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IAG Company's Optimal Capital Structure reached through a Multi-objective Genetic Algorithm

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If you don't go after what you want, you will never have it. If you don't ask, the answer will always be no. If you don't step forward, you will always be in the same place.

N. R.

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SUMMARY

Introduction

When it comes to making decisions in a business, the general objective of Corporate Finance is the maximization of the firm's value. Therefore, to assess the performance of the company in general, managers and stockholders need evaluation techniques which allow to address also the potential conflict between them. In this respect, the predominant purpose of Corporate Governance is the attempt at *"balancing the interests of power between the different stakeholders by management control, regulation of market discipline and transparency and quality of corporate disclosure"*[1]. The maximization of the firm's value is usually reached by adjusting the principal components of the company's capital structure[2] with the final aim of optimizing it. Companies always try to operate in an optimum range of capital structure and *"if they have to be excluded from this optimized range due to business conditions, they will return as soon as possible"*[3]. In such context, indeed, most of the economics books describe the structure parameter as the most effective parameter for the evaluation of companies operating in Capital markets.

Regarding the study of an optimal capital structure, several theories have been provided both by researchers and financial managers and its assessment dates back in late 1950s with the Modigliani and Miller Theory (1958), among the others: Jensen and Meckling with the *Static Trade-Off Theory* (1976) and the *Agency Cost Theory* (1976), Myers with the *Pecking Order Theory of Financing Choice* (1984), Jensen with the *Free Cash Flow Hypothesis* (1986), Baum and Crosby with the *NOI* (Net Operating Income) *Approach* (1988), Mundy with *NI* (Net Income) *Approach* (1992), Baker and Wurgler with *Market Timing Theory* (2002) and more dynamic *Trade-off Models* such as the ones proposed by Brennan and Schwartz (1984).

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[1] ECB. (2015). *The evolving framework for corporate governance.* Monthly Bulletin. May 2005, pages 88-90.

[2] The Capital Structure is defined as the firms' combination of different securities (Debt, Equity and mixes of the two), used to finance their projects and, more generally, all their operations and growth.

[3] MUELLER, E., SPITZ, A. (2006). *Managerial ownership and company performance in German small and medium-sized private enterprises.* German Economic Review. 2, pages 2-18.

These various capital structure theories try to establish a relationship between the financial leverage of a company (i.e., the proportion of debt in the company's capital structure) with its market value. Although these theories provide different financing methods (behaving as it is shown in Table 0.1), the results are controversial, and no one seems, actually, to provide a real optimized model. Table 0.1 summarizes all the previously mentioned theories and, for each one, compares the relative effects on: Leverage (L), Cost of Capital (K), and Expected Return (R) with respect to the final Value (V) of the company under analysis. In the table the symbol ↑ stands for increase, while ↓ stands for decrease.

Theory / Approach	**Effect (1)**	**Effect (2)**	**Results**
Net Income Approach (NI)	L↑	K↓ P↑	V↑
Net Operating Income Approach (NOI)	L↑	K	V
Modigliani & Miller Theory (Non-debt tax shield)	L↑	K↑ R↑	V↓
Modigliani & Miller Theory (Debt tax shield)	L↑	K↓	V↓
Static Trade Off Theory	L↑	K↓ Financial Distress ↑	V↑

Pecking Order Theory	First internal sources, then external sources ↑	Endeavor to invest on positive net present value projects ↑	First benefit for present shareholders, then an opportunity for new investors ↑
Agency Cost Theory	Conflict of interest between management, shareholders and creditors ↑	-----	V↓
	Conflict of interest between management, shareholders and creditors ↓	-----	V↑
Free Cash Flow Hypothesis	L↑ Dividend ↑	Agency Cost ↓	V↑
Dynamic Trade Off Theory	Correct future forecasting ↑	-----	V↑
	Incorrect future forecasting ↑	-----	V↓
Market Timing Theory	Overvalue of shares ↑	Issuing new shares	V↑
	Undervalue of shares ↑	Buyback their shares	V↑

Table 0.1: *Theories and approaches of Capital Structure*

**Source:* AFRASABI, J., AHMADINIA, H., HESAMI, E. (2012). *A Comprehensive Review on Capital Structure Theories.* The Romanian Economic Journal. Vol XV(45).

Furthermore, since the determination of the optimal capital structure belongs to the family of the prescriptive theories (we are looking for a target debt ratio in order to find the optimal mix of debt and equity that is applicable in the real world) and since it is based on the partial equilibrium[4], other researchers tried to develop new theories and/or to assess the target debt-equity ratio through new determinants.

Due to the deficiencies of traditional methods, as highlighted by the controversial results in the above Table, this thesis aims at providing a model that works on a proper mix of debt and equity such that it is possible to reach the optimal capital structure with some pre-determined specific objectives. Indeed, the proposed model is drawn considering the profitability maximization (from the equity holder's point of view), meanwhile keeping a proper level of debt repayment ability, in order to better balance the interests of the company's shareholders and the ones of the debt-holders.

In investment analysis the use of accounting measures of return, such as Return on Equity (ROE) or Return on Capital (ROC), still continue to prevail (partially because of their intuitive appeal for both investors and analysts, and partially because financial managers are reluctant to abandon familiar measures). However, here, the model is implemented through the use of the Genetic Algorithms (GAs) specific research technique, which is a trial-and-error stochastic search optimization algorithm used to solve complex optimization problems. In other words, GAs is a method for optimization and it can utilize as variables these kind of accounting measures. The choice of this specific search-metaheuristic, inspired by Charles Darwin's theory of natural evolution, comes from various reasons such as its ease of implementation (GA indeed provides a problem-independent method for solution searching), its equilibrium between exploration and exploitation[5] (achieved by the proper setting of the parameters) and, in addition, because it is one of the pioneer evolutionary algorithms and uses a mathematical and logical reasoning which allows GA to be

[4] Partial equilibrium is a condition that takes into account the impact only of a part of the market variables to reach equilibrium, ignoring therefore secondary variables (i.e., the ones which are assumed to have a small or null impact in any other market).

[5] Exploration focuses on the research of current good solutions in a local region, instead the exploitation generates different solutions in order to explore the search space on a global scale.

applied to different types of optimization problems in any field. Furthermore, Genetic Algorithm has stood out for its strong robustness and good convergence[6].

Corporate financial theory is primarily focused on stock price maximization, mainly because stock prices are constantly updated perceivable measures that could be used to evaluate the performances of firms and because, if investors were rational and markets efficient, stock price would reflect the long-term effects of the firms' choices (regarding for example the pick-up of specific projects or the way in which these projects are financed). However, are we sure that the aggregation of rational individuals creates a "rational" environment? Every economics' concept starts from the assumption that the economic agents behave rationally. Interestingly, a singular agent behaviour can be defined as rational, even if sometimes he/she takes some non-rational decision. But what happens if many agents take non-optimal and non-fully utility-maximizing decisions simultaneously? More often than we might think, this happens in the financial markets.

If we want to start from the assumption of markets following non-predictable movements (random patterns), an interesting suitable approach could be the implementation of a Genetic Algorithm model, since it is an optimization algorithm that starts from a batch of random solutions as well. The algorithm firstly finds the answer between random potential solutions (called *chromosomes*) through the basic operations of *selection, crossover* and *mutation*, then gradually improves the fitness function of the offspring[7], which gradually gets better than the one of the parents, and ultimately gets the optimal solution for the problem. The Genetic Algorithm stem, indeed, from the Darwinian idea of the *"survival of the fittest"* (natural system which follows the natural pattern of growth and reproduction).

[6] For more information, see: IYER, K. C., SAGHEER, M. (2012). *Optimization of bid-winning potential and capital structure for build-operate-transfer road projects in India.* Journal of Management in Engineering. 28(2), pages 104–113.

[7] The fitness function represents the score assigned to each point (i.e., solution) of the offspring generation, that is the transformation (through the *selection, crossover* and *mutation* operators) of the old generation into the new one.

This application of the GA in order to find the optimal capital structure will be performed to the specific case study of *International Consolidated Airlines Group S.A.* (aka *IAG*). IAG is a publicly traded holding company born out in 2011 from the merge of the UK's largest airlines company *British Airways* (aka *BA*) and the Spanish airlines company *Iberia,* joined later also by the Spanish company *Vueling Airlines* in 2013. IAG Company provides transportation services, offering both domestic and international air passenger and cargo transportations.

CHAPTER 1

Firms' Capital Structure

The Capital Structure of a company represents the way the company finances its assets. In this first chapter, the commonly pursued objective of firms is introduced and explained through the estimation of the Cost of Capital. The Cost of Capital is then split and analysed across all of its components.

1.1 The classic businesses' objective

In the corporate finance field, the classical objective among firms is the *value maximization*. There are although some disagreements whether this maximization should be meant from the point of view of the stockholders or from the point of view of the firm (which includes not only the stockholders but all stakeholders and the other financial claim holders like debt holders who bought some bonds issued by the company). The optimal or target capital structure is the one which simultaneously maximizes the firm value, minimizes the weighted average cost of capital and maximizes the market value of its stock. The goal of managing capital structure is maximizing the value of the firm.

Since the decisions in a business are generally taken by managers, rather than the owners, they will decide the way in which raising new funds for investments and, therefore, where to invest. Here comes the point. If the objective is stated in terms of stock price maximization, the managers will take the choice among different alternatives and the way to pick projects analysing which one will increase the stock price more.

The main reasons to choose as objective solely the stock price maximization are:

- They reflect the long-term effects of the choices taken by the firm, since they are a function not only of current operations (which can be analysed simply looking at the financial statement or other accounting measures) but also of the future chances of success and stability;
- They are a clear target for mangers and they could be seen as a proxy of the performance of publicly traded firms;
- They are constantly updated and always observable. Looking at their trend, managers could extrapolate the feelings of the investors regarding their choices. In other words, they can be seen as investors' feedbacks.

Focusing solely on the narrower objective of stock price maximization, with respect to the firm value maximization, we are implicitly assuming that stock prices are reasonable and unbiased estimates of the real value of that firm.

Indeed, the primary goal of financial managers is the maximization of stockholders' wealth, and this is reached by maximizing the value of the firm (or, equivalently, minimizing the WACC). Why is it equivalent to talk about maximizing the value of the firm and minimizing the Weighted Average Cost of Capital? Since the WACC is considered the most appropriate discount rate for the risk of the firm's assets, we can use it to get the firm's value by discounting its expected future cash flows. Firm value will be therefore maximized when the WACC is minimized, since value and discount rates move in opposite directions.

1.2 Estimation of the Cost of Capital

The Cost of Capital is the cost of a company's funds and it is commonly defined as the minimum required rate of return (also known as *hurdle rate*) that a firm must earn on its investments (assets) to satisfy its owners, the creditors and the other providers of capital, or they will invest elsewhere. Indeed, investors (who provide capital to the companies) consider an investment worthwhile if the expected Return on Capital (RoC) is higher than the Cost of Capital.

This concept is a basic input information in capital investment decision and its importance encompasses different managerial decisions such as:

- Capital Budgeting Decisions (the firm invests in projects that provides a satisfactory return, at least greater than the Cost of Capital of that project. In other words, there is a settlement of a benchmark that a new project has to meet)
- Corporate Financial Structure Design (managers need to change the methods of financing in order to increase the market price and the EPS – Earnings per Share. To maximize its value, the firm should minimize the cost of all its inputs)
- Management Performance Measures (financial performances could be evaluated through a comparison of the profitability of the projects with the planned overall cost of capital)
- Others (dividend decisions, working capital policy, bond refunding etc.)

In addition, the Weighted Average Cost of Capital (WACC) can be used by investors to choose the best corporate-investments and to calculate Discounted Cash Flow (DCF) valuation of companies. Indeed, *"the most widely used technique for financial* *evaluation is discounting the cash flow by weighted average cost of capital both in literature and practice"*[8].

Aswath Damodaran, professor of Corporate Finance and Valuation at the Stern School of Business at New York University, wrote in his book *Applied Corporate Finance*: *"The cost of capital is the weighted average of the costs of the different components of financing (equity, debt and hybrid securities) used by a firm to fund its financial requirements"*[9]*.* Therefore, to compute the firm's Weighted Average Cost of Capital, one has to estimate the costs of individual financing sources and the market value weights of each of the components. Companies try to keep the share of the sources of financing in optimal proportions. The formula can be written as follows:

$$WACC = K_E * \left[\frac{E}{D+E+PS}\right] + K_D * \left[\frac{D}{D+E+PS}\right] + K_{PS} * \left[\frac{PS}{D+E+PS}\right]$$

where:
E, D, PS stand for Equity, Debt and Preferred Stock, respectively (book values);
K_E, K_D, K_{PS} stand for Cost of Equity, Cost of Debt and Cost of Preferred Stock, respectively.

Affecting the Cost of Capital there are, however, some factors over which the companies have no control. These are, for example, the level of interest rates (which affect the Cost of Debt and, potentially, the Cost of Equity) and the tax rates (which affect the after-tax Cost of Debt). Companies will work therefore on the maintenance of the share of the individual components (in other words the sources of financing) in optimal proportions.

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[8] BABUSIAUX, D., PIERRU, A. (2001). *Capital Budgeting, Investment Project Valuation and Financing Mix: Methodological Proposals*. European Journal of Operational Research. 135, pages 325-337.

[9] DAMODARAN, A. (2015). *Applied Corporate Finance*, Chapter 4, Wiley (Fourth edition).

1.3 Cost of Equity

Generalizing, we can say that there exist two types of risk in the corporate finance field. The first, which is addressed in this section, is called equity risk and regards all the investments in which there are no promised cash flows (but rather, just expected ones). The second one, instead, regards investments with promised cash flows and will be addressed in the next section.

The Cost of Equity is the rate of return that investors require to invest in the equity of a firm (i.e. the compensation for bearing the risk they undertake by investing their capital and owning the asset). The computation of the cost of equity can be performed through different techniques which require some specifications (key variables): the risk-free rate, the risk premium, as well as the risk parameters. Here, the Cost of Equity is computed through the CAPM (Capital Asset Pricing Model) formulation. The Capital Asset Pricing Model assumes that investors are rational and risk-averse, that all assets are available for trading, that there are no transaction costs, that each investor can buy any fraction of these assets (in other words, the assets are perfectly divisible), that investors are price takers (which means they can not influence prices), that they can lend and borrow at the risk-free rate, and that there is no private information. In the CAPM, investors reflect their risk aversion simply by adjusting the proportions of their investments in the market portfolio (i.e., the portfolio which includes all the available risky assets) and in the riskless asset (i.e., an asset with guaranteed returns in a specified time horizon). Furthermore, two additional assumption in the CAPM framework are the existence of a risk-free asset and the possibility for investors to lend and borrow at a risk-free rate in order to get to their optimal allocations.

1.3.1 Risk-free assets

An asset must satisfy two requirements to be considered risk-free:

1. It must have not default risk (i.e., it has to be issued by a government);
2. There should not be uncertainty regarding reinvestment rate (i.e., there are not intermediate cash flows).

To meet such conditions, usually the risk-free rate is the expected return on a zero-coupon bond issued by a government whose sovereign rating is Aaa (on the Moody's rating scale or, equivalently, an AAA on the S&Poor's scale).
If one is analysing a company located in countries where the possibility of default is not so far remote (their governments could default on local currency debt), it is important to state that, in order to get to the real risk-free rate, we should adjust their government long-term bond rate by the estimated default spread (again looking at the local currency sovereign ratings).

1.3.2 Risk Premiums

What do we mean, in the CAPM framework, by saying that investors adjust for their risk preferences? We are talking about the risk premium, which is a measure of the extra return required by investors to change the proportions invested in the risk-free asset shifting to the market portfolio and it can be defined as a function of two variables, the risk aversion of the investors (they require higher premiums as they become more risk-averse) and the riskiness of the average risk investment (investors require higher premiums if the riskiness of the average risk investment increases). Since both the two just mentioned variables are subjective, the equity risk premium (ERP) is a weighted average of individual premiums, weighted on the basis of the wealth each investor brings to the market.

These Risk Premiums could usually be estimated through three potential approaches:

1. Survey Premiums (the most influent investors are surveyed about their expectations for the future);
2. Historical Premiums (computed through historical data as $average\ return\ on\ stocks - average\ return\ on\ riskless\ asset$);
3. Implied Premiums (the required return on equity is obtained from the current level of a stock on the market and it is adjusted by subtracting the risk-free rate).

The two clearest drawbacks of the first approach are the fact that survey premiums are too much volatile and reactive to market movements and the fact that they tend to be short term.

One drawback of the second approach is that Historical Risk Premiums can differ among different computations because of different choices on the time period to use, on the risk-free securities (as riskless rate) and on the use of arithmetic (simple mean) as opposed to geometric averages when computing returns (the geometric average considers the compounded returns).

The Implied Risk Premium approach assumes that the stock market is correctly priced. It is market-driven (in other words it changes in response to changes in the market conditions) and forward-looking. Another advantage is that it does not require any historical data. If we start from the simple formula (here below) used for the evaluation of stocks, we notice that the only unknown variable is the required return on equity and so, solving for it, we get an implied expected return on stocks.

$$Value = \frac{Expected\ dividends\ of\ next\ period}{(Required\ return\ on\ equity - Expected\ growth\ rate\ in\ dividends)}$$

where *Value* is meant as the current level of the market price and, as well as for the *expected dividends* and the *expected growth rate in dividends*, it does not need to be estimated because it's a known amount.

This formula can be seen also as the present value of dividends growing at a constant rate. Starting from this formula and subtracting then the riskless rate, we arrive at an implied equity risk premium.

$$Implied\ Equity\ Risk\ Premium = Required\ return\ on\ equity - Risk\ free\ rate$$

The three approaches just presented yield different estimates due to several reasons and, consequently, one should decide which one to use.

In order to choose, the main aspects to take into consideration are the *predictive power*, the *beliefs about the market* (in the sense of suspicion whether the market is undervalued or overvalued) and the *purpose of the analysis*. Following the results of an analysis[10] carried out by the professor Aswath Damodaran on these different approaches (except for the survey premium since it was not possible to get data so far in time) from 1960 to 2012, it results that the best predictor for the premiums of the next periods was the implied equity risk premium. With regards to the *beliefs about the market* the choice depends whether one believes markets are efficient or not: in the first case the current implied ERP would be the best choice. Instead, if one believes that markets are erroneous, the historical ERP or at least an average implied ERP become a better choice, while if one considers the markets totally unpredictable and has no faith at all, the choice will fall on the survey premium. With regards to the purpose of the analysis, if one is, for instance, interested in acquisition valuations, the use of the current implied equity risk premium is suggested, while instead in corporate finance (where the objective is to arrive at the cost of capital in order to plan and decide about the firm's long-term investments) the suggested choice should fall on long-term average historical premium.

[10] For more information, see: DAMODARAN, A. (2015). *Applied Corporate Finance*. Wiley (Fourth edition). Pages: 104-106.

1.3.3 The Beta risk measure

Now, a question arises: how can, instead, be measured the risk[11] of each asset? For an investor such risk is the risk added on by the single asset to his/her portfolio and it is statistically measured by the covariance of the return of this asset with the return of market portfolio. To get the beta of the asset the risk measure should be standardized by dividing the covariance by the variance of the market portfolio.

$$\beta_i = \frac{Cov(i, M)}{Var_M}$$

where:

β_i is the beta of the asset *i;*

$Cov(i, M)$ is the covariance of the asset *i* with the market portfolio;

Var_M is the variance of the market portfolio.

The *beta* is the only firm-specific input in the CAPM equation for the expected return on an asset.

As for the Risk Premium, there are three potential approaches to estimate the beta:

1. Historical Market Beta (which implies to use historical market prices for each asset);
2. Fundamental Beta (which implies to use fundamental decisions the firms undertake on which type of business to enter and on the level of operating leverage to keep);

[11] Risk is usually addressed with the Greek letter *beta (β)* and will therefore be used as risk measure.

3. Accounting Beta (which implies to use accounting earnings of the companies).

1.3.3.1 Historical Market Beta Approach

With regards to the first approach (Historical Market Beta) there are two ways (two different regressions) to estimate the beta. For both the two, the slope (*beta*) of the regression represents the riskiness of the asset, while the intercept (*alpha*) measures the stock price-performance (relative to CAPM expectations), where a positive value means that, during the timeframe considered by the regression, the stock outperformed with respect to expectations, and vice versa for a negative value.

⇒ The first way is to regress the excess return of the investment (with respect to the risk-free rate) against the excess return on the market. The formula is the following:

$$\left(R_i - R_f\right) = \alpha + \beta_i(R_m - R_f)$$

where:
R_i is the expected return on asset *i*;
R_f is the risk-free rate;
R_m is the expected return on the market portfolio;
β_i is the beta of that individual investment;
α is the Jensen's alpha (which measures if the asset performed better or worse than the market).

In this excess return regression, a positive (or, respectively, negative) Jensen's alpha means that the asset outperformed (underperformed) the expectations. In the situation of exact return-prediction by the CAPM formula, the Jensen's alpha will be zero.

⇒ The second way to estimate the beta, following the Historical Market Beta approach, is to regress the raw return on the investment (stated in other terms, the return not adjusted for the risk-free rate) against the raw return on the market. The formula is the following:

$$R_i = \alpha + \beta_i * R_m$$

In this raw return regression, the Jensen's alpha should be compared to the predicted intercept:

- If $\alpha > R_f(1-\beta)$ then the asset performed better than expected;
- If $\alpha < R_f(1-\beta)$ then the asset performed worse than expected;
- If $\alpha = R_f(1-\beta)$ then the asset performed as well as the prediction.

When using both these kinds of regressions, there are three aspects to take into consideration:

1. Length of the estimation period (the longer it is, the more observation we will have, but we need to keep in mind that during this time span the risk characteristics might have changed);
2. Return interval (daily, weekly, monthly, etc.);
3. Choice of the market index to use in the regression.

1.3.3.2 Fundamental Beta Approach

With regards to the second approach for the beta estimation (Fundamental Beta approach), it is important to underline which are the determinants for this beta:

- Type of business (the more sensitive a business is to market conditions, the higher its beta in absolute terms);
- Degree of operating leverage, which is meant as relationship between fixed and total costs (the higher the operating leverage and hence the proportion of fixed costs with respect to total costs, the higher the variance in operating income. High degree of operating leverage translates into a high beta). This beta's determinant is approximated as:

$$Degree\ of\ operating\ leverage = \frac{\%\ Change\ in\ operating\ income}{\%\ Change\ in\ sales}$$

  where the $\%\ Change\ in\ operating\ income$ is usually measured by the EBIT (Earnings before interest and taxes);
- Degree of financial leverage (a high leverage means high variance in earnings per share and, therefore, high risk in equity investments in the firm. This translates in a high beta).

1.3.3.3 Accounting Beta Approach

The third approach (Accounting Beta approach) implies the use of the regression of the changes in a firm's earnings against the changes in earnings for the whole market to get an estimate of the *market beta* for the CAPM formula. Nonetheless, such an approach presents some drawbacks. Firstly, earnings can be affected by the different accounting choices. Secondly, the regressions have often not enough observations since the accounting measures are usually measured just once per year. Lastly, the accounting earning, compared to the underlying value of the firm, are usually smoothed out too much.

1.3.4 Cost of Equity Formulation

Recapitulating, now that we have analysed all the determinants, we can get the CAPM formula for the Cost of Equity which is, as already stated, simply the expected return from investing in the equity of the firm or, stated in other terms, the rate they require in order to be compensated for the risk assumed for investing in the equity of the firm. From mangers' perspective, the Cost of Equity can be defined as the return they should manage to reach in order to satisfy the investors.

$$Expected\ return = Risk\ free\ rate + Beta * ERP$$

When estimating the Cost of Equity using the beta in the CAPM formula as measure of risk, we implicitly assume that the marginal investor[12] is a well-diversified investor. Nevertheless, in private firms we can not make this assumption (since usually the owner of a private firms invests the majority of his/her wealth in his own company) and therefore it is suggested either to add a premium to the Cost of Equity to reflect the higher risk (given the fact that the investor most probably lacks the possibility to diversify), either to adjust the beta in order to reflect the total risk (instead of the market risk only) simply by dividing the market beta by the square root of the R-

[12] The marginal investor is an investor who owns a large portion of the equity and trades it frequently and is considered, therefore, to be the investor in the firm who will be the buyer/seller on the upcoming trade.

squared statistic, which is a statistic measure of the goodness of fit of the regression but in this specific economic framework it represents an estimate of the proportion of the risk that could be imputed to market risk[13].

Summarizing, the Return on Equity depends both on business and on financial risk. We refer to business risk (inherent in the operations of the firm) when it is a risk that depends on the systematic risk of the assets, while we refer to financial risk when it is an extra risk to stockholders which results from debt financing and so when it depends on the level of leverage (Debt/Equity ratio).

There exist, however, other models to calculate the Cost of Equity (depending on the type of Cost of Equity one wants to consider). One almost equally viable alternative to the Capital Asset Pricing Model could be represented by the Dividend Capitalization Model[14], which estimates a future dividend stream based on the firm's dividend history (assuming a constant growth rate) looking for the market capitalization rate that match the current market price. To accomplish this, the Dividend Capitalization Model is based on the following formula:

$$R_e = \frac{D_1}{P_0} + g$$

where:

R_e is the Cost of Equity;

D_1 is the Dividend per share of next period;

P_0 is the current share price;

g is the expected dividend growth rate.

[13] Hence, in such an economic framework the $(1 - R^2)$ can be seen as the firm-specific risk.

[14] Also called Gordon Growth Model (GGM) from its author Myron J. Gordon who published it along with Eli Shapiro n 1956. To deepen: Gordon, M. J., Eli Shapiro (1956), "*Capital Equipment Analysis: The Required Rate of Profit",* Free Press, Management Science, 3(1), 102-110.

Since this model is applicable in case of payments of dividends on the shares and assuming that they will grow at a constant rate, in the case of private firms without dividends' distribution, the firm's ability of apportion of profits through dividends is assessed looking at the Net Income and cash flows and then compared to the dividends paid out by firms of analogous dimensions.

Anyway, for the analysis that we are going to perform in the next chapters we will use the CAPM rather than this Dividend Capitalization Model.

Even if in investment analysis what is commonly used as hurdle rate is the Cost of Capital, there exist situations in which the use of the Cost of Equity could be more suitable. For example, if investors want to measure the returns made on their equity investments (in other words in projects or the entire business of a company) the most appropriate hurdle rate to consider is the Cost of Equity.

1.4 Cost of Debt

The idea behind comes from the possibility that, when an investor lends to a firm, there exists the likelihood that the borrower could default on the principal and the interests of the loan. In such investments (investments with default risk), the risk is indeed represented by the likelihood that the promised[15] cash flows might not be delivered. Since nothing is to be taken for sure, we should talk about expected return. The expected return on bonds issued by companies is meant to be the reflection of its firm-specific default risk. The current cost to the firm of borrowing funds to finance the projects is commonly known as Cost of Debt. It is a model based on the default risk and it depends on:

[15] It is called *promised* because investing in bonds issued by a company means that the coupons are fixed at the time of the issue. These coupons are the promised cash flows.

- Default risk[16] of the firm (if it increases, lenders will charge higher interest rates to reflect the new further risk they are undertaking);
- Current level of interest rates (investments with higher default risk should have higher interest rates and if interest rates rise, the Cost of Debt rises as well. If interest rates decrease, the Cost of Debt decreases as well);
- Tax advantage (since interest is tax-deductible, as tax rates increase, the after-tax Cost of Debt will be lower than the pre-tax Cost of Debt)

$$After\ tax\ Cost\ of\ Debt = \\ (Risk\ free\ rate + Default\ spread) * (1 - Marginal\ tax\ rate)$$

where the default spread is a representation of the default risk of the firm and it is exactly the premium investors demand over the risk-free rate.

Looking at it from a different perspective, we can say also that borrowers with higher default risk should pay higher interest rates on their borrowing than those with lower default risk. With respect to the risk and return models (used in the Cost of Equity) which assess the effects of the market risk on expected returns, models of default risk gauge the effects of individual firms' default risk on pledged returns.

Rating agencies, using a mix of both public and private information (mainly financial ratios), transform these assessments into measures of default (under the name of *bond ratings*), which could be considered by investors as a shorthand measure of default risk. The two most known and reliable rating agencies are *Standard & Poor's* and *Moody's*. In Table 1.1 below it is depicted the way in which they assign these bond ratings.

[16] The default risk is a function of different elements. Firstly, it is function of the firm's capacity to generate stable cash flows from operations and financial obligations. Secondly, it is function of its assets' liquidity since it would become easier to liquidate them in crises times when there is the need to meet debt obligations.

INDEX OF BOND RATINGS			
STANDARD & POOR'S		**MOODY'S**	
AAA	Highest debt rating assigned. The borrower's capacity to repay debt is extremely strong	Aaa	Best quality with a small degree of risk
AA	Capacity to repay is strong and differs from the highest quality only by a small amount	Aa	High quality but rather than Aaa because margin of protection may not be as large or because there may be other elements of long-term risk
A	Strong capacity to repay but the borrower may be susceptible to adverse effects of changes in circumstances and economic conditions	A	Bonds possess favorable investment attributes but may be susceptible to risk in future
BBB	Adequate capacity to repay, but adverse economic conditions or circumstances are more likely to lead to risk	Baa	Neither highly protected nor poorly secured. Adequate payment capacity
BB, B, CCC, CC	Predominantly speculative (BB the least speculative, CC the most)	Ba	Bear some speculative risk.
		B	Generally lacking features of a desirable investment, probability of payment small
D	In default or with payments in arrears	Caa	Poor standing and perhaps in default
		Ca	Very speculative; often in default
		C	Highly speculative; in default

Table 1.1: *Index of Bond Ratings*

Ratings range therefore between AAA (or Aaa for *Moody's*) to D (or, equivalently, C for *Moody's*), but we can make a division between Investment Grade and Junk Bonds. Bonds with a rating above BBB (or Baa for *Moody's*) are defined as *investment grade*, which means there is a very low likelihood of default. On the other hand, bonds with a rating below that are called *junk-bonds* or high-yield bonds (since they should promise high yields, given the risk investors bear lending to the companies that issued them).
Some examples of the financial ratios utilized by the rating agencies to determine whether the companies are able to meet debt obligations and whether they have stable positive cash flows are: *Pre-tax Interest Coverage Ratio*, *EBITDA Interest Coverage Ratio*, *Free Operating Cash Flow over Total Debt*, *Operating Income over Sales*, *Total Debt over Capitalization*. Nevertheless, rating agencies do not rely solely on these financial ratios when assigning grades to a company, but they rather consider also expectations in future performances (which are kind of subjective evaluations).

Furthermore, the default risk determines the level of the interest rate on corporate bonds (high rated bonds should yield lower interest rates with respect to lower rated ones). The default spread is function of the interest rates in the sense that it is computed as the difference between the interest rate on a corporate bond (bearing some kind of default risk) and a default-free government bond. This is displayed in Table 1.2.
The default spread is itself function of the bond's maturity (showing evidence that short-term default risk is greater than long-term default-risk) and of economic conditions (revealing that default spreads increase during economic slowdowns). This implies a drawback: default spreads for bonds must be re-evaluated quite often.

Rating is:	***Spread 2018***	***Spread 2017***	***Spread 2016***	***Spread 2015***	***Interest rate on bond***
Aaa/AAA	0,54%	0,60%	0,75%	0,40%	2,95%
Aa2/AA	0,72%	0,80%	1,00%	0,70%	
A1/A+	0,90%	1,00%	1,10%	0,90%	

A2/A	0,99%	1,10%	1,25%	1,00%	3,34%
A3/A-	1,13%	1,25%	1,75%	1,20%	
Baa2/BBB	1,27%	1,60%	2,25%	1,75%	3,68%
Ba1/BB+	1,98%	2,50%	3,25%	2,75%	
Ba2/BB	2,38%	3,00%	4,25%	3,25%	4,33%
B1/B+	2,98%	3,75%	5,50%	4,00%	
B2/B	3,57%	4,50%	6,50%	5,00%	5,82%
B3/B-	4,37%	5,50%	7,50%	6,00%	8,29%
Caa/CCC	8,64%	6,50%	9,00%	7,00%	
Ca2/CC	10,63%	8,00%	12,00%	8,00%	10,63%
C2/C	13,95%	10,50%	16,00%	10,00%	
D2/D	18,60%	14,00%	20,00%	12,00%	

Table 1.2: *Default Spreads for Rating Classes*

Source: http://www.bondsonline.com (NYU Stern University – Datasets – Bond spreads)

Coming back to the estimation of the Cost of Debt, it is extremely important to underline that it should be based on actual market interest rates and not on book interest rates[17] since we are investigating whether the projects under analysis earn more than alternative investments of equivalent risk and since the Cost of Debt is not the rate at which the firm was able to borrow at in the past.

In the situation where a company issues long-term bonds[18] which are liquid and frequently traded (it happens usually with big companies with large capitalization), the Cost of Debt can be estimated through the market price of these bonds adjusted for their coupons and maturity. Indeed, the expected return on corporate bonds displays the firm-specific default risk of the company that issued the bonds. In the case in

[17] Book interest rates are also called *coupon rates* and are the rates that are fixed at the time of the bonds issue from the company.

[18] We are referring to long-term bonds because we want that the rate reflects the cost of long-term borrowing since this is the hurdle rate investors want for their long-term investments to overcome.

which a rated company issues long-term bonds but they are not frequently traded, the Cost of Debt can be estimated using the firm's rating and its default spread.
The situation becomes a little bit more difficult if the company is not rated. In this case, we can look at the recent borrowing history to get the default spreads charged (using the inverse of the formula for the pre-tax Cost of Debt), or we can estimate a synthetic rating through the so-called *interest coverage ratio* (the operating income over the interest expense) even though we incur in some risks using just this ratio. The drawbacks are that we may miss some important information that is not included in it and the fact that the estimation can be biased considering only the operating income of last year. Although, the analysis can be improved, and these drawbacks overcome if we compute the interest coverage ratio over a sufficient long period of time and if we include additional financial ratios.
The pre-tax cost of debt can be computed as:

$$Pre\ tax\ Cost\ of\ Debt = Long\ term\ riskless\ rate + Default\ spread$$

Anyway, since the most important feature of the debt is the so-called *tax shield* (which is nothing more than the already mentioned advantage of the tax deductibility) we should be aware that most of the time what is reported as *Debt* (both short-term and long-term borrowings must be considered) in the balance sheet may not realistically represent the true borrowings.
In the corporate financial analysis framework Aswath Damodaran suggests that *"we should treat all lease payments as financial expenses and convert future lease commitments into debt by discounting them back to the present, using the current pre-tax cost of borrowing for the firm as the discount rate. The resulting present value can be considered the debt value of operating leases and can be added on to the value of conventional debt to arrive at a total debt figure".*[19]

[19] DAMODARAN, A. (2015). *Applied Corporate Finance.* Wiley, Fourth edition. Chapter. 4, page 140.

Regarding the *tax shield*, one should bear in mind that, in order to exploit this advantage, a firm must be profitable (in other words it should have not operating losses) and that this advantage needs to be computed via the marginal tax rate since interest expenses offset the marginal dollar of income.

$$After\ tax\ Cost\ of\ Debt = Pre\ tax\ Cost\ of\ Debt\ (1 - Marginal\ tax\ rate)$$

The formula here above is valid only if the operating income is greater than zero, otherwise the pre-tax and after-tax Cost of Debt are exactly equivalent.

1.5 Hybrid Securities

Pursuing the optimal proportion of the financing components, one should consider also the so called *"hybrid securities"* in the WACC computation. The term *"hybrid"* means that some sources of funds present some specific features of equity and some others of debt. The simplest way to address the problem of the calculation of the cost of such securities is to divide them into the equity and debt components, and, then, compute the cost of each component independently.

The most known hybrid securities are *convertible bonds* and *preferred stocks*. A convertible bond is a mix of a straight bond (debt part) and of a conversion option (equity part).

If the hybrid security is publicity traded, Aswath Damodaran suggests for its computation two alternatives, both straightforward:

1. Evaluate the convertible bond as if it were a straight bond (taking as interest rate on it the pre-tax cost of debt) and compute the value of the conversion

option simply as the difference between the price of the convertible bond and the value of the straight bond;

2. Evaluate the conversion option through an option-pricing model and associate the leftover value of the bond as debt component.

A preferred stock has elements common to debt (the preferred dividends, which are disbursed before common dividends and are pre-determined fixed amounts) and to equity (the payments of preferred dividends are not tax-deductible). The shareholder of this kind of security are not the owner of the firm. In order to use this source of fund (sometimes named also as *preference share capital*), the firm has to pay out dividends at a fixed rate.

Assuming that the preferred stock is not associated with special characteristic (like, for example, callability or convertibility) and that the dividend will be forever constant in the currency of issue terms, the computation of the Cost of Preferred Cost can be achieved through the following formula:

$$K_{ps} = \frac{Preferred\ dividend\ per\ share}{Market\ price\ per\ preferred\ share}$$

1.6 Respective components' weighting

The WACC, as already stated, is the Weighted Average Cost of Capital and is called like that because each category of capital is proportionally weighted with respect to the firm's total capital. Therefore, once the costs for each component (equity and debt) have been computed, what we have to do is to assign the weights on each of them. The alternatives are to choose book value weights or to estimate their relative market values. Usually the second option is preferable since market value weights reflect the forward-looking feature of the cost of capital and also because the raise of new capital is carried out using the actual prices on the market. The market value of equity and debt are computed as:

$$Market\,Value\,of\,Equity = Number\,of\,\,shares\,outstanding * Current\,stock\,price$$

$$Market\,Value\,of\,Debt = Book\,Value\,of\,Debt$$

The market value of debt is equivalent to its book value since usually debt is not traded under the form of bonds in the market. However, this is commonly acceptable only for mature firms in developed markets. If this is not the case, it is possible to convert book value debt into market value considering the entire debt as a coupon bond, whose coupon is set equal to the interest expenses on all of the debt and whose maturity is set equal to the face value weighted average maturity of the debt.
Regarding equity, the market values of all types of shares outstanding (including also non-traded shares or particular types of equity claims such as conversion options or warrants) have to be aggregated and estimated.

Once all the market value weights (relative to each component) have been determined, together with their costs, the Cost of Capital can be computed. For simplicity, the formula is reported again here below.

$$WACC = K_E * \left[\frac{E}{D+E+PS}\right] + K_D * \left[\frac{D}{D+E+PS}\right] + K_{PS} * \left[\frac{PS}{D+E+PS}\right]$$

As we discussed in the Cost of Equity section, the correct hurdle rate to consider during investment analysis could be either the Cost of Equity or the Cost of Capital, depending on the perspective one wants to adopt.

From the perspective of whom wants to measure the composite returns to all claimholders (therefore the ones based on the earnings prior to payments of debt-holders and preferred-stockholders), the most appropriate hurdle rate to consider is the Cost of Capital.

CHAPTER 2

Genetic Algorithms

Genetic Algorithms (GAs) is a population-based evolutionary metaheuristic, usually applied to solve global unconstrained optimization problems.

2.1 Optimization background

The strive for efficiency belongs to the main areas of human interest. In computer sciences *efficiency* translates in the attempt of programming computers in order to compute algorithms and complete in a faster way the tasks, which involves also less power (in terms of energy) needed. This *efficiency*-search is generally pursued through the *optimization* which can be described as the research process to get to the best solution among all the available ones. Mentioning optimization, in this thesis we refer to minimizing or maximizing some functions relative to some set, often representing a range of choices available in a certain situation. The function allows comparisons of the different choices for determining what might be the best solution. Here we want to analyse a *finite-dimensional optimization problem,* where the choice of the values is among a finite number of real variables, named *decision variables*. Referring to optimization techniques, under this thesis' interest (branch of the numeric and approximated methods), we can define optimization as *"fine-tuning the inputs of a process, function or device to find the maximum or minimum output(s). The inputs are the variables, the process or function is called objective-function, cost function or fitness-value (function) and the output(s) is fitness or cost"*[20].

[20] HAUPT, R. L., S. E., and WILEY, A. J. (2004). *Algorithms: Practical Genetic Algorithms*. John Wiley & Sons, Inc., Hoboken, New Jersey.

Kalyanmoy Deb, in his book *"Multi-objective Optimization Using Evolutionary Algorithms"*[21], defines the evolutionary algorithms as methods that start from a bunch (i.e., population) of random solutions, which are then updated at each iteration. Belonging to such evolutionary methods we have the Genetic Algorithms. However, before examining more in depth the GAs, it is necessary to have, at least, an understanding of what is a metaheuristic.

2.2 Heuristics and Metaheuristics

Metaheuristics have been proposed since 1980 to bypass the issues of the Heuristic methods in general and could be considered as the development of the latter. Heuristics are pretty simple rules (usually iterative algorithms) which, in reasonable times, produce good solutions to a tough optimization problem. The way in which iterative algorithms work is the search, at each step (i.e., iteration), for the new best solution among the previous (already found) best set of solutions. The algorithm then provides a good[22] solution and stops either when some appropriate stopping criterion is met (i.e., the algorithm has run all the iterations set at the beginning), either when it finds near-optimal solutions through the reach of a satisfactory fitness level. Anyway, there are some disadvantages in their usage due to precise features:

- Problem-specific;
- Generation, at various times, of a limited number of different solutions;
- Possibility of stop at poor-quality local optima.

[21] DEB, K. (2001). *Multi-objective Optimization Using Evolutionary Algorithms.* John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex PO19 8SQ, England.

[22] The choice of the term "good", referring to the solutions, here is not casual. There is no guarantee at all to reach the optimal solution in heuristic sciences.

Metaheuristic is defined as the *"general and high-level problem-independent algorithmic framework which, providing a set of guidelines or strategies, can be applied to different optimization problems with relatively few modifications".* In other words, metaheuristics are generally non-deterministic strategies that guide the search optimization process with the aim to explore the search space (so as to find near-optimal solutions) and they are not problem-specific.

Generally, metaheuristics are classified basing on their behavior (exploration or exploitation) and the initial number of solutions (trajectory-based or population-based). Exploration focuses on the research of current good solutions in a local region, instead the exploitation is meant to generate different solutions in order to explore the search space on a global scale. Trajectory-based metaheuristics start from a single solution and replace the current solution with a better one at each step of the process. Population-based metaheuristics instead start from a set of solutions, randomly chosen, and, going through an iterative process, replace part of it or even the entire population with the newly generated individuals, which are better than the previous.

2.3 GAs Overview

One of the most known and applied metaheuristic is Genetic Algorithms (GAs). It was created and described for the first time between 1950 and 1960 from John H. Holland and then developed between 1960 and 1970 from Holland and his colleagues. One of the most important events during this search path is the publication, in 1975, of the book "Adaption in natural and artificial system"[23], in which we find the fundamentals of the evolutionary theory applied to artificial intelligence and the concept of adaption as it is used in GAs.

Holland's method was a method for classifying objects, selecting breeding with these objects to produce new ones. Its name refers to the genetics since this technique follows the fundamentals of natural evolution (such evolutionary growth could be

[23] See: HOLLAND, J.H. (1992). *Adaption in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence.* MIT Press.

described as of Figure 2.1 below). The Genetic Algorithms stem, indeed, from the Darwinian idea of the *"survival of the fittest"* (natural system which follows the natural pattern of growth and reproduction). Literally, this specific methodology generates a population of *chromosomes,* which are strings made of bits' sequences (single bits are called *genes*), and, through the use of the *selection, crossover and mutation* operators, it transforms the old generation into a new one. The chromosomes could be interpreted as the potential solutions. They are called potential solution because they are candidates to the resolution of the optimization problem the algorithm tries to solve and because, before being effective, they must go through and survive all the steps of the process. The values of the bits are named *alleles* and usually the binary system is used to define these values[24]. Then, a fitness function is assigned, and each chromosome is evaluated on its fitness score (according to the goodness of the solution for the given optimization problem, which means that the assignment of the score depends on how well they perform compared to either the goal and/or the rest of the individuals in the population). Typically, individuals have as domain a set of binary strings of prefixed length $L < +\infty$.

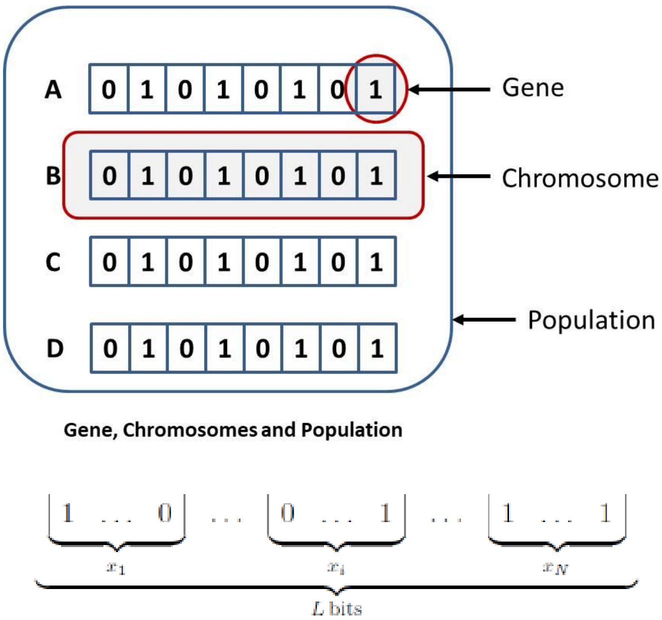

Figure 2.1: *GA's elements*

[24] In this introductory phase, the space of the solutions is given by the set of binary strings. Later on, we will see that for specific problems (such as the analysis of company bankruptcies) the space of the solutions will be restricted and will be constituted by a limited number of elements using real numbers.

Recapitulating, the binary string is called chromosome and each of the *L*-element (Figure 2.1 above) constituting the chromosome is called gene, which can assume two values (0 and 1). In the chromosomes, some sequences of genes may be particularly significant as they may be a piece of the searched solution. These sequences are called *schemata* (which is the plural of *scheme)*[25].

Then, each chromosome is evaluated on its fitness score (according to the goodness of the solution for the given optimization problem). This fitness function must be specified for the problem to solve. The single numerical fitness scored of each chromosome indicates the degree of utility or ability of the individual which that chromosome represents. In other words, we can state that the fitness function transforms a measure of performance into an allocation of reproductive opportunities. Even though evolutionary algorithms evolved during the last years and the GAs assumed different forms (sometimes slightly diverging from the original formulation of Holland), the Figure 2.2 below gives an overview of the process since the basic idea is still the same.

```
Create a population of objects (creatures)        //Initialization
Evaluate the fitness of each object               //analyzing
While the population is not fit enough            //Fitness check
{
Delete (kill) all unfit objects                   //Removing unfit objects
While population size <max:                       //size check
{
Select two best populations
Create new objects
Random mutations
Evaluate and place in population                  //breeding
}
}
```

Figure 2.2: *Evolutionary growth*

[25] The name refers to the *Schemata Theorem,* which was presented for the first time in 1975 by J. H. Holland.

Systematically, the steps of the GAs can be summarized as follow:

- Initialization phase: random creation of the potential solutions;
- Evaluation phase: the fitness function is evaluated to get information regarding the potential solutions;
- Selection operation: selection of which chromosomes (parents) should be used for the creation of the new generation (offspring). Higher is the fitness value, higher is the probability for the potential solutions to be chosen more than once for the mating-pool;
- Crossover operation: recombination of individuals parents to generate individuals-children (offspring), taking advantage of the features possessed by the good members of the previous generation;
- Mutation operation: random alteration of the genes;
- Substitution phase: the new generation of potential solutions replaces, partially or even totally, the initial population.

The process repeats the passages from the evaluation to the substitution phases until one stopping condition is met, or one optimal solution is eventually reached. This allows spreading good characteristics throughout the entire population: the mixing (i.e., selection) and exchange (i.e., crossover and mutation) of good characteristics among individuals are essential parts of the biological process.

Coming back to the above-mentioned *Schemata Theorem*, it is important to underline that each *scheme* can assume, in addition to 0 and 1 values, also a jolly value (*) which could be both 0 or 1. Through such theorem, Holland proves that the potential solutions with higher fitness value tend to increase esponentially in the next generation and quantifies the minimum number of *schemata* in this new generation.
The *schemata* features are the *order* $o(S)$ and the *defining length* $\delta(S)$. The *order* is the number of alleles in the scheme, while the *defining length* is the distance between

the first and last "defined"[26] allele in the scheme. Generally, a solution of length L can be represented by 2^L schemata.

Considering $N(S,t)$ as the number of schemata in the t-generation and $N(S,t+1)$ as the number of schemata in the next *(t+1)*-generation, Holland provided a formula to compute this number for the successive generation. The minimum number of *schemata* in the t-th generation $N(S,t)$ will be equal to 2^L (case when the *n*-chromosomes are identical between each other) and the maximum will be $n * 2^L$ (case when all *n*-chromosomes are different between each other).

$$N(S,t+1) \geq N(S,t) * \left[\frac{f(S,t)}{f_{avg}(t)}\right] * [1 - p_c * \frac{\delta(S)}{L-1} - p_m * o(S)]$$

where:

$f(S,t)$ is the average fitness of the solutions represented by the scheme *S* at time *t*;

$f_{avg}(t)$ is the average fitness of the solutions in the t-generation;

p_c and p_m are the crossover and mutation probability, respectively;

$o(S)$ and $\delta(S)$ are the *order* and the *defining length*, respectively.

Analysing the formula, it is possible to identify the role played by the so-called *building blocks*, which are the schemata with low probability of not being selected for the successive generation due to some specific features (high fitness value and small order and defining length). In other words, the *building blocks* are the ones that, in each iteration, spread across the population with greater ease. The term $\frac{f(S,t)}{f_{avg}(t)}$ is the only element that can determine the increase of the number of solutions represented by the scheme *S* in the population. Notice, indeed that, if $f(S,t) > f_{avg}(t)$, then such specific scheme *S* will be present in the next generation, while the crossover and mutation operators worsen this probability of being considered for the new generation. In particular, a high p_c and p_m values mean, respectively, that there is a

[26] "*Defined*" means that the allele assumes a value of 0 or 1. The jolly value is not defined.

high probability that the crossover will take place on this *S*-scheme and that there is a high probability that the genes of the *S*-scheme will be subject to potential mutations. At equal p_c value, a high *defining length* value (i.e., a high number of genes between the first and the last gene in the *S*-scheme) increases the probability of being subject to both crossover and mutation.

Additionally, it is possible to affirm that when $o(S)$ and $\delta(S)$ are small, the fitness value $f(S,t)$ is high and, consequently, the number of schemata *S* in the generation *(t+1)* will be high too.

Like every computation technique, Genetic Algorithms has pros and cons. Some advantages are for example: flexibility, speed and ease of use. All the potential variations in the GAs' parameters increase the flexibility. Regarding this flexibility feature, together with the computational speed one (GAs are able to explore rapidly even a very wide solution space), one can refer to the series of influential articles of Richard Bauer, in which he shows why finance professionals should add such computerized decision-making tools, focusing his attention to Genetic Algorithms[27]. In addition, GAs do not need any specific probability distribution for its data, unlike other statistical techniques. Other advantages of the evolutionary algorithms in general are mentioned by Goldberg[28] and are: the requirement of little prior knowledge about model characteristics, easy implementation, robustness and the ability to be carried out in parallel.

A couple of drawbacks that could be associated with GA are, first, the fact that the founding of optimum solutions is not guaranteed and, second, the *overfitting* problem. With regards to the first drawback it is, however, important to notice that the problem of focusing on solutions which are just local-optima is reduced (with respect to other algorithms) since GAs consider more regions of the solution space at the same time. The *overfitting* issue can occur when the algorithm works just following his memory of the data (starting from a *training-set* which is a sample that includes some data-example of already-solved problems in order to teach the algorithms how to select the

[27] See: BAUER, R. J. JR. (1994). *Genetic Algorithms and Investment Strategies.* Wiley.

[28] See: GOLDBERG, R., A. A., J. L. (2005). *Evolutionary Multi-objective Optimization: Theoretical Advances and Applications*. Springer London Ltd., 1st edition.

best solutions to that kind of problem). This could happen especially if the *training-set* is too small (containing too few examples) or if the teaching process has been iterated too many times.

2.4 Reproductive-inspired operators

As already stated, usually the numeric system used to define the values of the bits (genes) is the binary one. In order to explain the different versions of the following operators, we will consider potential solutions whose structure is elementary (in other words, chromosomes are encoded through a string of binary digits that is a list of zeros and ones).

2.4.1 Selection Methods

Selection focuses its efforts in choosing individuals from the current generation (parents) with the highest qualifications (i.e., those with high fitness scores). Some of these methods are analysed by Deb K.[29] in 2001 and are: tournament selection, roulette wheel selection (RWS), ranking selection, proportionate selection and stochastic universal selection (SUS). These can be grouped into two different families of selection methods: one does not directly consider the absolute value of the fitness function and relativizes this value with respect to the values belonging to the other chromosomes of the population, while the other one works by directly comparing the fitness values of the chromosomes of the population.

One method belonging to the first type of selection-methodology's family is the so called *"roulette-wheel selection"*. The name comes from the idea that to each chromosome is assigned a sector (a sub-interval) whose dimension is proportional to

[29] DEB, K. (2001). *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons Ltd. The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, England.

its fitness value. In order to get the exact sector that should be assigned to one specific chromosome, one has to relativize the weight of its fitness value with respect to the sum of the fitness value of all the other chromosomes in the population. The relative fitness value is computed through the following formula:

$$p_i = f_i \Big/ \sum_{i=1}^{n} f_i$$

where:

p_i is the relative fitness of *i-th* chromosome;

f_i is the fitness value associated to the *i-th* chromosome.

Consider an interval [0,1]. This interval is then divided into as many sub-intervals as the individuals in the population. The number of the individuals generated in the population are as many random numbers uniformly distributed in [0,1]. Greater is the fitness value associated to a chromosome, higher will be its relative fitness and greater will be the sub-interval filled in [0,1]. Consequently, the probability that the extracted random uniform number would fall into the interval that identifies such chromosome will be higher.

Regarding the second type of selection-methodology's family, two examples of methodologies belonging to this family are the "*tournament selection"* and the "*truncation selection"*.

The *tournament selection* provides that *s*-chromosomes are randomly chosen in the population and compared one to each other. The chromosome of the group with the highest fitness value is selected for the group which is meant to reproduce. Usually, s=2 such that the process has to be repeated n-times in order to obtain a population of *n* genes (i.e., individuals) ready for the reproduction phase. The *truncation selection* sorts the chromosomes of one population in a ranking, where the first chromosome is the one with the highest fitness value. Then, a specific portion *p,* with *p=1/2, 1/3, etc.,* of the chromosomes with high fitness value is selected and reproduced for *1/p* times in the mating pool.

Since the objective of the selection operator is to keep and duplicate the solutions with high fitness value while removing the poor chromosomes and maintaining the size of the population constant, we can not say that this operator takes part in the reproduction phase (it does not create new chromosomes from the initial population, but it produces only copies of good solutions). The reproductive phase will be performed by the crossover or mutation operators, starting from the parents which have been selected.

2.4.2 Crossover

There are not specific steps in the crossover operation, because the algorithm adapts itself to the features of the specific problem to solve. Crossover emulates the exchange of chromosomes having already better traits than their parents (according to the basic Darwinian theory that the fittest individuals tend to survive and mate to form the next generation) to generate an offspring that, in terms of fitness, is stronger. After parents' selection, a random uniform number u is generated and compared to the crossover probability p_c (it is common to set a value of 0.7). If $u > p_c$ then the parents are simply placed into the new generation without undergoing the crossover. Otherwise, if $u < p_c$ then crossover takes place.

In order to determine the differences between the various crossover operators, one should look at the ways in which the group of genes has to be changed between the selected chromosomes and, in some cases, also look at the position in which the selected genes are reinserted in the next generation of chromosomes (i.e. the offspring).

One kind of these crossover methods is the so called "*single-point crossover".* Given two chromosome-parents, crossover cuts with a given probability the two chromosomes at the same gene chosen at random. Acting in that way, it is ensured that the number of genes at the right of the crossover point (i.e. the tail) in the first of the two parents (from now on it will be called G1) is equal to the number of genes at the right of the crossover point in the second (from now on it will be called G2). Then, the tail of G1 is cut and merged with G2 and, simultaneously, the genes at the left of the crossover point (i.e. the head) in G2 are cut and merged with G1. To have a better understanding, the graphical representation is in Figure 2.3 below (here we can see both the crossover with one single crossover point and with two crossover points).

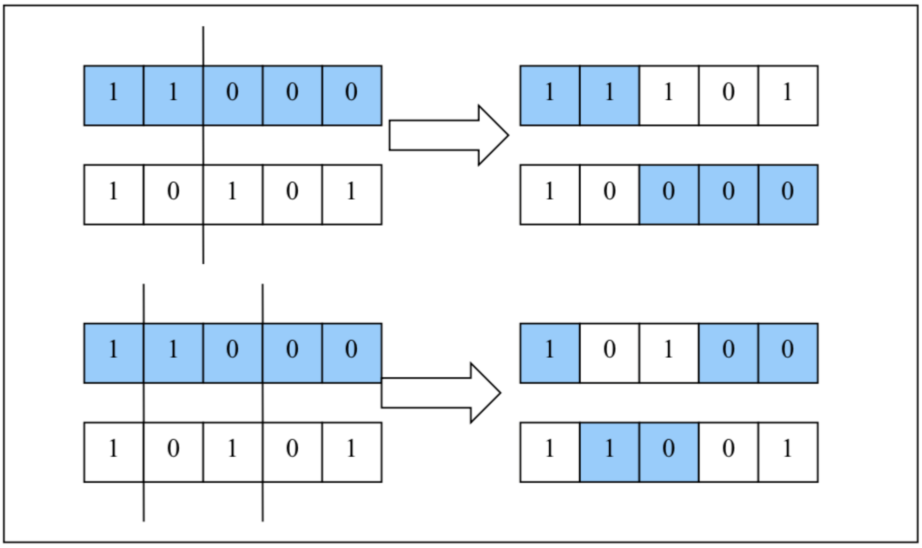

Figure 2.3: *Single and Double-point crossover*

Another kind of crossover method is the so called *"uniform crossover".* The two selected chromosome-parents are considered separately (i.e., one by one gene), which means that each gene belonging to G1 will be exchanged with the correspondent gene of G2 with a certain exchange probability p_s (usually set equal to 0.5). Then, a uniform random number u is generated in order to be compared with p_s in the same way we have seen it is compared to p_c (crossover probability).

An alteration of the *"uniform crossover"* method is the *"order based uniform crossover".* Consider again G1 and G2 as the chromosome-parents, and a string of zeros and ones (randomly ordered) of the same length of G1 and G2. The chromosome-children will be called from now on F1 and F2. This *"order based uniform crossover"* method ensures that F1 has the same gene (and therefore the same allele) of G1 if the string has value one in the same position of that gene, or, otherwise, the gene is not assigned (just temporarily). Then, to complete F1, the alleles of F1 to whom are assigned a zero value have to be obtained from G2 and placed in F1 in the same position they appear in G2. Of course, the same procedure is meant to be applied for F2. Again, to have a better understanding, the graphical representation is in Figure 2.4 below.

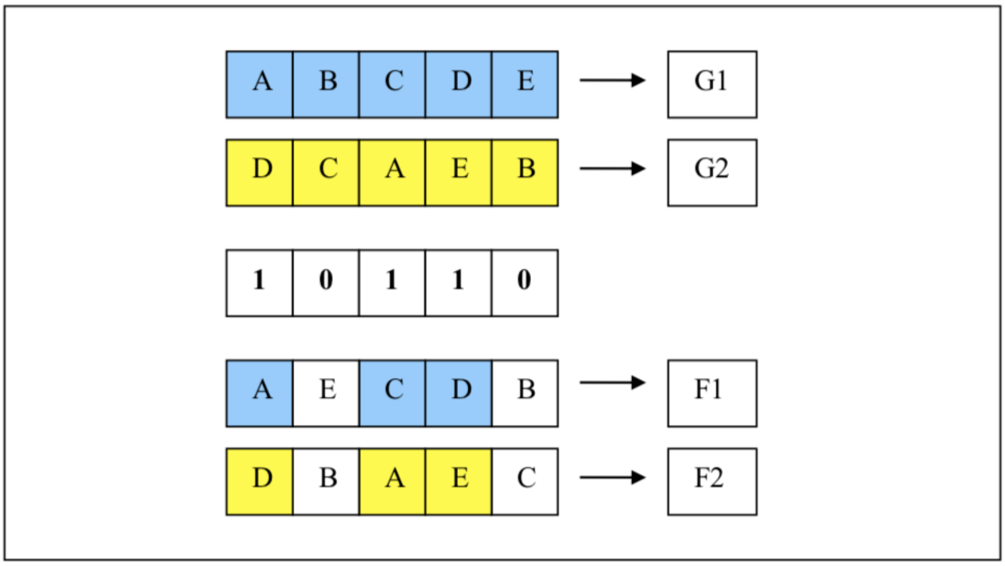

Figure 2.4: *Order based uniform crossover*

Another kind of crossover method is the so called *"partially recombined crossover".* The process starts again from two randomly selected G1 and G2 and two crossover points. Considering the genes in between the two crossover points in G1 and looking at the corresponding genes in between the two crossover points in G2, we can say that, ideally, some couples are created (one for each gene in between the crossover points). Specifically, in G1 (G2) it is possible to find the same alleles that are also in G2 (G1), even if in different positions, recreating therefore the couples defined through the comparison between G1 and G2. The next step is the exchange of the alleles of one couple of genes in G1, generating in this way the chromosome-child F1. The same works for the creation of F2 from G2. In this way, the exchange is done just at chromosome level (i.e. the genes exchanged belong to the same chromosome). The passages described until now can be seen in Figure 2.5.

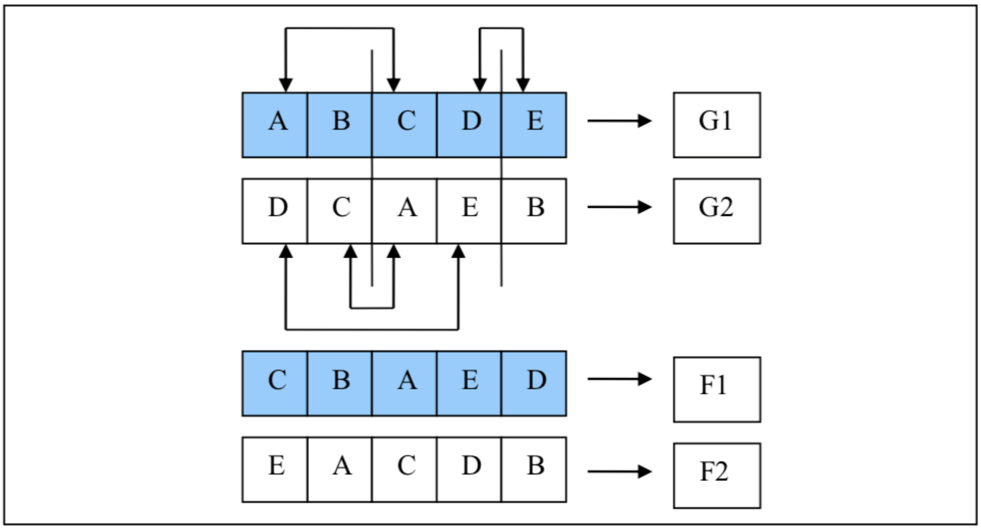

Figure 2.5: *Partially recombined crossover*

Then, the process is repeated to identify the last allele assigned to F1: the new starting point is the allele copied in F1. One should look which is the position of such allele in G1 and consider the gene of G2 in the same position of the last allele in G1. Once this gene in G2 has been identified (and consequently its relative allele), it has to be copied in F1 in the same position of the same allele in G1. This process is repeated cyclically until we return back to the first gene considered or, alternatively, until all the genes and relative alleles of G1 have been considered. The genes that were leaved unfilled are then filled with the correspondent alleles of G2 (Figure 2.6). The same is true for F2, but with G2 instead of G1.

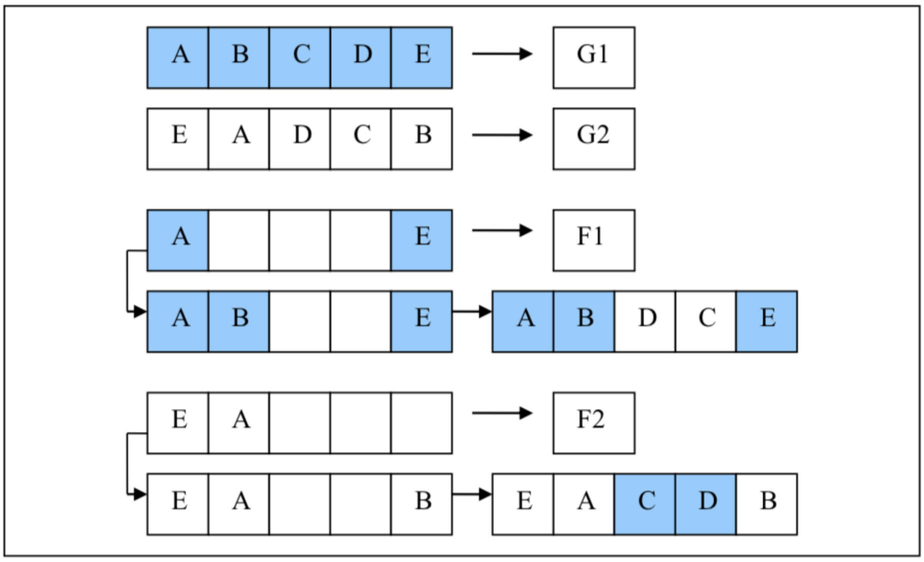

Figure 2.6: *Cyclical crossover*

A drawback of the crossover operator is that allows to exclusively recombine genes which are already in the population of the potential solutions and, therefore, it is not possible to explore the space of the solutions in depth[30]. To accomplish such task, the mutation operator is applied.

2.4.3 Mutation

The aim of this operation is to perturb the individuals-solution of the new population. Literally, new alleles, which are not present in the initial chromosome-population, are introduced in the population. Mutation allows to explore new sub-intervals of solutions on which will then be performed a research in depth through the crossover operator. Looking at the mutation operator from this perspective, we can say that it is a "secondary" operator and Holland itself indeed wrote *<<Mutation is a "background" operator, assuring that the crossover operator has a full range of alleles so that the*

[30] Exploring the space of the solutions in depth implies limiting the risk that the Genetic Algorithm got stuck in regions of local optimum. The GA with only the crossover operator faces the risk of providing solutions with low explanatory power, since they would come from a limited sub-interval of solutions.

adaptive plan is not trapped on local optima […]. The mutation serves some enumerative function, producing alleles not previously tried[31]*>>.*

As for the crossover operation, a certain probability p_m is defined. Such probability represents the probability that each allele of the chromosome taken into consideration will change its value. Anyway, p_m is lower than p_c.

There are different forms of mutation, depending on whether the representation is binary or non-binary. For binary representations, mutation is from 0 to 1 or vice versa. Instead, for non-binary representations, mutation is much more complex but usually the recommended way of proceeding is to add a zero mean Gaussian number to the original values.

There is no guarantee that this operation will provide better results, but at least we are sure that, changing some part of the chromosomes, something new is created. According to Gen and Cheng[32] this operator helps in exploring new regions of the multi-dimensional solution space.

Anyhow, besides the traditional reproductive-inspired operators of Selection, Crossover and Mutation, there exist other less conventional operators such as *Inversion* (discussed by Holland) and the *Lamarckian operator* (proposed by Gen and Cheng).

2.5 Substitution phase

After the evaluation phase and the application of the different operators, with the final objective of obtaining a new and better population, there exist mainly three methods

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[31] See: HOLLAND, J.H. (1992). *Adaption in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence. Page 111.* MIT Press.

[32] See: GEN, M., CHENG, R. (1997). *Genetic Algorithm and Engineering Design.* John Wiley & Sons, Inc. New York.

of substitution of the population from which the chromosome-parents were selected with the one of the chromosome-children:

1. The *"delete-all"* substitution provides that the new population is constituted merely by the chromosome-children and all the chromosome-parents are completely deleted;
2. The *"steady-state"* substitution provides that the new population is constituted both by the chromosome-parents and the chromosome-children. One parameter needs to be set to define the proportion between parents and children into the new population (i.e. how many parents need to be removed). The criterion to choose which of them should be removed is based again on the fitness score;
3. The *"steady-state without duplicates"* substitution is an amended version of the "*steady-state"* substitution. The unique difference is that the duplicates of chromosomes are deleted.

2.6 Constraint Handling

All kind of optimization problems present constraints too. Constraints could be both in equality or inequality form and could be divided in hard and soft constraints. Hard ones must be satisfied, instead soft one could be relaxed in order to accept a solution. Constraints which are in equality-form can rather be easily translated into inequality through the following formula:

$$|h(x)| - \varepsilon \leq 0$$

where $h(x)=0$ is the equality constraint and ε is a small value amount.

In literature there exist different constraint handling methods when using metaheuristics, which are usually classified in five different types:

- Methods based on preserving the feasibility of solutions;
- Methods based on penalty functions;
- Methods biasing feasible over infeasible solutions;
- Methods based on decoders;
- Hybrid methods.

For the aim of this thesis' optimization problem, the suitable method could be the penalty function method, which practically transform a constrained optimization problem into an unconstrained one through the use of an additive penalty term or of a penalty multiplier. Furthermore, these penalty methods can be grouped into seven categories:

- Death Penalty;
- Static Penalties;
- Dynamic Penalties;
- Annealing Penalties;
- Adaptive Penalties;
- Segregated GA.

Again for this thesis purpose, the most suitable choice with regards to the different penalties is the static penalties approach because the penalty parameters do not change within generations and because they are applied to infeasible solutions only. There are different ways to approach this method and here we present one of the most known. It was initially presented by Morales A. K.[33] in 1997, and it is based on the penalization of the fitness function of infeasible solutions by using the information on the number of violated constraints.

[33] MORALES, A. K., QUEZADA, C. V., BATIZ, J. D., LINDAVISTA, C. (1997). *A Universal Eclectic Genetic Constrained Optimization Algorithm for Constrained Optimization.* Optimization, pages 2-6.

$$F(x) = \begin{cases} f(x), & if\ x\ is\ feasible, \\ K - \sum_{i=1}^{s} \left[\frac{K}{m}\right], & otherwise. \end{cases}$$

where:

s is the number of non-violated constraints;

m is the total number of constraints;

K is a large pre-defined positive constant.

The value of *K* is generally chosen (by Morales et al.) equal to $1x10^9$. The aim of this large enough pre-defined penalty factor is to ensure the assignment of larger fitness values to infeasible solutions compared to feasible solutions.

CHAPTER 3

Case Study: IAG – WACC computation

In this chapter the International Consolidated Airlines Group, S.A. (IAG) company is introduced as the company chosen for the specific case-study. There are both the industry and the company overview. Then, there are the calculations of the different components of the Cost of Capital, which will be used as model's inputs for the implementation of the Genetic Algorithm in the next chapter.

3.1 The Airline Industry Landscape

The company operates primarily in the Aviation, or Air Transport, industry, which includes both passenger and cargo transportations.

This sector of the market consists of over 2000 airlines, providing services to nearly 3700 airports around the world and operating more than 23000 aircrafts on a daily basis. In the last few decades, international airlines established different kinds of alliances in order to expand and reach a global presence. Air Transport industry, as well as airlines' profitability, is highly correlated to economic, political and social factors and, consequently, it is considered one of the most volatile industries. It is an increasingly competitive and fast paced environment. Therefore, in order to strengthen their very low profit margins[34] and to keep such challenges under control, airlines need constant improvements. Even though the overall profitability of the world airlines lost 14.3% in 2018, analysts from S&P Global Ratings are broadly optimist regarding the growth prospects of the airline industry, mainly thanks to the

[34] The main reason is due to the extremely fast-growing low-cost carriers (LCCs), which fosters competition primarily in the pricing polices (hence lowering the profit margin).

rising desire of the young generation to travel more and more around the globe, together with higher levels of spending on travelling costs among the older generation. The major market drivers are the growing demand for air travel, the accessibility to air travel thanks to low-cost carriers, the burgeoning e-commerce, the opening of new routes and the opportunities offered by the incorporation of new technologies[35]. The challenges, instead, are represented mainly by increasing fuel prices, labour expenses and of course political uncertainties.

The European Union is currently home of 135 airlines, while America doesn't count more than 59 companies (*American Airlines Group* is the biggest one by revenues, fleet size and passengers carried, while *Delta Airlines* is the largest airline by assets value and market capitalization). Focusing on Europe, instead, the top 5 Airlines or Airline Groups by passengers carried are: *Lufthansa Group, Ryanair, International Airlines Group (IAG), Air France – KLM, and easyJet.*[36]

3.2 Company Overview

The International Consolidated Airlines Group (IAG) is an Anglo-Spanish registered airline company in Madrid (Spain) with its operational headquarters based in London (United Kingdom), whose shares are traded on the London Stock Exchange and Spanish Stock Exchanges. It is one of the world's largest airline groups with almost 600 aircraft flying to more than 250 destinations, carrying around 113 million passengers each year. IAG's CEO is Willie Walsh.

[35] The use of Biometrics and RFID for self-check-in, passport control, baggage tracking and security. In addition, airlines and airports are planning to use Artificial Intelligence (AI) and Blockchains according to SITA (Société Internationale de Télécommunications Aéronautiques - the world's leading specialist in air transport communications and information technology).

[36] Source: World Air Transport Statistics (WATS 2018), IATA – International Air Transport Association.

3.2.1 History

IAG is an Anglo-Spanish company registered in Madrid and was incorporated on April 8, 2010. The launch of the *International Consolidated Airlines Group S.A.* (hereinafter "*International Airlines Group"* or "*IAG"*) company dates back to January 2011 after the merger between *British Airways* and *Iberia*, the leading airline companies of United Kingdom and Spain, respectively. Also, *British Airways World Cargo* and *Iberia Cargo* merged, forming IAG Cargo. In December of the same year IAG made a deal with *Lufthansa*, as cleared by the European competition authorities, for the acquisition of *British Midland International* (BMI), whose fleet and routes were integrated with those of *British Airlines*. IAG acquired BMI effectively in 2012. In addition, in 2013 IAG bought *Vueling*, a leading Spanish short-haul airline. After the rejection of two offers, in 2015 IAG managed to acquire also the Irish airline *Aer Lingus*.
Furthermore, in response to increased competition in the low-cost long-haul market, in the late 2017 a new subsidiary company, called *LEVEL*, was created.
Hence, IAG has become the parent company of *British Airlines, Iberia, Vueling, Aer Lingus* and *LEVEL*.

3.2.2 Business Model

IAG's vision is to be the world's leading airline group and to maximize the creation of sustainable value for both shareholders and customers.
Although the Group portfolio consists of distinct operating companies (from full service long-haul to low-cost short-haul carriers, each targeting specific customer needs and geographies), IAG relies on a common integrated platform which allows the Group to exploit revenue and cost synergies (this would not be achievable in the case of operating companies working alone), while maintaining simultaneously simplicity, efficiency and their unique identities. This pursuit of gradually increasing value and sustainable growth allows the Group to reduce costs and improve efficiency. IAG takes advantage of these synergies opportunities by leveraging its scale, by engaging itself

with new innovation strategies and by increasing external B2B services. All these strategies enhance productivity and create value for the customers. Figure 3.1 below is a graphical representation of its business model.

Figure 3.1: *IAG Company's Business Model*

Source: IAG Company's website – Business Model & Strategy section

Furthermore, *British Airways* and *Iberia* are members of *Oneworld* alliance, which brings together 13 of the world's leading airlines and around 30 affiliates, allowing a cooperative approach in different fields (such as scheduling and pricing) and combining destinations spread all over the world. Some examples are the alliance between *British Airways, Iberia* and *American Airlines* that connects Europe with the United States of America, Canada and Mexico, or the one between *British Airways, Finnair, Iberia* and *Japan Airlines* that connects Europe to Asia and Japan, or the one between *British Airways* and *Qatar Airways* that connects the UK with Doha. This alliance produces operating efficiencies and improves customer convenience and choice, also allowing mixing and matching flights to get the best deals.

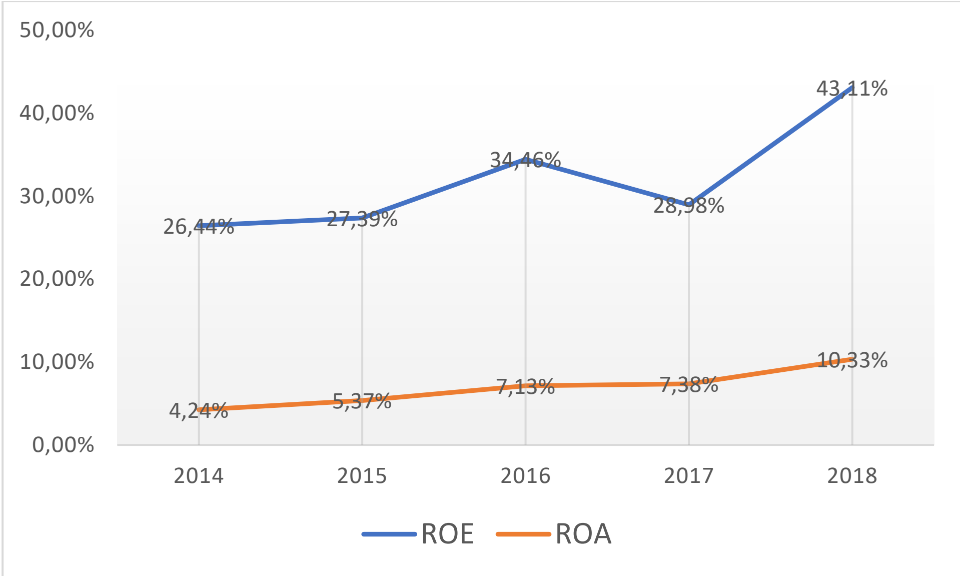
3.2.3 Profitability, financial and structure ratios

In a glance, looking at the profitability and main financial ratios of IAG (in Table 3.1), it is possible to observe that the company had a great improvement in the efficiency of the management and in terms of profitability moving from year to year. Nevertheless, the situation changes focusing just on the ratios between 2016 and 2017, where the company appears to make worse its efficiency and also profitability indicators become lower. However, this was a really slight worsening and after that the company started to recover again.

	2014	2015	2016	2017	2018
ROE	26,44%	27,39%	34,46%	28,98%	43,11%
ROA	4,24%	5,37%	7,13%	7,38%	10,33%
EBITDA margin	10,64%	15,86%	16,71%	15,06%	20,21%
EBIT (operating) margin	5,10%	10,14%	11,01%	9,89%	15,07%
Net Profit Margin	4,97%	6,63%	8,65%	8,78%	11,87%
Current ratio	0,76	0,80	1,05	1,01	0,91
Leverage	6,24	5,10	4,83	3,93	4,17
Tot. Liabilities/Tot. Assets	0,84	0,80	0,79	0,75	0,76

Table 3.1: *Main IAG's Ratios*

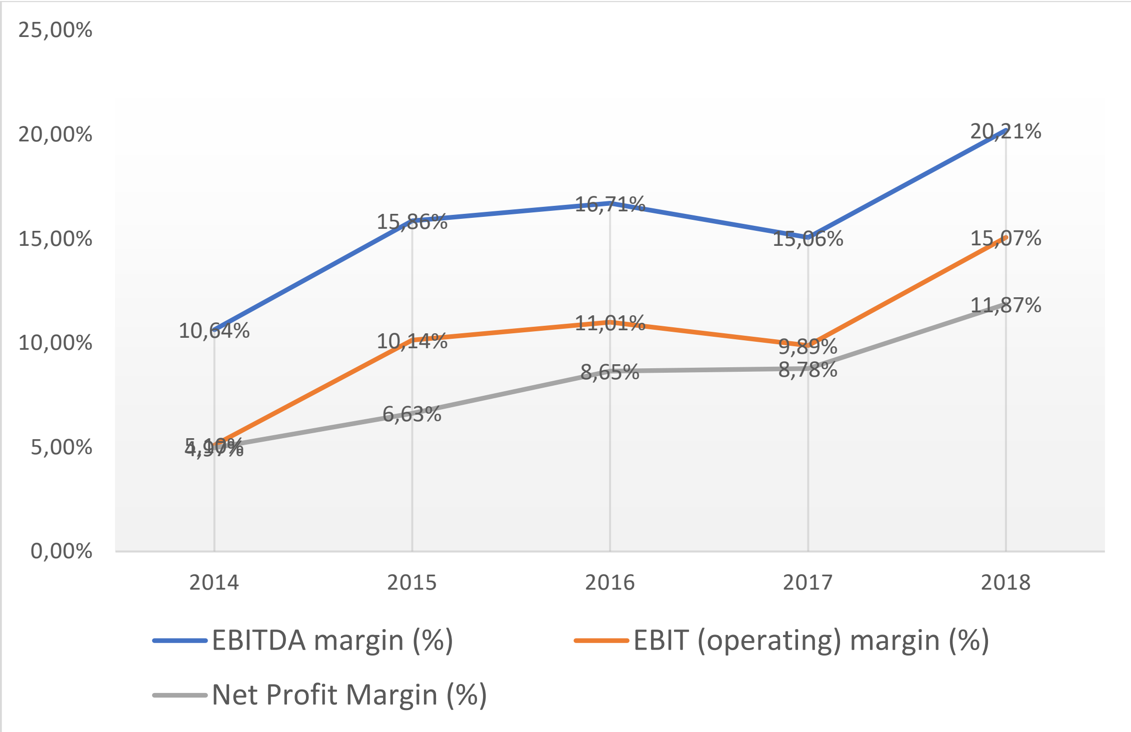
In detail, the ROE (Return On Equity) of the company had a huge increase during the last year, which means that it had a great improvement in terms of profitability, productivity and management efficiency. Also the ROA (Return On Assets), which shows the percentage of how profitable the company's assets are in generating revenue, followed the same path of the ROE. The behavior of both the ratios could be seen in Graph 3.1.



Graph 3.1: *ROE & ROA ratios over last 5 years*

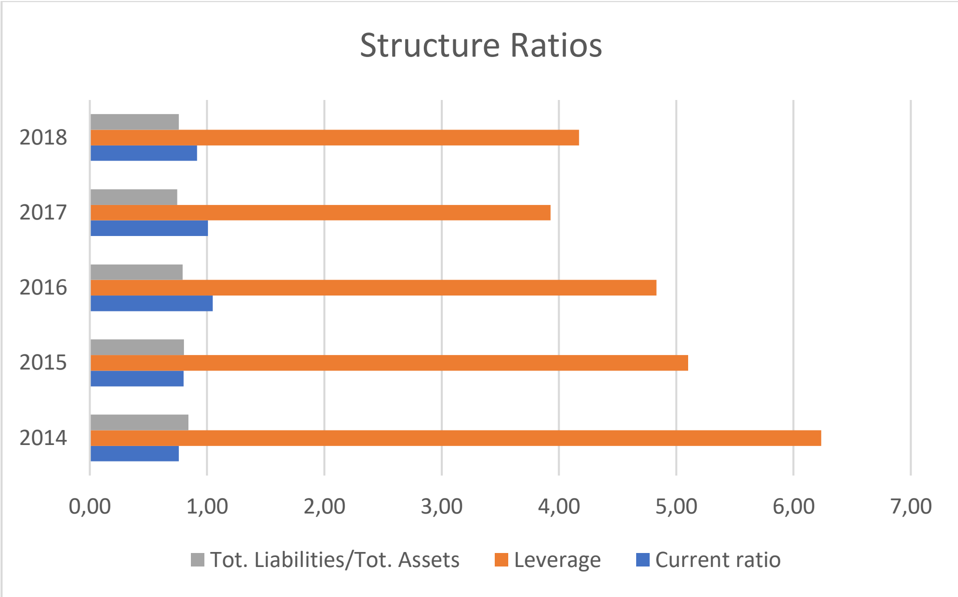
For the other financial ratios considered, the situation is the same: the *EBITDA* and *EBIT margin* variations suggest that the company improved its capacity to generate value through the operational management (with the only already mentioned exception between 2016 and 2017).

In Graph 3.2, the bottom line of the Consolidated Income Statement is the *Net Income*, which reflects the total amount of revenue left over after all expenses and additional income streams are accounted for, including also interests from debts and taxes. Dividing it by the revenues and multiplying by 100, we obtain the *Net Profit Margin*, which reflects a company's overall ability to turn income into profit.
Here again the behavior is in line with the other financial ratios, as we can see from Graph 3.2.



Graph 3.2: *IAG's financial ratios over last 5 years*

Finally, looking at the structure ratios, we can notice that the *Current ratio*, which is the ratio between current assets and current liabilities and measures whether a firm has enough resources to pay its debt over the next 12 months, slightly decreased during the last year compared to the previous two but it is still around a value of 1. Usually it is considered to be positive when its value is greater than 1. Anyway, this is counterbalanced by the *Leverage ratio* (=Total Assets/Equity), which shows a really good value (usually a good value is considered to be between 1 and 3). Another ratio that gives useful information is the ratio between total liabilities and total assets. Since it is a leverage ratio, the higher it is, the higher the risk is as well. For IAG Company the value of this ratio is small enough to guarantee a good stability. The path of these three ratios during the last 5 years can be seen in Graph 3.3.



Graph 3.3: *IAG's structure ratios over last 5 years*

3.3 IAG's Weighted Average Cost of Capital

From now on, the chapter will focus on the derivation of the Weighted Average Cost of Capital. The costs of individual financing sources, precisely the Cost of Debt and the Cost of Equity, have to be both estimated (since they are the only components of the firm's total capital) and weighted. To recap, the general formula of the Weighted Average Cost of Capital (i.e., WACC) is:

$$WACC = K_E * \left[\frac{E}{D + E + PS}\right] + K_D * \left[\frac{D}{D + E + PS}\right] + K_{PS} * \left[\frac{PS}{D + E + PS}\right].$$

In chapter two we defined the hybrid securities and, therefore, it is important to say that since Convertible Bonds were already included in the consolidated financial statement of the company under the voice of *"Long-term borrowings – Bank and other loans"* and since the amount of Preferred Stock is equal to zero in 2018, for the following analysis just Debt and Equity will be considered as the components of the firm's Capital. Considering such adjustment, the formula of the WACC can be simplified in the following way:

$$WACC = K_E * \left[\frac{E}{D+E}\right] + K_D * \left[\frac{D}{D+E}\right]$$

IAG operates in different countries and so it has to cope with different currencies. However, the analysis is completely led in euros since all foreign operations are translated into euros at the rate of exchange ruling at the balance sheet date. In other words, all profits and losses coming from its operations are translated in euros at average rates of exchange during the year.

3.3.1 Cost of Debt

Regarding the Cost of Debt, the first step is looking at the rating of the company. In November 2018, S&P and Moody's assigned IAG with a long-term investment grade credit rating with stable outlook (BBB). From Table 1.2 of chapter one it is possible to retrieve the default spread corresponding to the company's rating (1.27%), to which the risk-free rate is added in order to arrive at the real cost of debt. Since the firm is settled in Spain, but operates in many different countries, in order to have a proxy of a risk-free rate of the Euro-zone, the choice has been to take the mean of the 10-years-maturity Treasury Bonds of the countries which IAG considers for taxation purposes (precisely: UK, Spain and Ireland). This choice brings to a yield of 0.40%.
Hence, adding this risk-free rate to the firm's specific default rate we get a pre-tax Cost of Debt equal to 1.67%.
Since the vast majority of the Group's activities are taxed in the countries of effective management of the main operations (UK, Spain and Ireland with corporation tax rates during 2018 of 19%, 25% and 12.5% respectively), the marginal tax rate chosen for the

analysis is the one reported by IAG itself as *"The Group's effective tax rate for the year"* and it is equal to 16.9%.

The after-tax Cost of Debt can be formalized as follows:

$$K_D \; after \; tax = (1 - tax \; rate) * K_D.$$

Considering 16.9% as the final marginal tax rate for the entire Group and applying it to the formula above, we get to an after-tax Cost of Debt of 1.39%.

3.3.2 Cost of Equity

The Cost of Equity, computed through the usual CAPM formula, depends on three key components: the risk-free rate, the equity risk premium (ERP) for the firm and its levered beta.

While all that does concern the risk-free rate has already been explained, some attention on the other two components is needed.

In order to calculate the proper ERP, firstly, the chosen approach is the long-term average historical approach because in this thesis we assume that the belief that investors are rational is erroneous and because the purpose of the analysis regards corporate finance (and the average historical is the best recognized approach in this field). Secondly, the partitioning of revenues per country has been taken from the consolidated financial statement of the company. Lastly, the risk of the business has been considered as a weighted average of the risks of the countries in which IAG Company operates (weighting the various countries' equity risk premium by considering the contribution of each geographical subdivision to the revenues).

Country	Rating (S&P 500)	Default Spread	ERP	Revenues	% on the Total
UK	AA	0.56%	5.85%	€ 7 982.00	32.71%
SPAIN	A	1.80%	7.09%	€ 4 064.00	16.65%
USA	AA+	0.00%	5.29%	€ 4 093.00	16.77%
Rest of the World (mainly Asia Pacific, Latin America & Caribbean, Africa & Middle East)		4.06%	9.35%	€ 8 267.00	33.87%

Table 3.2: *IAG – Revenue Subdivisions*

For USA the final ERP is equal to the implied equity risk premium[37] of 5.29% (exploited by A. Damodaran's tables – NYU Stern University website, updated in December 2018), without therefore adding any default spread since it is considered as major developed country. Instead, for the other countries, sovereign default spreads are usually used as measures of additional country risk premiums that equity-analysts would demand for investing in the equity of those countries. Hence, for UK and Spain, a small default spread (based on the sovereign rating) is added to the implied equity risk premium, giving a final ERP of 5.85% for UK and one of 7.09% for Spain. For the *Rest of the World* the default spread has been computed as the median value of the various default spreads of the countries belonging to Asia Pacific, Latin America & Caribbean, Africa & Middle East (which are the countries, indicated by the firm itself, from where the other revenues come), bringing to a final estimate of the ERP equal to 9.35%. Weighting then these ERPs by the shares of revenues coming from the specific countries, the resulting weighted average equity risk premium for IAG Company is 7.15%.

[37] The implied equity risk premium is an alternative way to estimate risk premium. It does not require historical data or adjustments for country-specific risks. It is obtained from the difference between the present value of dividends growing at a constant rate and the risk-free rate. It will be our starting point.

Table 3.3 below incorporates the data of the selected countries belonging to the areas where the other revenues come from (specifically, Asia Pacific, Latin America & Caribbean, Africa and Middle East).

Country	Region	Moody's rating	Rating-based Default Spread	Country Risk Premium
Abu Dhabi	Middle East	Aa2	0.56%	0.69%
Angola	Africa	B3	7.34%	9.03%
Argentina	Central and South America	B2	6.21%	7.64%
Aruba	Caribbean	Baa1	1.80%	2.22%
Bahamas	Caribbean	Baa3	2.48%	3.06%
Bahrain	Middle East	B2	6.21%	7.64%
Bangladesh	Asia	Ba3	4.06%	5.00%
Barbados	Caribbean	Caa3	11.28%	13.87%
Belize	Central and South America	B3	7.34%	9.03%
Benin	Africa	B1	5.08%	6.25%
Bermuda	Caribbean	A2	0.96%	1.18%
Bolivia	Central and South America	Ba3	4.06%	5.00%
Botswana	Africa	A2	0.96%	1.18%
Brazil	Central and South America	Ba2	3.39%	4.17%
Burkina Faso	Africa	B2	6.21%	7.64%
Cambodia	Asia	B2	6.21%	7.64%
Cameroon	Africa	B2	6.21%	7.64%
Cape Verde	Africa	B2	6.21%	7.64%
Cayman Islands	Caribbean	Aa3	0.68%	0.84%
Chile	Central and South America	A1	0.79%	0.98%
China	Asia	A1	0.79%	0.98%
Colombia	Central and South America	Baa2	2.15%	2.64%

Congo (Democratic Republic of)	Africa	B3	7.34%	9.03%
Congo (Republic of)	Africa	Caa2	10.16%	12.50%
Costa Rica	Central and South America	B1	5.08%	6.25%
Côte d'Ivoire	Africa	Ba3	4.06%	5.00%
Cuba	Caribbean	Caa2	10.16%	12.50%
Curacao	Caribbean	A3	1.35%	1.67%
Dominican Republic	Caribbean	Ba3	4.06%	5.00%
Ecuador	Central and South America	B3	7.34%	9.03%
Egypt	Africa	B3	7.34%	9.03%
El Salvador	Central and South America	Caa1	8.46%	10.41%
Ethiopia	Africa	B1	5.08%	6.25%
Fiji	Asia	Ba3	4.06%	5.00%
Gabon	Africa	Caa1	8.46%	10.41%
Ghana	Africa	B3	7.34%	9.03%
Guatemala	Central and South America	Ba1	2.82%	3.47%
Honduras	Central and South America	B1	5.08%	6.25%
Hong Kong	Asia	Aa2	0.56%	0.69%
India	Asia	Baa2	2.15%	2.64%
Indonesia	Asia	Baa2	2.15%	2.64%
Iraq	Middle East	Caa1	8.46%	10.41%
Israel	Middle East	A1	0.79%	0.98%
Jamaica	Caribbean	B3	7.34%	9.03%
Japan	Asia	A1	0.79%	0.98%
Jordan	Middle East	B1	5.08%	6.25%
Kenya	Africa	B2	6.21%	7.64%
Korea	Asia	Aa2	0.56%	0.69%
Kuwait	Middle East	Aa2	0.56%	0.69%
Lebanon	Middle East	B3	7.34%	9.03%
Macao	Asia	Aa3	0.68%	0.84%
Malaysia	Asia	A3	1.35%	1.67%
Maldives	Asia	B2	6.21%	7.64%

Mauritius	Asia	Baa1	1.80%	2.22%
Mexico	Central and South America	A3	1.35%	1.67%
Mongolia	Asia	B3	7.34%	9.03%
Montserrat	Caribbean	Baa3	2.48%	3.06%
Morocco	Africa	Ba1	2.82%	3.47%
Mozambique	Africa	Caa3	11.28%	13.87%
Namibia	Africa	Ba1	2.82%	3.47%
Nicaragua	Central and South America	B2	6.21%	7.64%
Nigeria	Africa	B2	6.21%	7.64%
Oman	Middle East	Baa3	2.48%	3.06%
Pakistan	Asia	B3	7.34%	9.03%
Panama	Central and South America	Baa2	2.15%	2.64%
Papua New Guinea	Asia	B2	6.21%	7.64%
Paraguay	Central and South America	Ba1	2.82%	3.47%
Peru	Central and South America	A3	1.35%	1.67%
Philippines	Asia	Baa2	2.15%	2.64%
Qatar	Middle East	Aa3	0.68%	0.84%
Ras Al Khaimah (Emirate of)	Middle East	A2	0.96%	1.18%
Rwanda	Africa	B2	6.21%	7.64%
Saudi Arabia	Middle East	A1	0.79%	0.98%
Senegal	Africa	Ba3	4.06%	5.00%
Sharjah	Middle East	A3	1.35%	1.67%
Singapore	Asia	Aaa	0.00%	0.00%
Solomon Islands	Asia	B3	7.34%	9.03%
South Africa	Africa	Baa3	2.48%	3.06%
Sri Lanka	Asia	B1	5.08%	6.25%
St. Maarten	Caribbean	Baa2	2.15%	2.64%
St. Vincent & the Grenadines	Caribbean	B3	7.34%	9.03%
Suriname	Central and South	B2	6.21%	7.64%

	America			
Swaziland	Africa	B2	6.21%	7.64%
Taiwan	Asia	Aa3	0.68%	0.84%
Tanzania	Africa	B1	5.08%	6.25%
Thailand	Asia	Baa1	1.80%	2.22%
Trinidad and Tobago	Caribbean	Ba1	2.82%	3.47%
Tunisia	Africa	B2	6.21%	7.64%
Turks and Caicos Islands	Caribbean	Baa1	1.80%	2.22%
Uganda	Africa	B2	6.21%	7.64%
United Arab Emirates	Middle East	Aa2	0.56%	0.69%
Uruguay	Central and South America	Baa2	2.15%	2.64%
Venezuela	Central and South America	C	18.00%	22.14%
Vietnam	Asia	Ba3	4.06%	5.00%
Zambia	Africa	Caa1	8.46%	10.41%

Table 3.3: *Asia Pacific, Latin America & Caribbean, Africa & Middle East Risk Premiums*
Source: NYU Stern University – Datasets – Country Risk Premiums

Regarding the last component, we should introduce before the formula for the levered beta (under the assumption that all of the firm's risk is borne by the stockholders and that debt creates a tax benefit to the firm):

$$\beta_L = \beta_U * \left[1 + (1 - t) * \left({}^{D}/_{E}\right)\right] ,$$

where:

β_L is the levered beta for equity in the firm;

β_U is the unlevered beta of the firm (beta of the firm's assets);

t is the marginal tax rate for the firm;

$D/_E$ is the Debt/Equity ratio (market values).

The unlevered beta for a firm (also known as *asset beta)* depends on the types of businesses in which it operates and its operating leverage. This industry unlevered beta will be used later to re-lever the beta of the company, in order to have a more precise estimate of its equity-risk component.

For this reason, the analysis starts by selecting 21 listed comparable firms (meaning that they are mostly operating in the airlines industry). Table 3.4 below includes the names of the selected comparable firms, together with the prevailing country in which they operate (among these firms, 6 are European, 8 are American and 7 are Asian-Pacific) and the corresponding marginal tax rate.

Comparable Firm	Country (in which it mainly operates)	Marginal Tax Rate (KPMG)
RYANAIR	Ireland	12,5%
AIRFRANCE KLM	France	33,0%
LUFTHANSA GROUP	Germany	30,0%
WIZZ AIR	UK	19,0%
EASYJET	UK	19,0%
FLYBE	UK	19,0%
CATHAY PACIFIC	Honk Hong	30,8%
JET AIRWAYS	India	35,0%
THOMAS COOK India	India	35,0%
MALAYSIA AIR	Malaysia	24,0%
SINGAPORE AIRLINES	Singapore	17,0%
THAI	Thailand	20,0%
PEGASUS	Turkey	22,0%
DELTA AIRLINES	USA	27,0%
SKYWEST INC	USA	27,0%
UNITED AIRLINES - UNITED CONTINENTAL HOLDINGS	USA	27,0%

AMERICAN AIRLINES	USA	27,0%
SOUTHWEST AIRLINES	USA	27,0%
ALLEGIANT TRAVEL CO	USA	27,0%
SPIRIT AIRLINES INC	USA	27,0%
JETBLUE AIRWAYS CORP	USA	27,0%

Table 3.4: *List of IAG's Comparable Firms*

The unlevered beta for each company has been estimated according to the following formula:

$$\frac{levered\ beta}{1 + (1 - marginal\ tax\ rate) * \frac{D}{E}}\ .$$

The levered beta has been retrieved from Bloomberg (calculated against 5 years' worth of weekly data of the relevant stock index), while the marginal tax rate is the one of the countries each company belongs to and earns its operating income and it has been retrieved from KPMG website[38].

As stated in the second chapter, the analysis should be implemented with the use of market values (rather than the book values) for Equity and Debt. The market value of equity, which is usually the number of shares outstanding multiplied by the current stock price, has also been taken from Bloomberg[39].

Instead, as suggested by A. Damodaran, the estimation of the market value of debt could be formalized through the following formula.

[38] KPMG is a global network of professional firms providing Audit, Tax and Advisory services. To retrieve the table for corporate marginal tax rates visit: https://home.kpmg/xx/en/home/services/tax/tax-tools-and-resources/tax-rates-online/corporate-tax-rates-table.html

[39] This choice comes from the fact that using average stock prices over time is not good practice since we want to measure the cost of raising funds today. Bloomberg, instead, considers multiple classes of shares to provide its computations.

$$\frac{interest\ expenses\ *(1-(1+cost\ of\ debt)^{average\ maturity}}{cost\ of\ debt + book\ debt\ *\ (1+cost\ of\ debt)^{average\ maturity}},$$

where the *interest expense* and the *average maturity* have been again retrieved from Bloomberg.

Regarding the *average maturity*, the choice has been to take the *weighted* average maturity (on the specific-firm-debt) reported by Bloomberg.

This formula is the translation of the book value of debt into a single coupon bond, with the coupon being equal to the interest expenses on all debt and the maturity being equal to the weighted average maturity of the debt. Then, this coupon bond is valued at the current cost of debt for the firm. Such translation of book-value debt into this kind of coupon bond derives from the difficulty of obtaining a real market-value debt since it is impossible to have all the debt in form of bonds outstanding and trading on the open market (the debt is indeed mostly represented by bank-debt).

To sum up, to estimate the average unlevered betas for the comparable firms, the median value of all their unlevered betas (equal to 0.649) has been taken and considered as the industry unlevered beta because, for a small sample, the median describes better the typical value (since extremes, which distort the mean, are excluded).

Estimating in the same way of comparable firms, IAG's market value of debt resulted to be 7,112.68 (million euros), giving a current $D/_E$ of 52.14%. Being then the weighted average marginal tax rate equal to 16.9% and the unlevered beta equal to 0.649, the obtained levered beta is equal to 0.930. From these three elements, it is now straightforward to arrive at the Cost of Equity applying the CAPM formula:

$$K_E = risk\ free\ rate + (\beta_L * ERP)$$

Considering a risk-free rate of 0.40%, a levered beta of 0.930 and an ERP of 7.15%, the resulting Cost of Equity is 7.05%.

3.3.3 Weighted Average Cost of Capital

As already stated at the beginning of this analysis, if one wants to measure the composite return to all claimholders, the most correct hurdle rate to use is the Weighted Average Cost of Capital, which is computed by estimating the costs of individual financing sources (i.e., Debt, Equity and Preferred Stock) and by proportionally weighting them. Since for the year 2018 there were not any Preferred Stock or any other hybrid security (to tell the truth, there were a couple of Convertible bonds but their debt and equity components were already divided in the balance sheet and, hence, included in the respective costs calculation), once the costs of the various components of financing have been computed, the last step in order to get to the Weighted Average Cost of Capital is choosing the weights of its components. *As a general rule*, as underlined by A. Damodaran, *the weights used in the cost of capital computation should be based on market values. "This is because the cost of capital is a forward-looking measure and captures the cost of raising new funds to finance projects. Because new debt and equity has to be raised in the market at prevailing prices, the market value weights are more relevant".*

To recapitulate, considering:

$$E = 13{,}742.00$$

$$D = 7{,}112.68$$

$$K_E = 7.05\ \%$$

$$K_D = 1.67\ \%$$

$$K_D * (1 - tax) = 1.39\ \%$$

We finally get to a Weighted Average Cost of Capital for IAG Company equal to 5.11%, which is lower than the Cost of Equity since the Cost of Capital is a measure of the composite cost of raising money that a firm faces, and in this process the tax benefits should be taken into consideration.

CHAPTER 4

GA's Approach Specifics

In this chapter the calculations performed in the previous one for IAG Company are used as inputs for the specific Genetic Algorithm model applied. There are the mathematical functions used, together with the constraints and their relative explanations.

In our specific case, the structure of the potential solution will be a little bit more complex than the one presented in the last chapter, but the methodology through which the GA's operators work (with the aim of convergence to the global optimum) is the same. Furthermore, the role of the fitness function remains unchanged, but has to be adapted to the company's capital structure optimization problem. Instead, one difference regards the solution space: up to now there were no constraints, nor about the number of potential solutions, nor about the values that the optimum solution should have, but the use of GA for such kind of analysis requires to define ex-ante the space of the solutions. This simply means that one should provide a set of potential efficient variables to the algorithm. These variables are the ones that will be used to define the function and the initial population. Then, the GA will proceed trying to understand which ones have the most discriminatory power.

4.1 Data Collection

It is important to underline that the aim of this chapter is to provide a model for the optimal capital structure for IAG Company based on the implementation of the Genetic Algorithm and, therefore, we will start from the calculations performed in the previous chapter, even though there are little modifications. These variations refer to the weights of the different components of financing. Just to recap, the WACC has been computed by weighting the costs of equity and debt through their market values. Here, we will instead look at the Cost of Capital from a slightly different perspective, using book values both for equity and debt rather than their market values.

The decision of using book value weights is not applied to this thesis only, but it is rather a common choice also among many firms. The most common reasons reported by mangers or analysts of such firms are:

1. Since book values are not as volatile as the market values, they are considered more reliable;
2. Weighting by book values, managers obtain more conservative debt ratios[40];
3. Since accounting returns are based on book values, managers should be following a consistency principle/approach and use therefore book values.

These three argumentations have, however, a couple of drawbacks, in particular:

1. Since firms' value changes over time, especially when new information (regarding both the companies their selves, the specific sector and, more generally, the global economy as well) become available on the market, market values reflect in a better way such kind of changes (because they are constantly updated. Differently, book values are updated, at maximum, every quarter, and sometimes even just yearly);
2. Even if the book value of debt and its market value are often similar (also in our specific case, with the values of 7,112 and 7,509 million euros respectively), this is not true for the book and market value of equity, with the last being usually well above its book value. Hence, the Cost of Debt will be well below than the Cost of Equity, causing the Cost of Capital computed through book values being much lower than the one computed through market values;
3. There is no economic fundamental in using book values, since the alternative of investing in a company is to invest somewhere else on the market.

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[40] The debt ratio measures the percentage of funds provided by debt holders. Inversely, the equity ratio defines the ratio of equity to total capital.

Even though the choice of market values has been highlighted as preferable, for our specific analysis the use of book values is mainly because of two reasons. Firstly, a practical reason: the output of the GA will be a Debt ratio and it will be easier for managers to change the firm's structure starting from fixed values. Managers, of course, can not decide the firm's market values, but they can rather modify only the book value of debt (for example by issuing corporate bonds or taking out additional bank's loan) and/or the one of equity (for example by stock buybacks or by issuing new shares). Secondly, in our specific case, IAG's equity and debt (in book value terms) produce an higher WACC then the WACC computed with their market values (meaning, therefore, that the debt ratio is not more conservative).

4.2 Model Development

In most of finance applications and optimization problems there are more than one single objective, and they are often conflicting (i.e., maximize performance while reducing costs or different objectives between Shareholders and Managers). In order to find feasible solutions for such kind of multi-objective optimization problems several scientists suggested the use of Evolutionary Metaheuristics (methods that, starting from a bunch of random solutions, try to reach optimal solutions at each generation). In these cases, the optimal solution of one objective would be, most likely, not the best solution for the other objectives and, therefore, a set of solutions (representing the best trade-offs among the various objectives) is required to come up to the optimal solutions for all objectives. In multi-objective optimization problems, the values of the objective functions create a multi-dimensional space called objective space, in which each decision variable inside corresponds to a point. In a simple representation of decision making the trade-off curve tells that, taking the extreme optimal of one objective, a compromise in the other objective is required.
Hence, we will refer to the concept of *Domination* to compare the solution with respect to different objective functions:

A feasible solution x_1 is said to dominate another feasible solution x_2 (mathematically, in the case of a minimization problem: $x_1 \preccurlyeq x_2$) if and only if:

- The solution x_1 is no worse than x_2 with respect to all objective values;
- The solution x_1 is strictly better than x_2 in at least one objective value.

We can equivalently say that solution x_1 dominates solution x_2 or that solution x_1 is non-dominated by solution x_2.

A solution is *Pareto-optimal* if it is not dominated by any other solution in the decision variable space. The Pareto-optimal solution is the optimal one with respect to all objectives and no improvement can be achieved in any objective that does not lead to degradation in at least one of the remaining objectives[41]. The set of all feasible solutions which are not dominated by any others is called *non-dominated, Pareto-optimal set* or *Pareto front.* The main goal in multi-objective optimization problems is therefore the pursuit of a set of non-dominated solutions with the least distance to Pareto-optimal set. A graphical representation of the Pareto front can be seen in Figure 4.1.

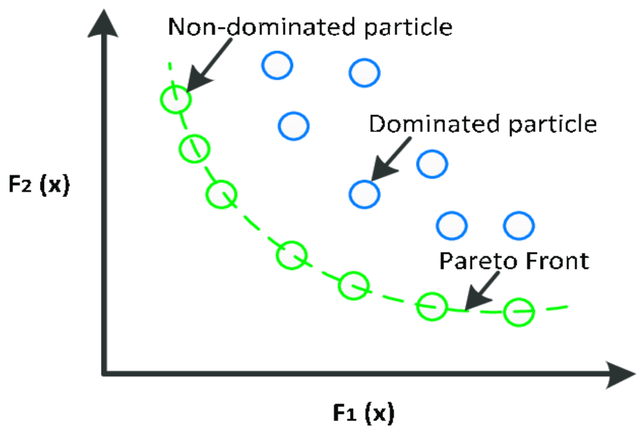


Figure 4.1: *Pareto Frontier*

[41] DEB, K. (2002). *Multi-objective optimization using evolutionary algorithms.* John Wiley and Sons, Inc.

Since evolutionary algorithms can generate a set of non-dominated solutions in each generation, in this thesis a MOGA (Multi Objective Genetic Algorithm), which was first introduced in 1993 by Fonseca and Fleming[42], is applied in order to generate the Pareto-optimal set for the research of the optimal capital structure for IAG Company. To illustrate which are the steps the MOGA goes through, we can refer to Figure 4.2. The figure shows the simple and logical flow chart of the algorithm, from the beginning of the process (data input) until the end (production of a Pareto frontier), going through the different steps (i.e., selection, crossover and mutation operations).

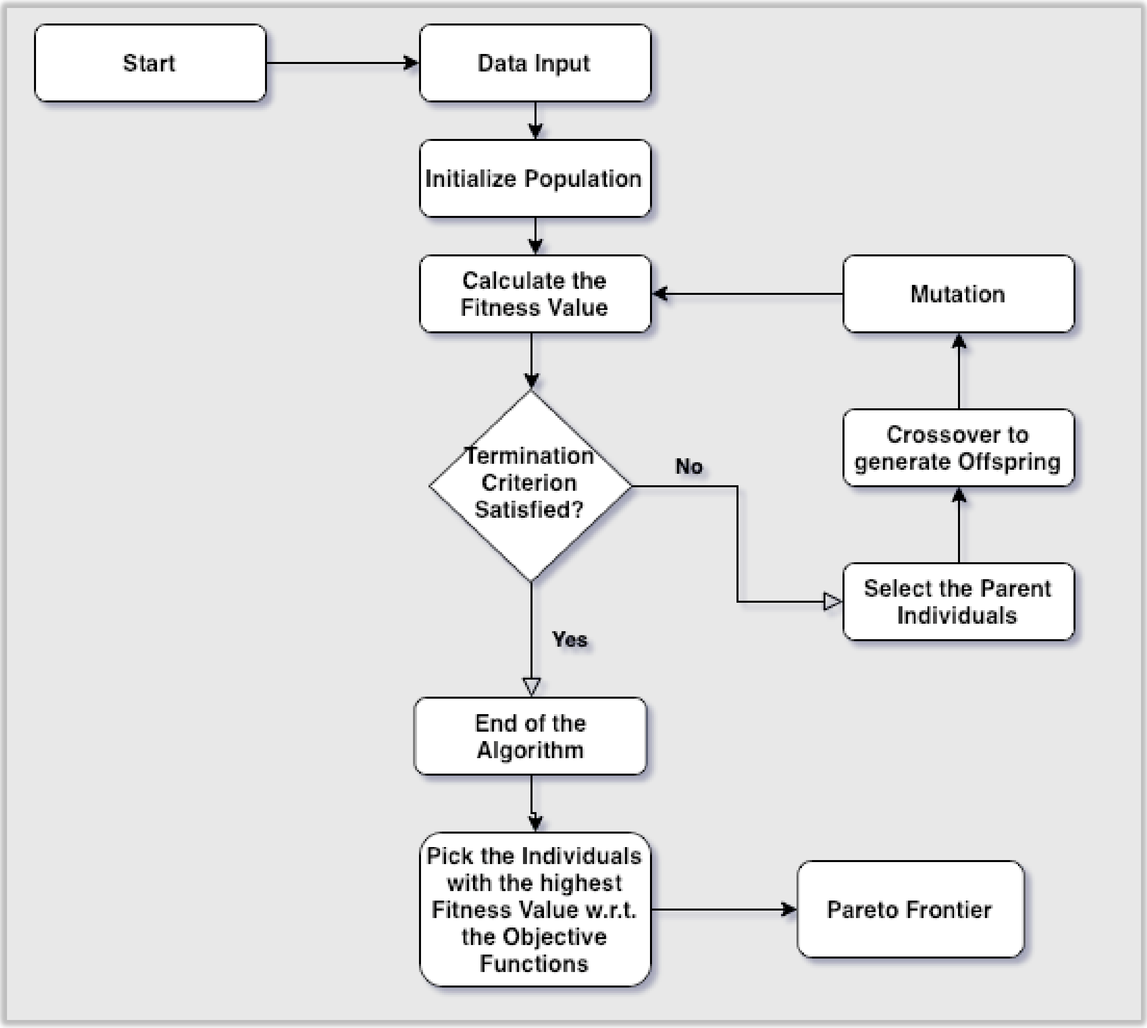


Figure 4.2: *Multi-objective Genetic Algorithm Flow Chart*

[42] FONSECA, C. M., FLEMING, P. J. (1993). *Genetic algorithms for multi-objective optimization: Formulation, discussion and generalization.* Proceedings of the 5th International Conference on Genetic Algorithms. Ubana-Champaign, I11. Pages 416-423.

The Multi Objective Genetic Algorithm tries to optimize the components of a vector-valued objective function, which, in our case, consists of two different objectives (i.e., the minimization of the Weighted Average Cost of Capital and the maximization of the Interest Coverage Ratio). Besides the setting of the objective functions, the optimal solutions that we want to find are also subject to some specific constraints (i.e., limits on the single financing components ratios) that have to be included in the configuration of the algorithm in MATLAB software. The problem can be formalized as in the following way:

$$\begin{cases} Minimize & WACC \\ Maximize & ICR \\ Subject\ to\text{:} & D + E = 1; \\ & 0.2 \leq D \leq 0.8; \\ & 0.2 \leq E \leq 0.8. \end{cases}$$

As already described in the first and third chapter, the first objective is represented by the minimization of the Weighted Average Cost of Capital function (i.e., WACC), while the second objective is the maximization of the Interest Coverage Ratio (i.e., ICR).
The Interest Coverage Ratio belongs to the coverage ratios' group, which is a group of measures of the firms' ability to service their debt and meet their financial obligations (i.e., interest payments and/or dividends). It is computed by dividing EBIT (i.e., Earnings Before Interest and Taxes) by Interest Expenses and it is commonly used to assess whether companies might be in troubled financial situations, measuring exactly their ability to pay the interest expenses on their debt. Its formulation is the following:

$$ICR = \frac{Total\ Revenues - Total\ Expenditures\ on\ Operations}{Interest\ Expense}$$

In the above described problem, the voice *"Subject to"* sets the constraints to be applied to this specific problem solver:

- $D + E = 1$ means that the sum of Debt and Equity constitutes 100% of the firm's Total Capital;
- $0.2 \leq D \leq 0.8$ fixes 20% minimum and 80% maximum levels on the debt ratio in order to guarantee repayment capacity and take advantage of the tax-shield;
- $0.2 \leq E \leq 0.8$ fixes a 20% minimum base and an 80% maximum limit on the equity ratio for the stability of the capital structure.

With regards to the ICR, the higher it is, the better it is, since it means more ease in the payments of dividends and/or interests on debt. Nevertheless, this measure alone can not be considered as an exhaustive indication of financial difficulty, and it should be instead evaluated through a deeper dive into the firm's financial statement (looking for example at other liquidity and solvency ratios). The choice, however, fell on the ICR because it represents the other side of the coin with respect to the WACC, in the sense that a higher Debt ratio leads to a decrease in the WACC (positive consequence) but also to a decrease in the ICR (negative consequence). Minimizing the first function and maximizing the second, this thesis aims at finding a set of optimal solutions combining these two conflicting objectives. When talking about *conflicting objectives* we refer to the fact that the equity-shareholders' target is not aligned with the one of the debt-holders. Indeed, the equity-shareholders' target is the pursuit of the company's economic interests, while debt-holders would like the company to keep a lower debt ratio in order to be able to repay it. Hence, firms have incentives in the reduction of the use of private equity for their operations in order to raise the rate of return. On the contrary, an increase in the use of private equity for operations would consequently lead to a decrease in the debt ratio, which would in turn lessen the burden of debt service and improve the firms' financing stability. The reduction in the private equity's utilization is instead carried out through an increase of the debt ratio, which will consequently reduce the WACC too. However, firms should pay attention at the impact of the financial leverage because, raising too much their debt, they risk to reach a point where the debt level becomes unsustainable (because of the too high interest rates). From such perspective, the Interest Coverage Ratio may be interpreted as a threshold ratio defining an acceptable level for lenders, who want to evaluate the amount of the loan that can be covered by the firm's cash flow or other financial resources. Indeed, companies that do not keep a proper level of debt ratio usually face two consequences:

1. Costs of financial distress;
2. Bankruptcy costs.

The costs of financial distress can be attributed to the inability to negotiate long-term supply-contracts in the future. Furthermore, at some point the additional value of the interest tax shield would be offset by the increase in expected bankruptcy cost. At this point the value of the firm will start to decrease as more debt is added and the most direct drawback will be the disincentive to debt financing.

In our specific IAG Company's case study, the EBIT can be found in the Income Statement of the company, retrieved in the section "Investors and shareholders – Results and reports" of their website, under the name of Operating profit ($Operating\ Profit = Total\ Revenue - Total\ expenditure\ on\ operations$) and it is equal to 3,678 million euros, while the Interest Expense can be found under the name of *finance costs*, again in the Income Statement. Such *finance costs* are mainly interest expenses on bank borrowings, financial leases or other borrowings. Since all these voices (i.e., interest expenses, financial leases) fall into the definition of debt and could be directly related to it, the income statement *finance costs*-voice could be described and represented as a function of debt. The approximation of this function has been computed in Excel taking into consideration debt and *finance costs* values of the last five years for IAG Company. A linear approximation results in the following function:

$$y = 0.0509x - 150.15,$$

where *x* represents the book value of debt.

Clearly, as debt increases, interest expenses increase as well. The parameter in front of x (i.e., debt value) is small because the amount of debt is expressed in million euros while the value of the interest expenses (i.e., *finance costs*) is in thousands.
The plot of the *finance costs* approximation as a function of debt can be seen in Figure 4.3 below, where also its R-squared value ($R^2 = 0.9869$) is reported.

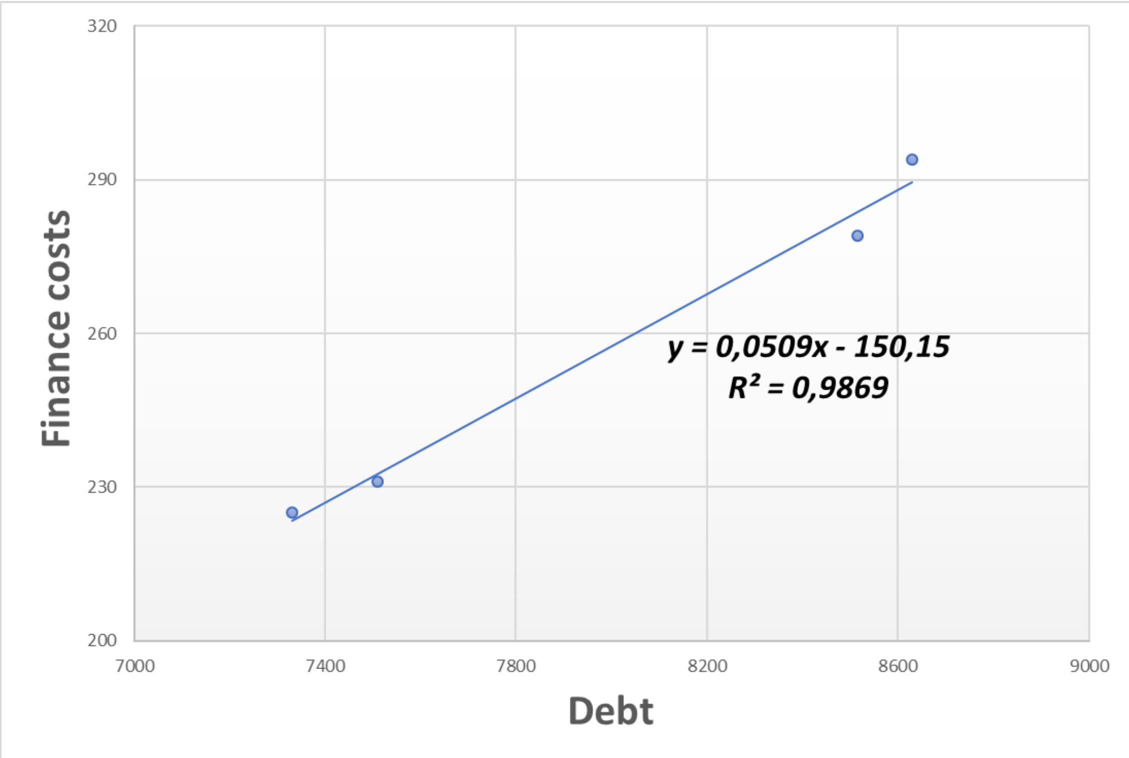


Figure 4.3: *Finance costs estimation as a function of Debt*

The scope of this thesis is the specification, through a Multi-objective Genetic Algorithm, of an optimized capital structure model in order to reach a balanced structure between the interests of the company itself and its creditors. This balanced structure synchronizes both *profitability* through the minimization of WACC (in the meaning of average rate of return the company expects to compensate to all its different investors) and *repayment capacity* through the maximization of the Interest Coverage Ratio, where the numerator (i.e., the EBIT) is implicitly assumed constant for the next calculations while the denominator (i.e., the Interest expense) is represented by the linear approximated function (depending on the Debt ratio). The reason behind this balanced structure is that any company needs to find an optimal point where its profitability is maximized considering also the lenders' perspective.

4.3 Model Specifics

In order to implement our specific Multi-objective Genetic Algorithm, the last updated version of the software MATLAB has been used, precisely version *MATLAB R2019b.* In this version there are different graphic user interfaces called *toolboxes.* To program a MOGA, one can use the *gamultiobj*-solver in the *OPTIMIZATION*-toolbox. Since the basic calculation of the MOGA in the *OPTIMIZATION*-toolbox in MATLAB is fitted to minimum-optimization, for the maximum-optimization of our second objective function (the ICR depending on the debt ratio) we need to multiply it by -1. The *gamultiobj*-solver, through a set of operators that work on the initial randomly generated population, computes the next generation of the population using the *non-dominated rank*[43] and a distance measure[44] of the individuals. Furthermore, this solver uses a controlled elitist[45] genetic algorithm. An elitist genetic algorithm favors the genes (i.e., the individuals) with better fitness value (i.e., lower rank). A controlled elitist genetic algorithm also favors individuals which can help in the population diversity's increase (even if they have lower fitness value). Maintaining the diversity in the population is important for the convergence to the optimal set of solutions on the Pareto front.

[43] The non-dominated rank is assigned to each individual using the relative fitness. Individual *p* dominates *q* (i.e., *p* has lower rank than *q*) if *p* is strictly better than *q* in at least one objective and *p* is no worse than *q* in all objectives. Individuals *p* and *q* are considered to have equal ranks if neither dominates the other.

[44] The distance measure of the individuals is used to compare individuals with equal rank. It is a measure of how far an individual is from the other individuals with the same rank.

[45] The concept of "elitism" is generally related to memory (in the sense of remember the best solution found). Particularly, for evolutionary algorithms like MOGA, *elitism* involves copying a small portion of the best solutions found into the next generation. The use of unchanged fittest candidates for building the next generation ensures that the algorithm does not waste time re-discovering previously discarded solutions.

Instead of using the *OPTIMIZATION*-toolbox and the *gamultiobj*-solver, one can write the code for the MOGA implementation. In a separated MATLAB file there are the two objective functions' formulation and the imputation of the number of variables, which in our case is just one (i.e., the debt ratio) since the other unknown component of the total capital (i.e., the equity ratio) can be retrieved simply by computing $1 - x$, where *x* is our decision variable. The structure of the code is set in the way shown in Figure 4.4.

```
FitnessFunction = @Objective_function;
numberOfVariables = ;
lb = []; % Lower bound
ub = []; % Upper bound
A = []; % Linear inequality constraints
b = []; % Linear inequality constraints
Aeq = []; % Linear equality constraints
beq = []; % Linear equality constraints
options = optimoptions();

[Outputs] = gamultiobj(FitnessFunction,numberOfVariables,A,b,Aeq,beq,lb,ub,options);
```

Figure 4.4: *Multi-objective Genetic Algorithm's typical code structure*

After defining which are the objective functions to minimize and how many decision variables have to be defined, the user can additionally set the lower and upper bounds to such decision variables and/or linear equality and/or inequality constraints. Regarding our decision variable, the lower limit of 20% and the upper limit of 80% has been chosen in order to take advantage of the tax-shield benefit on the debt level, considering however that a too high level would be unsustainable. For the IAG Company's Capital Structure Optimization nor linear equality nor linear inequality constraints have been set.

Then, to implement our Multi-objective Genetic Algorithm in MATLAB, the specific parameters (population size, generation number, maximum number of iterations, selection method, crossover and mutation functions) for optimizing IAG Company's capital structure have to be set. Table 4.1 below summarizes the basic settings used for this MOGA-model, which were selected following the guidelines provided in previous literature[46].

MOGA parameter	Value
Population size	100
Population type	Double vector
Maximum number of generations	150
Selection method	Tournament
Crossover function	Scattered
Crossover fraction	0.8
Mutation method	Adaptive feasible
Variable's lower bound	0.2
Variable's upper bound	0.8

Table 4.1: *Multi-objective Genetic Algorithm parameters*

Given these parameters, the algorithm starts from a population of 100 random solutions of our decision variable *x* (i.e., the debt ratio). The population type is then used to specify the input type for the fitness function. The default type is *doubleVector*, which represents the option used for mixed integer programming. The algorithm continues its process until it reaches one of the possible stopping criteria (that will be examined later on in this chapter). In our specific case we will see that the algorithm stopped because it exceeded the maximum number of generations (i.e., iterations).

The Selection method adopted for selecting the parents for the next generation is the *Tournament*-Selection with default size-value because it is the only available method for the *gamultiobj*-solver in MATLAB. The *Tournament*-Selection picks each parent by

[46] DEB, K., AGRAWAL, S., PRATAP, A., MEYARIVAN, T. (2000). *A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II.* Springer, Berlin.

randomly deciding *tournament size* players and then chooses the best individual out of them to be a parent.

The designated function picked for Crossover, which combines two individuals (i.e., the parents) to create a crossover child for the next generation, has been the *Scattered crossover* with the default crossover-probability. The *Scattered crossover* creates a random binary vector and selects the genes where the vector is equal to 1 from the first parent and the genes where the vector is equal to 0 from the second parent, combining then these genes to form the child. Most of the previous options (included *the Scattered crossover)* can not be used in the case in which linear constraints are present, because it may cause the population not to satisfy the constraints. Since we do not have linear constraints, we set the reproduction, which specifies how the Genetic Algorithm generates next generation's children, through the *CrossoverFraction* option. This option determines the fraction of the population at the next generation that is created by the crossover function.

The other Genetic Algorithm's operator is the mutation, which causes small random changes in the genes of the population in order to create mutated children. The option selected for our specific problem is the *Adaptive feasible,* which randomly generates directions that are adaptive with respect to the last successful generation. Usually the mutation probability is a constant value (i.e., all chromosomes have the same likelihood of being subject to mutation, irrespective of their fitness). With this specific option (i.e., keeping a step length along each direction), mutation becomes a function of fitness. In this way bounds and constraints are satisfied, the chance of disrupting a high-fitness chromosome is decreased and the exploratory role of low-fitness chromosomes is best exploited.

In addition, in table 4.2 are reported also all the options under which our specific MOGA-model has been programmed.

Multi-objective Options	Value
Pareto Fraction	0.2
Distance Measure Function	Phenotype
Function Tolerance	1e-4
Maximum Stall Generations	150

Table 4.2: *Multi-objective options*

The *Pareto Fraction* is a scalar from 0 to 1 which determines the fraction of the best fit genes to keep on the first Pareto front in order to maintain a diverse population. While its default value is 0.35 (i.e., the solver will try to limit the number of individuals in the current population that are on the Pareto front to 35% of the population size), for our specific optimization problem a 20% fraction has been decided in order to keep the most fit solutions down to a reasonable number, maintaining anyway a diverse population.

The *Distance Measure Function* is a measure of the concentration of the population computed by comparing the distance between individuals with the same rank in the function space (through the function handle *phenotype*). The distance function helps in maintaining diversity on the Pareto front by favoring the genes that are relatively far away on the front. These two options (*Pareto Fraction* and *Distance Measure Function*) together control the elitism of the Genetic Algorithm.

The *Function Tolerance* is a measure that is used to determine when the algorithm has to stop in the case in which the geometric average of the relative change in the spread value[47] of the Pareto solutions (i.e., the average relative change in the best fitness function value) over *Maximum Stall Generations*[48] option (default value is 100) is less than *Function Tolerance* option and the spread value is smaller than the average spread over the last *Stall Generations* option (in a simpler way we can say that the algorithm stops if the spread is small). The 0.0001 value specified for the Function

[47] The spread is a measure of the movement of the Pareto front. The coefficient for the geometric average of the spread-distance calculation is 0.5.

[48] The value of *Maximum Stall Generation* is based on a test (i.e., StallTest) for a geometric weighted average relative change.

Tolerance is the default value for Multi-objective optimization problems using Genetic Algorithms. The other potential conditions causing the stop of the solver are displayed at the end of the process as an output argument under the voice of *ExitFlag,* which is an integer. These additional stopping conditions could be the reach of the maximum number of generations, the achievement of an output function, the failed attempt of reaching a feasible point or the exceeding of the time limit.

4.4 Results

In the last line of the code reported in Figure 4.2 it is possible to see that in the square brackets the user has to insert the output arguments. The first two output arguments returned by *gamultiobj* are X (vector of the points of the decision variable on the Pareto front) and FVAL (the objective functions' values at the points' values X). The third and fourth output arguments *(exitFlag* and *Output)* return the reason why the MOGA stopped and information about the performance of the *gamultiobj* solver (i.e., the type of problem, the total number of generations, the total number of function evaluations, the average distance[49], the spread, the maximum constraint violation at the final Pareto set).

In the following table the *X* and *FVAL* values are reported. Each row of *FVAL* represents the WACC and ICR function values (in this specific optimization problem) at one Pareto-point in *X* (the set of the optimal selected debt ratios). In addition, there is also the respective equity ratio for each *x*-point. All the values, except the equity ratio that is simply computed by subtracting the *x*-value from 1, are the results of the optimization problem computed by the ga*multiobj* solver in MATLAB.

[49] The average distance is, by default, the standard deviation of the norm of the difference between Pareto front points and their mean.

The first line of table 4.4 has been added later and colored in light-blue in order to show which are the actual (2018) values of IAG company.

Debt_ratio	Equity_ratio	WACC	INT_COV_Ratio
52,77%	47,23%	5,56%	-15,92
52,09%	47,91%	5,65%	-22,02
66,89%	33,11%	4,40%	-12,49
59,75%	40,25%	4,93%	-18,02
57,18%	42,82%	5,14%	-19,91
79,94%	20,06%	3,19%	-5,79
60,19%	39,81%	4,89%	-17,13
69,97%	30,03%	4,05%	-10,04
74,21%	25,79%	3,83%	-7,90
57,24%	42,76%	5,12%	-19,90
53,89%	46,11%	4,96%	-19,88
63,20%	36,80%	4,80%	-15,14
79,99%	20,01%	3,15%	-5,77
65,49%	34,51%	4,46%	-13,63
61,38%	38,62%	4,91%	-16,91
70,44%	29,56%	4,02%	-12,42
74,68%	25,32%	3,61%	-6,99
79,61%	20,39%	3,25%	-6,13
69,09%	30,91%	4,21%	-11,34
52,13%	47,87%	5,65%	-22,28
73,29%	26,71%	3,69%	-6,35

Table 4.4: *Decision Variable and Objective function values on the Pareto front*

All the values of the ICR have to be read as positive values, since the negative sign is simply because the *gamultiobj* solver automatically minimizes the objective functions and so we had to multiply by -1 the ICR function to maximize it. It is possible to notice that the minimum optimum *x*-value is a Debt ratio equal to 52,19%, so well above the minimum set by the lower bound of 20%, while the maximum optimum is equal to 79,99% (the maximum set by the upper bound of 80%).

In figure 4.5 below is instead reported the pace of the debt ratio in each optimal solution of the Pareto front produced.

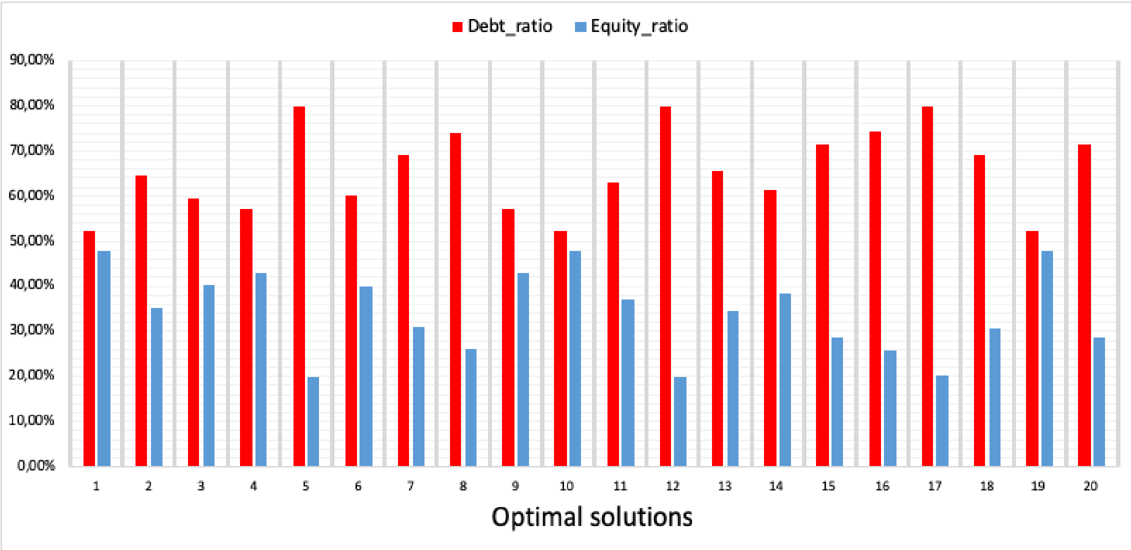


Figure 4.5: *Debt ratio values on the Pareto front*

As we have already described in the previous paragraph, the algorithm stops at some point when it meets one of the stopping criteria. The reason why our MOGA stopped producing the results of the above figure and table is reported in the following figure.

```
COMMAND WINDOW
Optimization terminated: maximum number of generations exceeded.
The number of points on the Pareto front was: 20
The number of generations was : 150
The average distance measure of the solutions on the Pareto front was: 0.0110635
```

Figure 4.6: *End of MOGA optimization*

The optimization process terminated because the algorithm reached the maximum number of generations (i.e. 150), returning the *x*-values on the Pareto front with an average distance between each other of approximately 0.011.

Through the *PlotFcn* option of the *gamultiobj* in MATLAB, the different data computed by the algorithm can be plotted in different ways showing different features. I selected the most interesting ones for this thesis's purpose in the figure below.

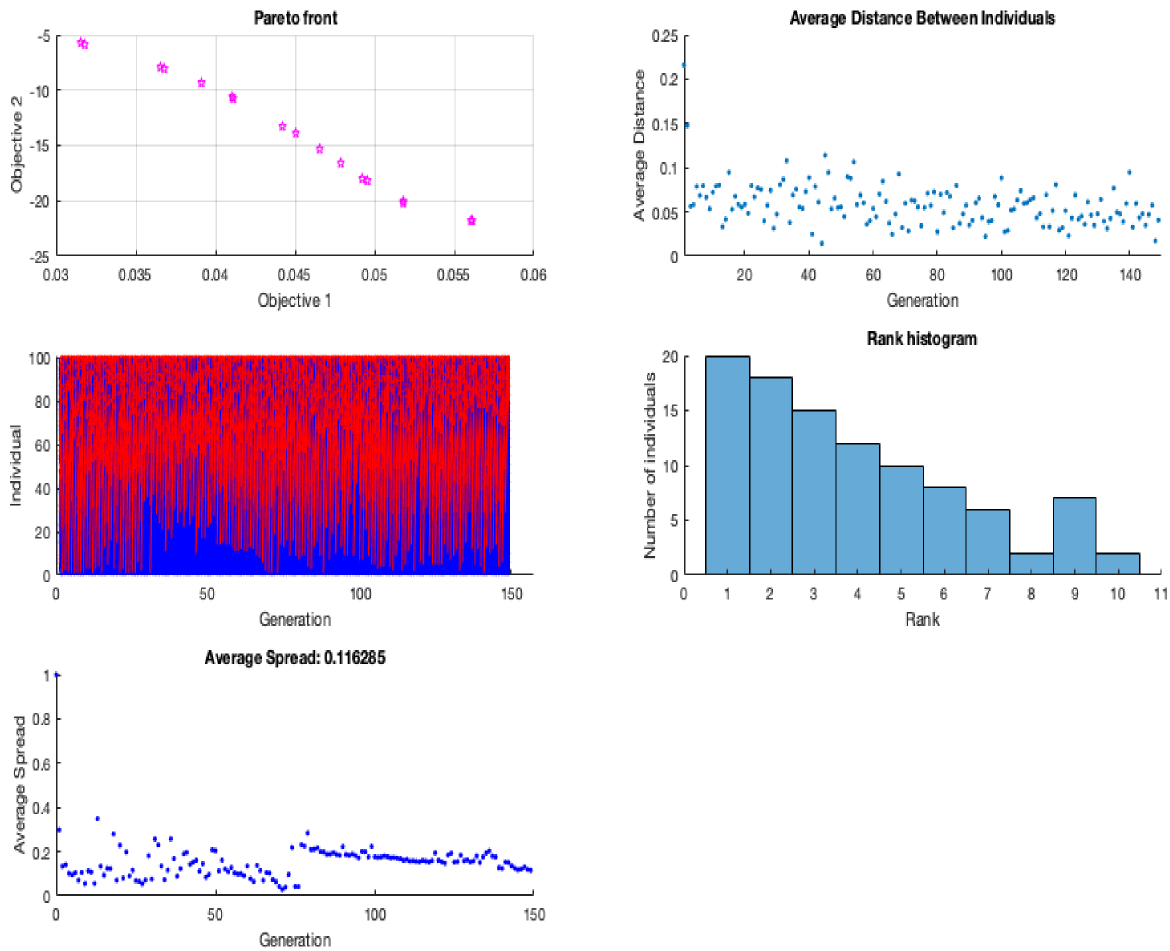
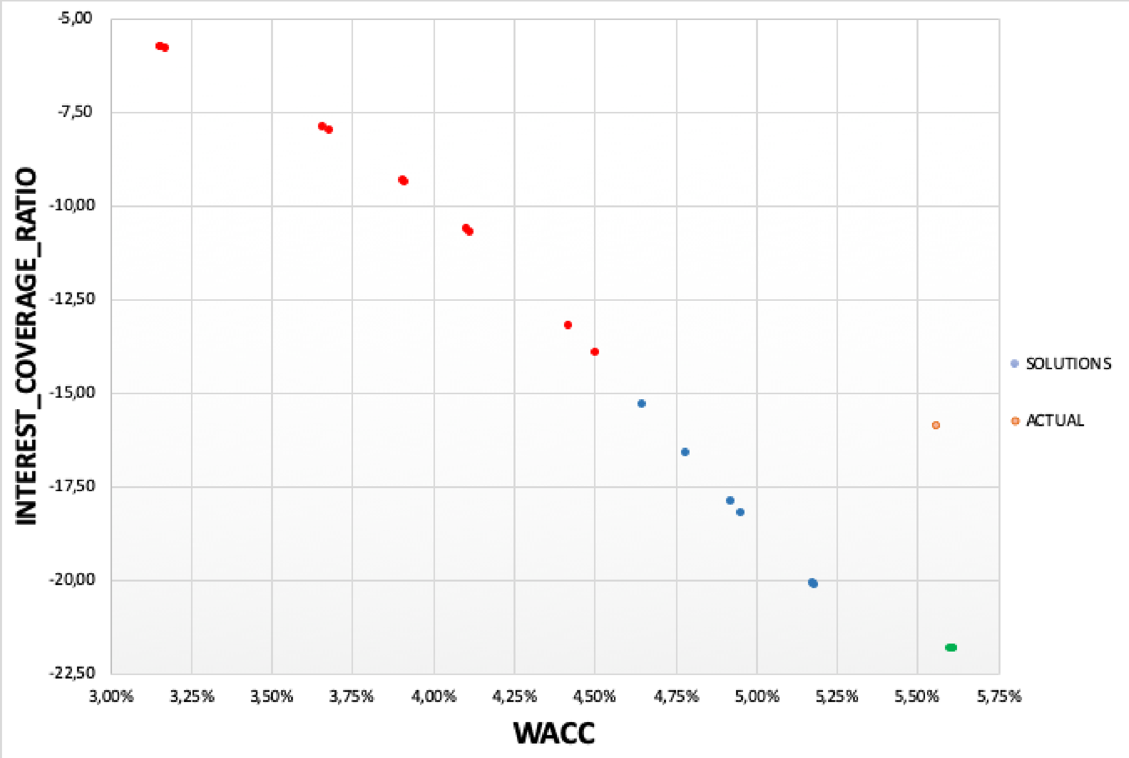


Figure 4.7: *gamultiobj plots*

The different plots above show different features for each graph:

- The graph on the top left corner is the Pareto front for the two objective functions;
- The graph on the top right corner shows the average distance between individuals at each generation;
- The graph on the middle-left is called *genealogy* of the individuals. The lines are colored between the generations, based on the operator that performed a change on the chromosomes of one generation going toward the next one: red lines represent mutated children, while blue lines represent crossover children;
- The graph on the middle-right shows an histogram of the initial population individuals' ranking. Individuals with rank equal to 1 are the ones on the Pareto front (i.e., 20 solutions), the individuals with rank equal to 2 are lower than at least one individual with rank equal to 1, therefore close to optimum but not on the optimal Pareto frontier, and so on;
- The graph on the bottom left corner represents the average spread (i.e., the measure of the movement of the Pareto frontier) as a function of the generations' number.

Adding the actual position of IAG Company (referring to the 2018 book values) with respect to the optimal solutions found by the algorithm it is possible to understand which are the changes that managers could make on the proportions of the company's capital components in order to reach the optimal capital structure analysed (maximizing therefore the company's profitability, while keeping an optimal level of debt-repayment capacity). Such analysis can be performed looking at the graph 4.1 below, where the yellow point shows IAG's actual capital structure.



Graph 4.1: *IAG Company's Pareto front with WACC minimization and Interest_Coverage_Ratio as objective functions*

On the x-axis and y-axis of the above graph there are the first and the second objective functions: the WACC and the ICR, respectively. The blue, red and green points constitute the Pareto optimal set (i.e., the Pareto front) found by running the Multi-objective Genetic Algorithm. These optimal solutions are signed with three different colors because I wanted to further skim the solutions. Even if, usually, in the economic literature, an ICR of value of three (and above) is considered optimal, the additional skimming is made clear through the red points, which denote the solutions found by the algorithm for which the ICR is below 15 (that is the nearest rounded value to the actual ratio of 15.42 for IAG Company). The reason behind such assignment of a minimum floor for the ICR is that an higher ICR allows to take into consideration also the involvement of different uncertainties in the financial environment. This skimming is, however, performed just on the ICR since the value of the WACC changes less than proportionally with respect to the ICR when there are changes in the debt value. Indeed, the minimum and the maximum values of the WACC on the Pareto front are

3.15% and 5.65% respectively, while the minimum and maximum values for the ICR are 5.77 and 22.28.

The green point represents instead the closest point (in debt-change absolute terms) to the actual position of IAG Company. IAG's actual position in the above graph is signed with the yellow color and shows the company's debt level at the end of 2018. Given the vector of solutions provided by the MOGA, the additional skimming on the corresponding ICRs and considering the closest position from the actual one in debt-change absolute terms, our suggestion is to move to this point (signed with the green color) as soon as possible. At this proposed optimal setting, 47.87% of the total capital should be funded by equity investment, and consequently the remaining part (52.13%) would be financed with bank debt. Since actually (at the end of December 2018) debt amounts to 7.5 billion, in order to reach the proposed optimal point (that is the second to last row in Table 4.4) the managers of IAG Company should decrease debt by 0.64%. A decrease of this kind may be carried out either by repaying part of its bank loans, either by buying back some corporate bonds for a value of 48 million euros.

Conclusion

The proposed model is built to deal with the optimization of IAG Company's capital structure satisfying the two objectives of minimizing the WACC (i.e., Weighted Average Cost of Capital) and maximizing the ICR (i.e., Interest Coverage Ratio).

Precisely, it should be noted that only the available book value data for the last year have been considered (with the only exception for the estimation of finance costs as a function of debt, for which five years book values have been utilized) and therefore the analysis is centered only on the relative debt and equity weights.

In order to make an adequate evaluation (i.e., in the proper environment) and a precise analysis, corporate analysts should consider first the firm's life-cycle stage. The usual companies' life-cycle is shown in Figure 5.1.

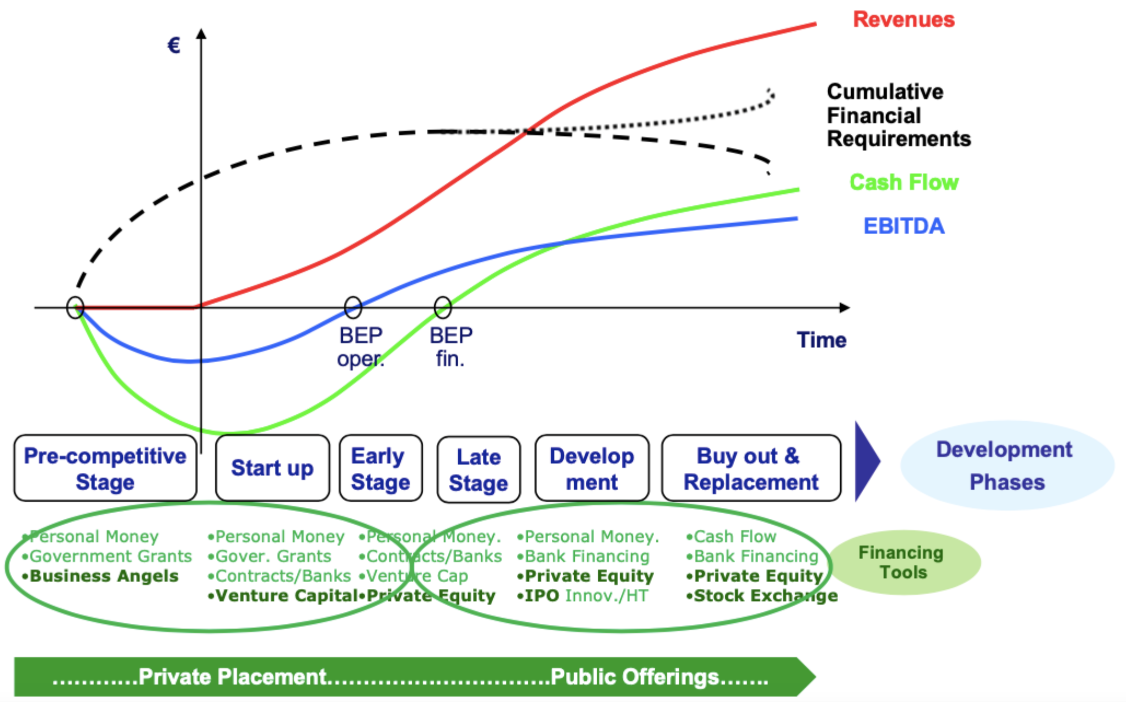


Figure 5.1: *Company's Life Cycle*

It is clear that, for example, at the Start-up or Early Stage (when the company is just at the beginning of its business' development), revenues could be negative (hence, the company can not take advantage of the tax-shield triggered by the leverage effect) and that the financing requirements are obviously high. Consequently, the debt ratio will be high too. When the firm goes toward the end of its life-cycle (i.e., in the Development or Replacement Stage through an hypothetical IPO – Initial Public Offering), most likely, there would be enough money (coming from the Cash Flow and from Private Equity) to cover the various activities and investments. This will in turn produce a decrease in the debt ratio.

Secondly, corporate analysts should perform economic and qualitative analysis on the company of interest and on the sector or business environment in which it operates, before running the Genetic Algorithm to find the optimal solutions.

In addition, it is important to underline that, for optimizing a capital structure, the objective functions must be chosen carefully because this is not a model ready to be applied to any company or to address any business-environmental condition. Indeed, setting for example two objectives that are not conflicting between each other (depending on the decision variables chosen) one runs the risk of incurring in results that are not really significant. For the sake of clarity, keeping the debt ratio as decision variable and setting as objective functions the minimization of WACC and the maximization of ROE (i.e., Return on Equity), the specific MOGA-model described in this thesis produces just a singular result. In fact, running the algorithm with these two objective functions, the Pareto front collapses in one single point (i.e., the sole optimal solution). The output after the run of the MOGA with these two objective functions is illustrated in Figure 5.2.

The reason behind the production of a solution solely derives from the fact that, increasing the debt-ratio, the *WACC*-function declines while the *ROE*-function rises (i.e., the two objective-function lines intersect). WACC declines because of the tax shield-effect generated by debt, while ROE increases since the denominator of this ratio decreases as the debt value rises, as demonstrated by its formula, that is displayed below.

$$ROE = \frac{NI}{E}$$

where *NI* and *E* represent *Net Income* and *Equity,* respectively.

In the IAG-specific case-study Equity is equal to $1 - Debt$[50].

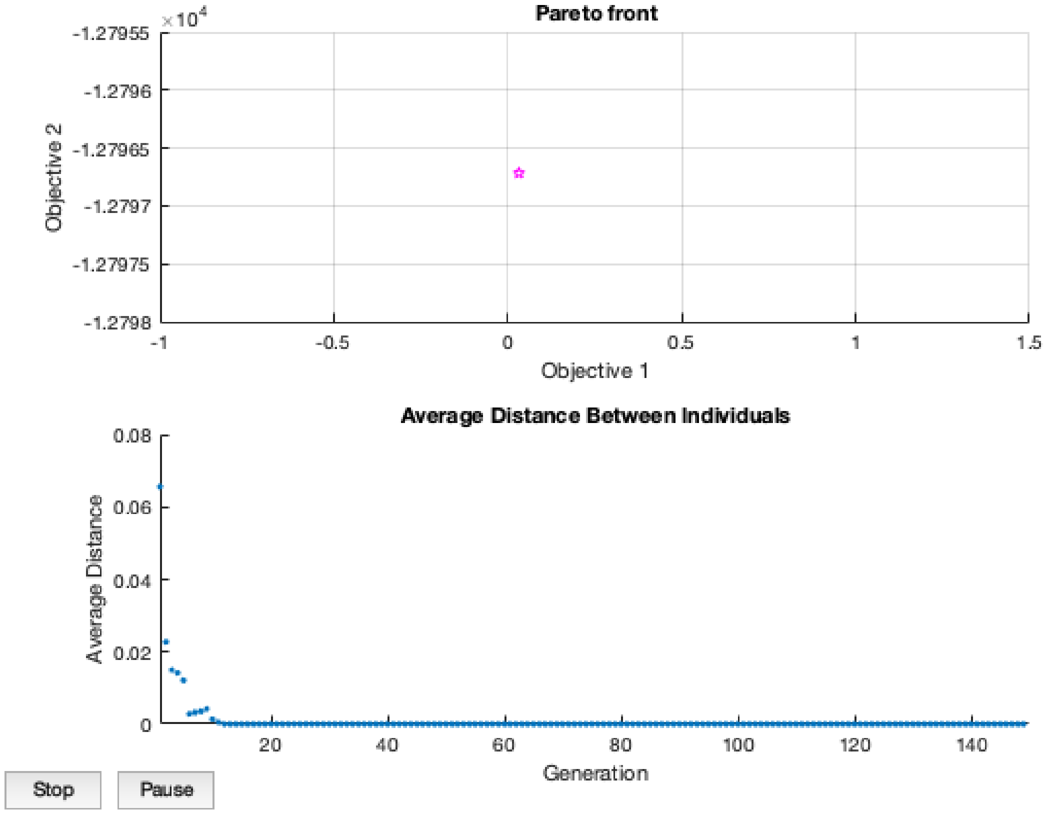


Figure 5.2: *gamultiobj plots with WACC-minimization and ROE-maximization as objective functions*

However, the model presented in this thesis bears some limitations. The most relevant one is given by the fact that there are many assumptions. First, modifying the debt and equity values (or, equivalently, their weights) also the other voices of the Income Statement and of the Balance Sheet will change consequently. Second, it is not

[50] This is true only in our specific case study because the capital of the company under analysis is actually composed only by Equity and Debt and there are not mixes of the two (i.e., hybrid securities).

reasonable to assume that the EBIT (i.e., the Operating Income) remains stable over the years because it is subject both to movements of revenues and costs as its formula demonstrates:

$$EBIT = Revenues - Costs.$$

In addition, the ICR should be evaluated together with other ratios possibly indicating financial difficulty (i.e., liquidity and/or solvency ratios) and should not be considered alone as exact indicator of financial trouble.

Last but not least, the main limitation is the availability of data. This analysis has been led using only the results and the data of the Company published on its website, through which it is not possible to have real and exact function estimations. Indeed, in order to have a more precise estimation, the knowledge of the different terms of loan contracts stipulated with the banks which finance the firm could be helpful for the calculation of the real interest expense. A greater availability of data may also bring to an improvement of the finance costs' (i.e., interest expenses) linear approximation by adding further points, accomplished for example by considering a wider time-horizon. Moreover, the fact that the firm operates in different countries (with different taxation regimes and even different currencies) adds even more difficulties in the estimation of the different functions.

Given the fact that here everything is assumed to remain stable and that there could not be an exact prediction about the revenues a company could earn during the year following, what can be tested in possible future researches is the combination of such MOGA-model with some forecasting techniques (i.e., Monte Carlo simulations with regards to the revenues or cash flows) in order to better estimate the potential changes that companies might undertake.

Further improvements may be brought also by substituting the maximization of ICR with some other objective functions making use of different kinds of ratios (the alternatives could be represented by profitability ratios or turnover ratios) and setting other solvency ratios or debt ratios as model's constraints.

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