
Total Assets, All Commercial Banks	FREDII
Commercial and Industrial Loans, All Commercial Banks	FREDII
Loans and Leases in Bank Credit, All Commercial Banks	FREDII
Total Savings Deposits at all Depository Institutions	FREDII
Loans to deposits ratio	FREDII
Consumer Credit Outstanding (Levels)	FREDII

TABLE C.2: Components of RF_t and their factor loadings

Abbreviation	**Indicator**	**Loading**
RPI	Real Personal Income	0.0413
W875RX1	Real personal income ex transfer receipts	0.0569
DPCERA3M086SBEA	Real personal consumption expenditures	0.0366
CMRMTSPLx	Real Manu. and Trade Industries Sales	0.0725
RETAILx	Retail and Food Services Sales	0.0578
INDPRO	IP Index	0.0127
IPFPNSS	IP: Final Products and Nonindustrial Supplies	0.0122
IPFINAL	IP: Final Products (Market Group)	0.0109
IPCONGD	IP: Consumer Goods	0.0754
IPDCONGD	IP: Durable Consumer Goods	0.0673
IPNCONGD	IP: Nondurable Consumer Goods	0.0502
IPBUSEQ	IP: Business Equipment	0.0117
IPMAT	IP: Materials	0.0111
IPDMAT	IP: Durable Materials	0.0119
IPNMAT	IP: Nondurable Materials	0.0853
IPMANSICS	IP: Manufacturing (SIC)	0.0132
IPB51222S	IP: Residential Utilities	0.0167
IPFUELS	IP: Fuels	0.0128
NAPMPI	ISM Manufacturing: Production Index	0.0138
CUMFNS	Capacity Utilization: Manufacturing	0.0142
HWI	Help-Wanted Index for United States	0.0633
HWIURATIO	Ratio of Help Wanted/No. Unemployed	0.0112
CLF16OV	Civilian Labor Force	0.0517
CE16OV	Civilian Employment	0.0110
UNRATE	Civilian Unemployment Rate	0.0858
UEMPMEAN	Average Duration of Unemployment (Weeks)	0.0562
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	0.0827
UEMP5TO14	Civilians Unemployed for 5-14 Weeks	0.0415
UEMP15OV	Civilians Unemployed - 15 Weeks & Over	0.0899
UEMP15T26	Civilians Unemployed for 15-26 Weeks	0.0547
UEMP27OV	Civilians Unemployed for 27 Weeks and Over	0.0716
CLAIMSx	Initial Claims	0.0143
PAYEMS	All Employees: Total nonfarm	0.0170
USGOOD	All Employees: Goods-Producing Industries	0.0792
CES1021000001	All Employees: Mining and Logging: Mining	0.0218
USCONS	All Employees: Construction	0.0107
MANEMP	All Employees: Manufacturing	0.0153
DMANEMP	All Employees: Durable goods	0.0147
NDMANEMP	All Employees: Nondurable goods	0.0123
SRVPRD	All Employees: Service-Providing Industries	0.0151
USTPU	All Employees: Trade, Transportation & Utilities	0.0156
USWTRADE	All Employees: Wholesale Trade	0.0160
USTRADE	All Employees: Retail Trade	0.0132

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Table C.2 – continued from previous page

Abbreviation	Indicator	Loading
USFIRE	All Employees: Financial Activities	0.0122
USGOVT	All Employees: Government	0.0432
CES0600000007	Avg Weekly Hours : Goods-Producing	0.0491
AWOTMAN	Avg Weekly Overtime Hours : Manufacturing	0.0339
AWHMAN	Avg Weekly Hours : Manufacturing	0.0499
NAPMEI	ISM Manufacturing: Employment Index	0.0144
HOUST	Housing Starts: Total New Privately Owned	0.0140
HOUSTNE	Housing Starts, Northeast	0.0122
HOUSTMW	Housing Starts, Midwest	0.0135
HOUSTS	Housing Starts, South	0.0121
HOUSTW	Housing Starts, West	0.0122
NAPM	ISM : PMI Composite Index	0.0124
NAPMNOI	ISM : New Orders Index	0.0125
NAPMSDI	ISM : Supplier Deliveries Index	0.0129
NAPMII	ISM : Inventories Index	0.0870
AMDMNOx	New Orders for Durable Goods	0.0118
AMDMUOx	Unfilled Orders for Durable Goods	0.0158
BUSINVx	Total Business Inventories	0.0135
ISRATIOx	Total Business: Inventories to Sales Ratio	0.0119
M1SL	M1 Money Stock	0.0128
M2SL	M2 Money Stock	0.0535
M2REAL	Real M2 Money Stock	0.0118
AMBSL	St. Louis Adjusted Monetary Base	0.0105
TOTRESNS	Total Reserves of Depository Institutions	0.0536
NONBORRES	Reserves Of Depository Institutions	0.0461
BUSLOANS	Commercial and Industrial Loans	0.0113
REALLN	Real Estate Loans at All Commercial Banks	0.0661
NONREVSL	Total Nonrevolving Credit	0.0964
CONSPI	Nonrevolving consumer credit to Personal Income	0.0982
S&P 500	S&P's Common Stock Price Index: Composite	0.0792
S&P: indust	S&P's Common Stock Price Index: Industrials	0.0748
S&P div yield	S&P's Composite Common Stock: Dividend Yield	0.0103
S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio	0.0108
FEDFUNDS	Effective Federal Funds Rate	0.0204
CP3Mx	3-Month AA Financial Commercial Paper Rate	0.0551
TB3MS	3-Month Treasury Bill:	0.0103
TB6MS	6-Month Treasury Bill:	0.0102
GS1	1-Year Treasury Rate	0.0628
GS5	5-Year Treasury Rate	0.0838
GS10	10-Year Treasury Rate	0.0899
AAA	Moody's Seasoned Aaa Corporate Bond Yield	0.0590
BAA	Moody's Seasoned Baa Corporate Bond Yield	0.0576
COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS	0.0625
TB3SMFFM	3-Month Treasury C Minus FEDFUNDS	0.0492

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Table C.2 – continued from previous page

Abbreviation	Indicator	Loading
TB6SMFFM	6-Month Treasury C Minus FEDFUNDS	0.0228
T1YFFM	1-Year Treasury C Minus FEDFUNDS	0.0210
T5YFFM	5-Year Treasury C Minus FEDFUNDS	0.0138
T10YFFM	10-Year Treasury C Minus FEDFUNDS	0.0452
AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS	0.0430
BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS	0.0583
TWEXMMTH	Trade Weighted U.S. Dollar Index: Major Currencies	0.0806
EXSZUSx	Switzerland / U.S. Foreign Exchange Rate	0.0101
EXJPUSx	Japan / U.S. Foreign Exchange Rate	0.0657
EXUSUKx	U.S. / U.K. Foreign Exchange Rate	0.0456
EXCAUSx	Canada / U.S. Foreign Exchange Rate	0.0672
PPIFGS	PPI: Finished Goods	0.0194
PPIFCG	PPI: Finished Consumer Goods	0.0724
PPIITM	PPI: Intermediate Materials	0.0713
PPICRM	PPI: Crude Materials	0.0959
OILPRICEx	Crude Oil, spliced WTI and Cushing	0.0573
PPICMM	PPI: Metals and metal products:	0.0441
NAPMPRI	ISM Manufacturing: Prices Index	0.0789
CPIAUCSL	CPI : All Items	0.0126
CPIAPPSL	CPI : Apparel	0.0879
CPITRNSL	CPI : Transportation	0.0437
CPIMEDSL	CPI : Medical Care	0.0646
CUSR0000SAC	CPI : Commodities	0.0120
CUUR0000SAD	CPI : Durables	0.0828
CUSR0000SAS	CPI : Services	0.0399
CPIULFSL	CPI : All Items Less Food	0.0537
CUUR0000SA0L2	CPI : All items less shelter	0.0772
CUSR0000SA0L5	CPI : All items less medical care	0.0773
PCEPI	Personal Cons. Expend.: Chain Index	0.0894
DDURRG3M086SBEA	Personal Cons. Exp: Durable goods	0.0873
DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods	0.0516
DSERRG3M086SBEA	Personal Cons. Exp: Services	0.0818
CES0600000008	Avg Hourly Earnings : Goods-Producing	0.0573
CES2000000008	Avg Hourly Earnings : Construction	0.0541
CES3000000008	Avg Hourly Earnings : Manufacturing	0.0153
UMCSENTx	Consumer Sentiment Index	0.0569
MZMSL	MZM Money Stock	0.0554
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding	0.0214
DTCTHFNM	Total Consumer Loans and Leases Outstanding	0.0439
INVEST	Securities in Bank Credit at All Commercial Banks	0.0223

Data source: Federal Reserve Bank of St. Louis, https://research.stlouisfed.org/econ/mccracken/fred-databases/

FIGURE C.1.1: Top 50 factor loadings of RF_t

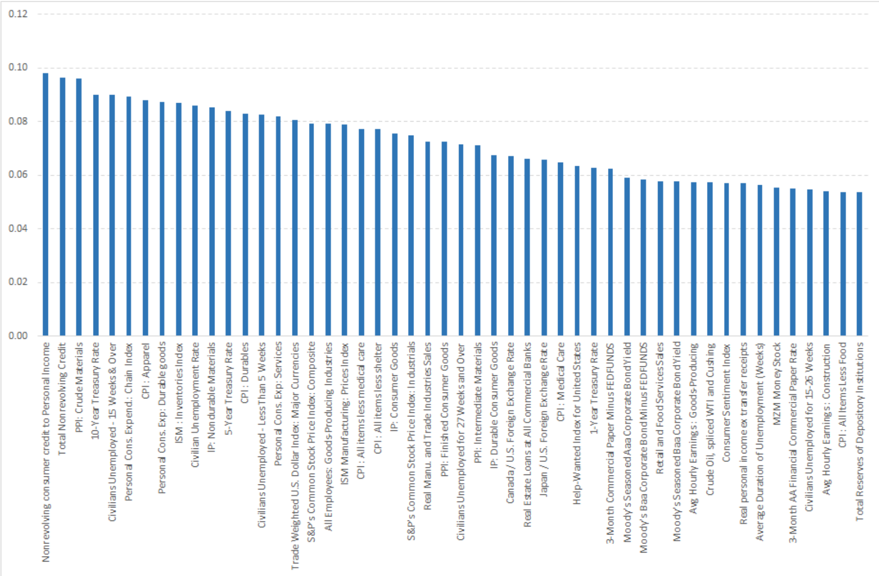
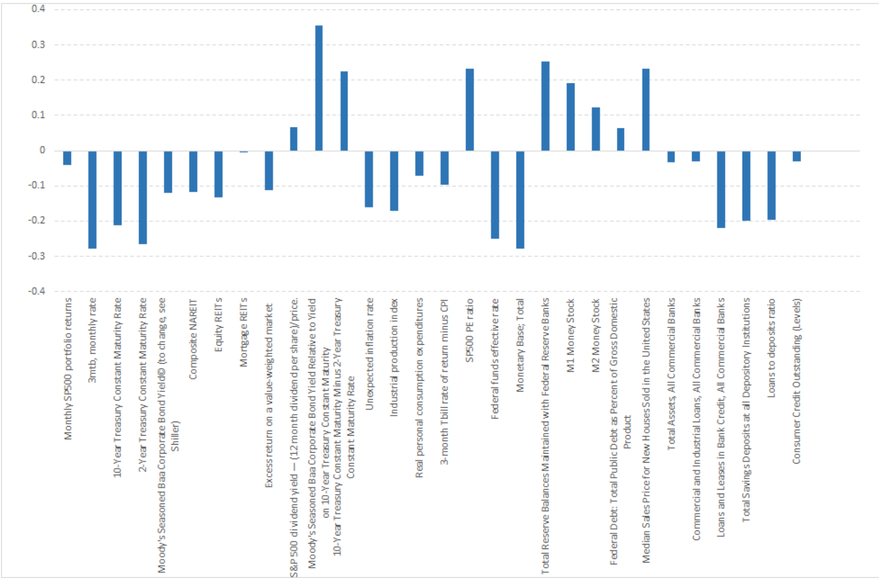


FIGURE C.1.2: Factor loadings of FF_t

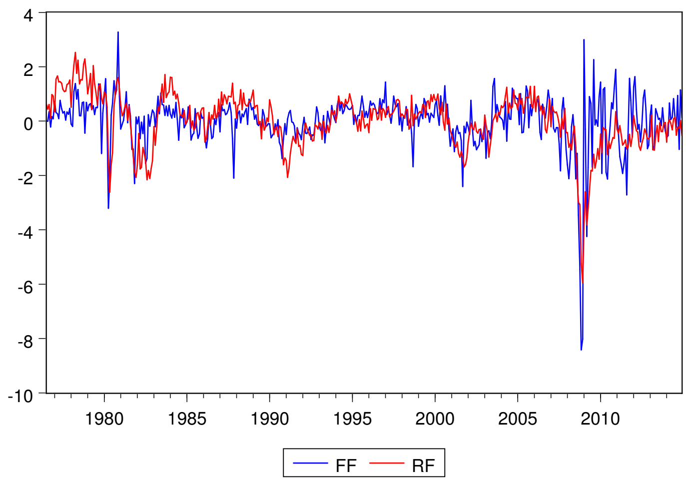


C.2 Dynamics of RF_t and FF_t

Figure C.2.3 shows the dynamics of the extracted factors RF_t and FF_t. Not surprisingly, it differs from the cycles proxies presented in Figure 5.1 since they correspond to the growth cycle rather than business cycle (defined as deviation from the trend).

As the state of financial downturn is usually characterized not only by low levels of the corresponding financial cycle indicator but also by its high volatility,[1] the usual correlation estimated on a moving window is not very informative as it would ignore the volatility aspect of the financial cycle. For this reason, to make a preliminary assessment of the interaction between the cycles approximated by RF_t and FF_t, we estimate the correlation between the signals of recession and financial downturn extracted from the factors[2] (see Figure C.2.4). As in Figure 5.2, the correlation is lower in the middle of the sample, although the period of high interaction starts a much earlier, indicating a possibility of a slightly different pattern of interaction for the growth cycle with respect to the business cycle.

FIGURE C.2.3: Dynamics of RF_t and FF_t

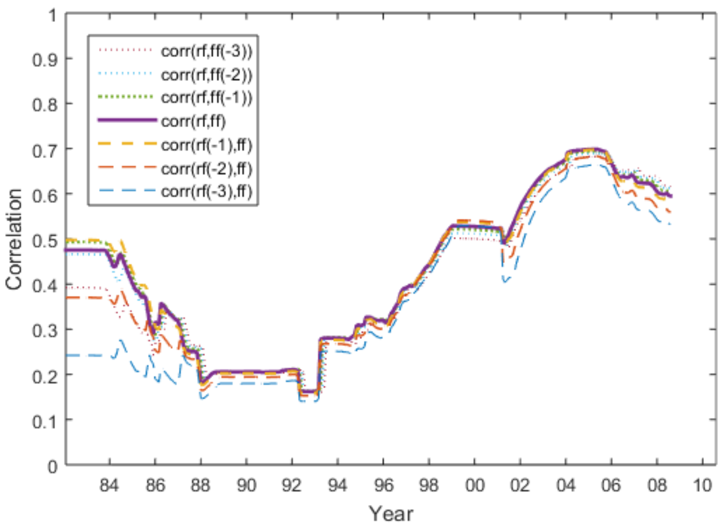


Note: blue and red line correspond to RF_t and FF_t respectively

[1]When the cycle is characterized by the growth rate series.

[2]we use smoothed probability of recession (financial downturn) estimated on RF_t (FF_t) with a standard Markov-Switching model à la Hamilton (1989).

FIGURE C.2.4: Cross-correlations between RF_t and FF_t



Note: Cross-correlations between the smoothed probability of recession (estimated on RF_t with a Markov-Switching model by Hamilton (1989)) and the smoothed probability of financial downturn (estimated on FF_t) computed on a moving window with width $w = 141$, i.e. a estimate for a date t is obtained using observations from $t - 70$ to $t + 70$.

C.3 Robustness check

C.3.1 Alternative dataset

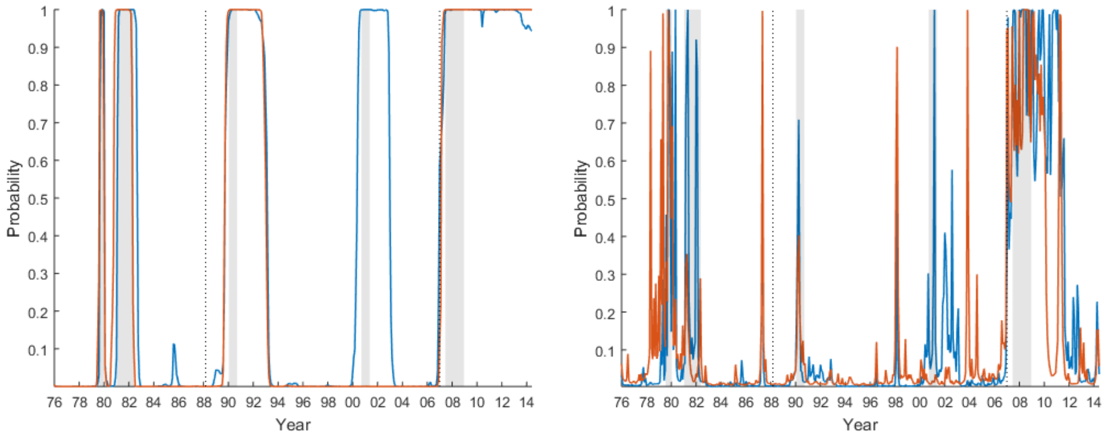
TABLE C.3: DI-MS-FM estimates on alternative datasets

	BL	RC1	RC2	RC3
Business cycle				
$\hat{\mu}_1$	0.57***	0.57***	0.58***	0.59***
$\hat{\mu}_2$	-0.85***	-0.85***	-1.19***	-1.19***
$\hat{\sigma}_1^2$	0.31***	0.31***	0.30***	0.29***
$\hat{\sigma}_2^2$	0.81***	0.81***	0.31***	0.31***
$\hat{D}_{11}^1$	0.99***	0.99***	0.99***	0.99***
$\hat{D}_{22}^1$	0.98***	0.98***	0.99**	0.99***
$\hat{C}_{11}^{12}$	0.91***	0.72***	0.93***	0.73**
$\hat{C}_{22}^{12}$	0.99*	0.85***	0.99***	0.87***
Financial cycle				
$\hat{\beta}_1$	0.16***	0.06***	0.15***	0.06***
$\hat{\beta}_2$	-0.86***	-0.39***	-0.97***	-0.37***
$\hat{\theta}_1^2$	0.27***	0.46***	0.29***	0.46***
$\hat{\theta}_2^2$	4.05***	4.39***	4.47***	4.26***
$\hat{D}_{11}^2$	0.99***	0.98***	0.99***	0.99***
$\hat{D}_{22}^2$	0.88***	0.36***	0.59**	0.39***
$\hat{C}_{11}^{21}$	0.75***	0.55***	0.98***	0.99***
$\hat{C}_{22}^{21}$	0.99***	0.99***	0.99***	0.99***
Influence regimes				
$\hat{R}^1$	$\begin{bmatrix} 0.98 & 0.06 \\ 0.02 & 0.94 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.04 \\ 0.01 & 0.96 \end{bmatrix}$
$\hat{R}^2$	$\begin{bmatrix} 0.86 & 0.81 \\ 0.14 & 0.19 \end{bmatrix}$	$\begin{bmatrix} 0.92 & 0.44 \\ 0.08 & 0.56 \end{bmatrix}$	$\begin{bmatrix} 0.91 & 1.00 \\ 0.09 & 0.00 \end{bmatrix}$	$\begin{bmatrix} 0.86 & 1.00 \\ 0.14 & 0.00 \end{bmatrix}$
$\hat{q}_{11}$	0.99	0.99	0.99	0.99
$\hat{q}_{22}$	0.97	0.95	0.99	0.95

Note: The parameters significant at 15%, 10% and 5% are marked with *, ** and ***, correspondingly. BL stands for baseline dataset, the description of cases RC1, RC2 and RC3 is given in Section 5.4.4.

In all figures below, grey shaded areas correspond to NBER recessions, dotted vertical lines mark the beginning of systemic banking crises as identified by Laeven and Valencia (2008 and 2010) and Reinhart and Rogoff (2008).

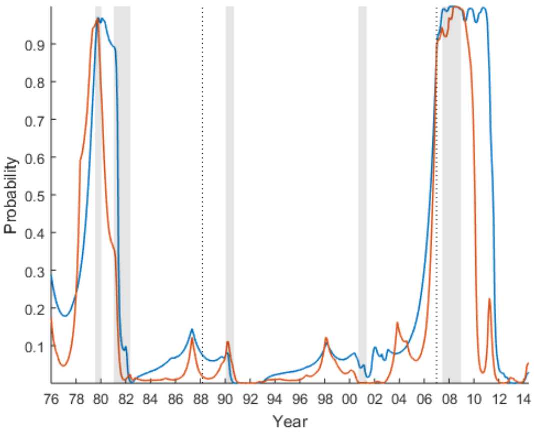
Figure C.3.1: Smoothed probability of recession and financial downturn



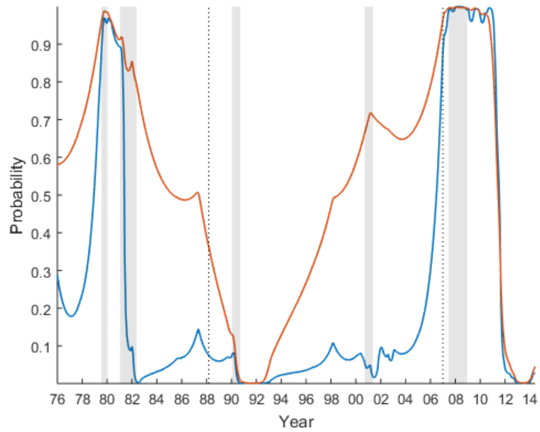
(a) Business cycle: RF_t vs HS (b) Financial cycle: FF_t vs $NAREIT$

Note: Blue line corresponds to the estimate of the smoothed probability of recession (financial downturn) in the baseline case (using RF_t and FF_t). The red line corresponds to the estimates obtained with alternative data (Number of housing starts HS_t for the business cycle, House price $NAREIT_t$ (NAREIT Composite index) for the financial cycle)

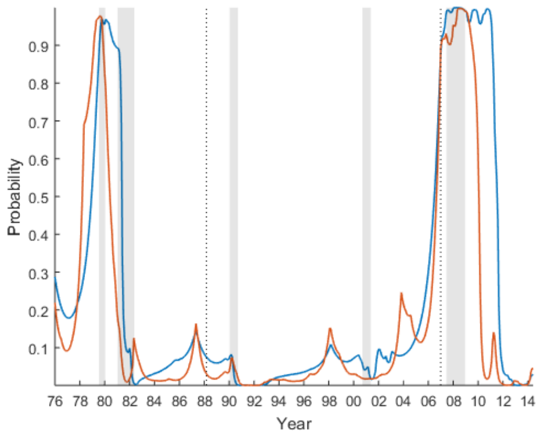
FIGURE C.3.2: Smoothed probability of high interaction regime



(A) Baseline case vs RC1 (RF + NAREIT)



(B) Baseline case vs RC2 (HS + FF)



(C) Baseline case vs RC3 (HS + NAREIT)

Note: Blue line corresponds to the estimate of the smoothed probability of "Interdependent cycle" regime in the baseline case (using RF_t and FF_t). The red line corresponds to the estimates obtained with alternative data (Number of housing starts HS_t for the business cycle, House price $NAREIT_t$ (NAREIT Composite index) for the financial cycle)

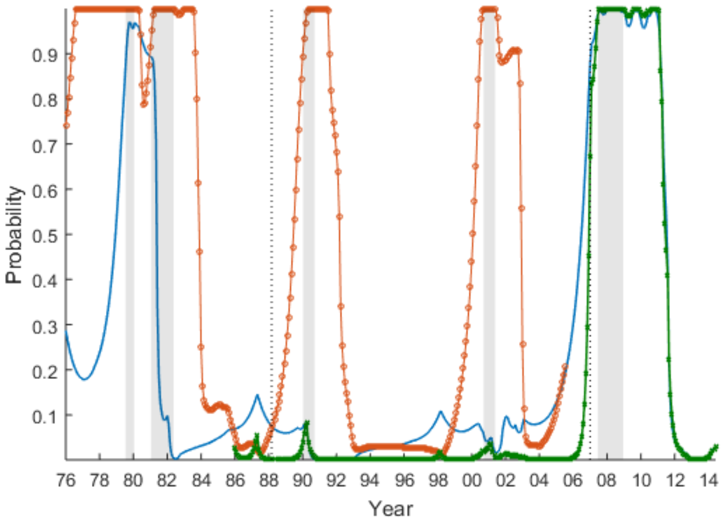
C.3.2 Estimation on subsets

TABLE C.4: DI-MS-FM estimates on alternative datasets

	Jul 1976-Dec 2014	Jul 1976-Aug 2006	Nov 1984-Dec 2014
Business cycle			
$\hat{\mu}_1$	0.57***	0.00***	0.00***
$\hat{\mu}_2$	-0.85***	-1.77***	-1.14***
$\hat{\sigma}_1^2$	0.31***	2.23**	5.74**
$\hat{\sigma}_2^2$	0.81***	0.30**	0.35**
$\hat{D}_{11}^1$	0.99***	0.99***	0.96***
$\hat{D}_{22}^1$	0.98***	0.87***	0.99***
$\hat{C}_{11}^{12}$	0.91***	0.62**	0.89***
$\hat{C}_{22}^{12}$	0.99*	0.13***	0.01***
Financial cycle			
$\hat{\beta}_1$	0.16***	0.37***	0.11***
$\hat{\beta}_2$	-0.86***	-0.52***	-1.20***
$\hat{\theta}_1^2$	0.27***	0.32***	0.31***
$\hat{\theta}_2^2$	4.05***	1.47***	5.15***
$\hat{D}_{11}^2$	0.99***	0.96***	0.52***
$\hat{D}_{22}^2$	0.88***	0.99***	0.99***
$\hat{C}_{11}^{21}$	0.75***	0.00***	0.00***
$\hat{C}_{22}^{21}$	0.99***	0.99***	0.00***
Influence regimes			
$\hat{R}^1$	$\begin{bmatrix} 0.98 & 0.06 \\ 0.02 & 0.94 \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$	$\begin{bmatrix} 0.95 & 0.16 \\ 0.05 & 0.84 \end{bmatrix}$
$\hat{R}^2$	$\begin{bmatrix} 0.86 & 0.81 \\ 0.14 & 0.19 \end{bmatrix}$	$\begin{bmatrix} 0.00 & 0.20 \\ 1.00 & 0.80 \end{bmatrix}$	$\begin{bmatrix} 0.79 & 1.00 \\ 0.21 & 0.00 \end{bmatrix}$
$\hat{q}_{11}$	0.99	0.98	0.99
$\hat{q}_{22}$	0.97	0.98	0.97

Note: The parameters significant at 15%, 10% and 5% are marked with *, ** and ***, correspondingly.

FIGURE C.3.3: Smoothed probability of "Interdependent cycles" regime



Note: Blue line corresponds to the smoothed probability "Interdependent cycle" regime in the baseline case (estimated on the whole period). Red and green lines correspond to the estimates obtained on the right and left subsamples (Nov 1984 - Dec 2014 and Jul 1976 - Aug 2006), respectively.

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Estratto per riassunto della tesi di dottorato

Studente: Anna Petronevich matricola: 956207

Dottorato: Ricerca in Economia

Ciclo: 29°

Titolo della tesi: Dynamic Factor Model with Non-linearities: Application to the Business Cycle Analysis

Abstract:

Questa tesi è dedicata allo studio di una classe particolare di Modelli a Fattori Dinamici non lineari, cioè i Modelli a Fattori Dinamici a cambiamento di regime di Markov (MS-DFM). Nel Capitolo 3, confronto le due tecniche di stima del MS-DFM, cioè il metodo ad una fase e quello a due fasi, e li applico al contesto francese per ottenere la cronologia dei punti di inversione del ciclo economico. Nel Capitolo 4, sulla base delle simulazioni Monte Carlo, studio la consistenza degli stimatori della tecnica selezionata - il metodo a due fasi - e analizzo il loro comportamento in piccoli campioni. Nel Capitolo 5, estendo la MS-DFM e suggerisco l'MS-DFM Dynamical Influence, che consente di valutare il contributo del settore finanziario alle dinamiche del ciclo economico e viceversa, tenendo conto che l'interazione tra questi può essere dinamica.

This thesis is dedicated to the study of a particular class of non-linear Dynamic Factor Models, the Dynamic Factor Models with Markov Switching (MS-DFM). In Chapter 3, I compare the two popular estimation techniques of the MS-DFM, the one-step and the two-step methods, and apply them to the French data to obtain the business cycle turning point chronology. In Chapter 4, on the basis of Monte Carlo simulations, I study the consistency of the estimators of the preferred technique - the two-step estimation method, and analyze their behavior in small samples. In Chapter 5, I extend the MS-DFM and suggest the Dynamical Influence MS-DFM, which allows to evaluate the contribution of the financial sector to the dynamics of the business cycle and vice versa, taking into consideration that the interaction between them can be dynamic.

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Firma dello studente