Università
Ca'Foscari
Venezia

Corso di Laurea magistrale (*ordinamento ex D.M. 270/2004*)
in Economics

Tesi di Laurea

–

Ca' Foscari
Dorsoduro 3246
30123 Venezia

A Bayesian MS-SUR Model for Forecasting Exchange Rates

Relatore
Ch. Prof. ROBERTO CASARIN

Correlatore
Ch. Prof. DOMENICO SARTORE

Laureando
IGOR ROTARU

Matricola 840658

Anno Accademico
2013 / 2014

Università
Ca'Foscari
Venezia

Contents

Abstract……………………………………………………………………......I

List of figures……………..……………………………………………..….II

List of tables……...………………………………………………………III

Introduction………………………………………………………..……..1

1. The models..3
    - 1.1. A standard SUR model..3
    - 1.2. A MS-SUR model..4
2. Posterior computation..8
    - 2.1. Likelihood functions..8
        - 2.1.1. SUR Model..8
        - 2.1.2. MS-SUR Model……………………………..………………8
    - 2.2. Posterior distributions..9
        - 2.2.1. Posterior distribution for the SUR model........................9
        - 2.2.2. Posterior distribution for the MS-SUR model…………11
3. Empirical results...17
    - 3.1. Preliminary analysis..17
    - 3.2. Descriptive statistics…………………………………………27
    - 3.3. Unit root and normality tests………………………………...31
    - 3.4. Heteroskedasticity and autocorrelation tests……………..33
    - 3.5. MCMC estimation results...36
4. Conclusions...48

References

Annexes

Università
Ca'Foscari
Venezia

Abstract

The thesis proposes a new Bayesian factor model in the forecasting exchange rates using an application of Markov chain Monte Carlo to Bayesian inference. First we describe the Zellner's Seemingly Unrelated Regression (SUR) multivariate model with ten macroeconomic fundamentals in order to forecast the six exchange rates over the years 2002-2014. Secondly, we assume a latent Markov switching process is driving the parameters of the SUR model in order to detect structural instabilities. We develop MATLAB code for analysing and forecasting monthly exchange rate series.

Graduation year 2014.

JEL Classification Codes: F31, F37, G17.

KEYWORDS: Foreign Exchange Markets; Forecasting Exchange Rates; Bayesian approach; MCMC, Gibbs Sampling; Markov Switching; Seemingly Unrelated Regression; MATLAB.

List of figures

Figure 1: Differentiation of the exchange rates……………………………………….....19
Figure 2: Different versions of computing interest rate differential……………….....24
Figure 3: Rolling window estimates………………………………………………….....25
Figure 4: Percentage statistics of the exchange rates……………………….………....26
Figure 5: Breakdown statistics by month and by year for EURUSD………….……....27
Figure 6: Histograms of the exchange rates and their densities……………………....28
Figure 7: Correlation matrix with histograms…………………………………………....31
Figure 8: ACF and PACF tests for EURUSD and GBPUSD……………….............34
Figure 9: ACF and PACF tests for SEKUSD and SEKCAD………………………....35
Figure 10: ACF and PACF tests for CADEUR and CADGBP……………………....35
Figure 11: Posterior means of the SUR model………………………………………...37
Figure 12: Posterior means of the MS-SUR model of state 1………………………...39
Figure 13: Posterior means of the MS-SUR model of state 0……………………..…..40
Figure 14: The switching regimes of MS-SUR…………………………………………....46

List of tables

Table 1: The list of fundamentals and their clusters……………………………………20

Table 2: Descriptive statistics of the monthly exchange rates……………………..29

Table 3: Correlation matrix of the exchange rates…………………………………….30

Table 4: Results of the stationary and normality tests………………………………..32

Table 5: Results of heteroskedasticity and autocorrelation tests…………………33

Table 6: The similarity of the SUR and MS-SUR state 1 estimates………………41

Table 7: MCMC estimates of posterior distributions of the first three exchange rates…………………………………………………………………………………………….42

Table 8: MCMC estimates of posterior distributions of the last three exchange rates…………………………………………………………………………………………….43

Table 9: Posterior variance-covariance matrix for both models………………….44

Table 10: Prior and posterior transition probabilities of the MS-SUR model….45

Introduction

Forecasting exchange rates was always in attention of econometricians and financial markets experts. In the early ages, was believed that time series follow a simple random walk model but afterwards, many other more complex models were proposed. The best known models for forecasting exchange rates were Purchasing Power Parity (which was developed by Cassel (1918) for the first time) and then followed by Uncovered Interest Rate Parity.

Knowing that exchange rates are sensible to some variables which are called fundamentals, we focused our attention to some researches on Seemingly Unrelated Regression model (SUR) developed by Zellner in 1962 which used macroeconomic fundamentals to improve estimation efficiency across equations. In this way, we followed some specialized websites in Foreign Exchange Markets which provided a full list of fundamentals so we gathered ten fundamentals for five countries to check their impact on six exchange rates.

After describing the SUR model, we decided to check if using Markov switching regimes in a SUR model would be useful for the extraction of switching states and for identification of structural breaks in the parameters.

In the literature, the Markov Switching is described as the best model for predicting the exchange rates (Lee and Chen, 2006) so we will test if combining the switching mechanism with a SUR model will provide better estimates.

As we could expect, our variables are non-stationary and we will try to deal with this problem. Many approaches have been proposed in the literature so we have compared two of them. One is a difference model specified in Frommel (2004) and the other one is the proportion model explained in Ghalayini (2014). The difference model is preferred in the literature as it provides the stronger relationship (higher correlation within dependent variables).

The thesis is structured as follows. In the first two chapters, we described in detail the Bayesian inference of our two models by establishing the prior distributions, the likelihood functions and computing the posterior distributions.

The third chapter contains a preliminary analysis of the spot exchange rates and the fundamentals. It is followed by an overview of the econometric methodology which includes some relevant tests (unit root, normality, heteroskedasticity and autocorrelation) in order to verify if the variables are correctly modelled and the models are estimated properly. This chapter also provides the estimation results with a discussion. The empirical analysis have been conducted using MATLAB.

The last chapter includes the main concluding remarks for both models while in the Annexes, there are all the results of the tests and the description of the explanatory variables.

1. The models

1.1. A standard SUR model

The Seemingly Unrelated Regressions (SUR) model was introduced by Zellner (1962). In order to improve estimation efficiency, Zellner combined several equations into one model and now this tool is used to study the impact of a wide range of phenomena, especially in econometrics and economics.

We start to present the SUR model by considering M equations written as:

$$Y_i = X_i\beta_i + \varepsilon_i \qquad i = 1,2,\dots,M$$

Where Y_i is a T-dimensional vector of observations on a dependent variable, X_i is a $T \times K$ matrix of observations on K nonstochastic explanatory variables, which does not include intercept. β_i is a K - dimensional vector of unknown coefficients that we wish to estimate and ε_i is a T-dimensional unobserved random vector.

We can compress our model with M=6 equations in this way:

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \end{pmatrix} = \begin{pmatrix} X_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & X_6 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix};$$

We can write the compact model in the vectorial form as:

$$\begin{array}{ccccccc} Y & = & X & \times & \beta & + & \varepsilon \\ T \times 6 & & T \times K & & K \times 6 & & T \times 6 \end{array}$$

We assume that the errors are heteroscedastic, correlated across equations and autocorrelated. Also they follow a normal distribution $\varepsilon_T \sim N\,(0\,,\ \Sigma \otimes I_T)$. $\otimes$ denotes the matrix Kronecker product also known as tensor product. Σ is the variance-covariance matrix of the error which is an M×M matrix and I_T is an identity matrix of order T×T.

We will assume the prior distribution of β and Σ to be a Normal-Inverse-Wishart (Zelner and Ando, 2010), that is:

$$\beta \sim N\left(\mu_\beta, \Sigma_\beta^2\right) \; \textit{(informative and proper prior)}$$

$$\Sigma \sim IW\left(d_\Sigma, \Omega_\Sigma\right) \; \textit{(informative and proper prior)}$$

The Inverse-Wishart distribution is frequently used as prior distribution the variance-covariance matrix parameter (Σ) of the multivariate distributions.

In the priors given above, μ_β, Σ_β^2, Ω_Σ and d_Σ are the hyperparameters. Depending on the values of Σ_β^2 and Ω_Σ, the degree of prior information can change. For example when Σ_β^2 is large, the prior is weakly informative.

One of the most popular approaches for estimating the SUR model in a Bayesian framework involves the use of Markov chain Monte Carlo (MCMC) method in order to compute posterior densities for parameters and predictive density functions. One of the MCMC methods is called Gibbs sampling algorithm introduced by Geman and Geman (1984). It is mainly based on simulating the full conditional distributions of each parameter vector conditioned on the remaining data parameters and computing posterior quantities of interest. In the following chapter, the SUR conditional distributions are computed with the Gibbs sampler approach.

1.2. A MS-SUR model

The Markov Switching mechanism identified through switching regression was first considered by Goldfeld and Quandt (1973). In 1989, Hamilton presented an analysis of Markov Switching model and its estimation method which expressed an extension of cases with dependent data such as autoregressions. The first Markov Switching paper for exchange rates modelling was first introduced by Engel and Hamilton (1990). Engle and Hamilton showed that there are persistence regimes ("long swings") in the log-exchange rates. Also both states are differentiated not only by their means but also by the variances of the conditional distributions.

The regime switching model became very popular in many fields of application as the switches of the two regimes states could correspond to episodes of an appreciation or a depreciation of the exchange rates over short periods. This would describe the declines, crisis, market crash or recovery. On the other side there is growth or expansion in dependence of the switching variables and the data of interest. All these switches at any given date are expected to be controlled by a hidden Markov chain. This model have attracted considerable attention in econometrics, biometrics and engineering. Nowadays, the researchers extend this model with different mixture referred also to Markov mixture models.

In this way, we will develop a new model where is applied the Markov switching approach to the SUR models. We initiate the presentation of the model and afterwards we continue with the Bayesian inference of MCMC process.

Consider the following process given by:

$$\begin{array}{ccccc} Y_t' & = & [\beta_1 S_t + \beta_0 (1 - S_t)] & X_t' & + \Lambda_t \varepsilon_t' \\ 6 \times T & & 6 \times K & K \times T & 6 \times T \end{array}$$

Where we define $\Lambda_t = \Sigma_1 S_t + \Sigma_0 (1 - S_t)$ as the variance-covariance matrix which depend on the states of the latent variable and $\varepsilon_t \sim N\,(0\,,\ I_t')$ is the Gaussian white noise.

The Markov Switching SUR (MS-SUR) model suggest the existence of latent variable S_t, for $t = 1, \dots, T$ which is the Markov chain process with values in $\{1,2\}$ (2-states Markov chain). We fixed just two states in our model but there is possibility of having finite regime states.

The different variance-covariance matrix in each state is represented by the identification constraint $\Sigma_{1,jj} < \Sigma_{0,jj}$ where $j = 1, \dots, M$ and $\Sigma_i \in R_+,\ i = 0{,}1$. This means that $\Sigma_{0,jj}$ would represent the bear market state which is a period of falling prices of the specific securities (in our case exchange rates). This market state is more volatile than the bull market because traders react faster to bad news. The investors close their orders quicker in case of sharp decrease of the prices in order to minimize the loss. Another explanation is the presence of the stop loss limits which is a tool of setting a boundary price during the trading activity.

The dynamics behind this model is known by transition probability which controls the probabilities of switching from one state to another thanks to the identification constraint that has been set on the variances $\Sigma_{1,jj} < \Sigma_{0,jj}$.

The difficulty of our model arises from the fact that the next probability is hidden and we define it as the following:

$$P(S_t = j|S_{t-1} = i) = p_{ij},\ i, j \in \{0,1\}$$

Or we can write it in the matrix form:

$$P(S_t = j|S_{t-1} = i) = \begin{bmatrix} P(S_t = 0|S_{t-1} = 0) & P(S_t = 0|S_{t-1} = 1) \\ P(S_t = 1|S_{t-1} = 0) & P(S_t = 1|S_{t-1} = 1) \end{bmatrix} = \begin{bmatrix} \theta_{00} & \theta_{10} \\ \theta_{01} & \theta_{11} \end{bmatrix}$$

We will assume it as a mixture of Bernoulli distributions:

$$S_t \sim Bern\,(\theta_{11} S_{t-1} + \theta_{01}(1 - S_{t-1}))$$

We add a constraint for the transition probability $\theta_{i1} + \theta_{i0} = 1$ with $i = 0,1$ and its density can be written as:

$$f(S_t|S_{t-1}) = {\theta_{11}}^{S_t S_{t-1}}(1 - \theta_{11})^{(1-S_t)S_{t-1}}{\theta_{00}}^{(1-S_t)(1-S_{t-1})}(1 - \theta_{00})^{S_t(1-S_{t-1})}$$

The transition probability of this process can have more parameters in case of more than two states but in the thesis we are having just two states so our transition probability have four probabilities of switching between states.

We will assume the Normal-Inverse Wishart-Beta prior distributions which are fairly informative. Koop (2004) suggested the use of fairly (weakly) informative priors if we have to compare two models with similar parameters. The advantage of using the informative is that the posterior standard deviations prior are slightly smaller than those using the non-informative prior. Also should provide some of the benefit of prior information while avoiding some of the risk from using information that doesn't exist.

For the MS-SUR model, we assume the following prior distributions for our parameters:

$p(\beta_1, \beta_0) \sim N(m_1, \Upsilon_1^2) N(m_0, \Upsilon_0^2)$ *(fairly-informative)*

$p(\Sigma_1, \Sigma_0) \sim IW(a_1, b_1) IW(a_0, b_0) I_{\Sigma_{1,jj} < \Sigma_{0,jj}}$ *(fairly-informative)*

$p(\theta_{ii}) \sim \mathcal{B}e(c_{ii}, d_{ii})$ *(fairly-informative)*

In the next chapter, we will continue our Bayesian inference with the specification of the likelihood functions and the computation of the full conditional distributions for our models.

2. Posterior computation

2.1. Likelihood functions

2.1.1. SUR Model

The likelihood function is the probability density function conditioned on a set of parameters. For the SUR model, the key parameters are β coefficients and Σ-variance-covariance matrix. The complete likelihood will be the following:

$$L(Y|\beta,\Sigma,X) = \frac{1}{(2\pi)^{\frac{MT}{2}}\,|\Sigma|^{\frac{T}{2}}} exp\left\{-\frac{1}{2}\,(Y - X\,\beta)'(\Sigma^{-1}\otimes I_T)(Y - X\,\beta)\right\}$$

2.1.2. MS-SUR Model

For the MS-SUR model, the complete likelihood function will be the product of likelihood function between the states conditioning on the parameters and the states' probabilities (weights):

$$\mathrm{L}\,(\mathrm{Y}_1,\dots,\mathrm{Y}_T,\mathrm{S}_1,\dots,\mathrm{S}_T|\beta_1,\beta_0\,,\Sigma_1,\Sigma_0,\mathrm{X}_1,\dots,\mathrm{X}_T,\theta_{11},\theta_{00}) =$$

$$= \sum_S \mathrm{L}\,(\mathrm{Y}_1,\dots,\mathrm{Y}_T|\beta_1,\beta_0\,,\Sigma_1,\Sigma_0,\mathrm{X}_1,\dots,\mathrm{X}_T,\mathrm{S}_1,\dots,\mathrm{S}_T)\;\; f(S_t|S_{t-1}) =$$

$$= \sum_S \left[\prod_{t=1}^{T} \frac{1}{(2\pi)^{\frac{MT}{2}}\,|\Lambda_t|^{\frac{T}{2}}} exp\left\{-\frac{1}{2}\;\left(Y_t - \left(\beta_1\,S_t + \beta_0\,(1-S_t)\right)X_t\right)'\,\Lambda_t^{-1}\,(Y_t - (\beta_1\,S_t + \beta_0\,(1-S_t))X_t)\right\}\,\theta_{11}{}^{S_tS_{t-1}}(1 - \theta_{11})^{(1-S_t)S_{t-1}}\;\;\theta_{00}{}^{(1-S_t)(1-S_{t-1})}(1-\theta_{00})^{S_t(1-S_{t-1})}\right]$$

Apart of the parameters from the SUR model, likelihood of MS-SUR model contains the latent variable $\theta_{jj}, j = 0{,}1$ which is the main advantage of this model. The calculation of the likelihood can be achieved by integrating out all possible regime paths along observed data ($Y_1,\dots,Y_T$ and $X_1,\dots,X_T$) taking in consideration the unobserved states $(S_1, \dots, S_T)$. However, we will consider the summation instead of integrating.

2.2.Posterior distributions

2.2.1. Posterior distributions for the SUR model

According to the Bayes' theorem, the joint posterior distribution for β, Σ is proportional to the product of likelihood and the prior distribution, that is:

$$p(\beta, \Sigma | Y, X) \propto L(Y|\beta, \Sigma, X)\; p(\beta, \Sigma)$$

$$\propto \frac{1}{(2\pi)^{\frac{MT}{2}} |\Sigma|^{\frac{T}{2}}} exp\left\{-\frac{1}{2}\,(Y - X\,\beta)'(\Sigma^{-1} \otimes I_T)(Y - X\,\beta)\right\} |\Sigma|^{-\frac{M+1}{2}}$$

$$\propto \frac{1}{(2\pi)^{\frac{MT}{2}} |\Sigma|^{\frac{T+M+1}{2}}}\; exp\left\{-\frac{1}{2}\,(Y - X\,\beta)'(\Sigma^{-1} \otimes I_T)(Y - X\,\beta)\right\}$$

$$\propto |\Sigma|^{-\frac{T+M+1}{2}}\, exp\left\{-\frac{1}{2}\,(Y - X\,\beta)'(\Sigma^{-1} \otimes I_T)(Y - X\,\beta)\right\}$$

Conditional posterior distribution for β it is the multiplication between the likelihood and the prior distribution as the following:

$$p(\beta | Y, X, \Sigma) \propto \mathrm{L}\; (Y|\beta,\, \Sigma, X)\; p(\beta)$$

$$\propto exp\left\{-\frac{1}{2}(\beta - \mu_\beta)'\Sigma_\beta^{-1}(\beta - \mu_\beta)\right\} exp\left\{-\frac{1}{2}\,(Y - X\,\beta)'(\Sigma^{-1} \otimes I_T)(Y - X\,\beta)\right\}$$

$$\propto exp\left\{-\frac{1}{2}\left(\beta'\Sigma_\beta^2\beta - 2\beta'\,\Sigma_\beta^2\,\mu_\beta\right) + \left[\beta'X'\,\left(\Sigma^{-1} \otimes I_T\right)X\beta - 2\,\beta'X'\left(\Sigma^{-1} \otimes I_T\right)Y\right]\right\}$$

$$\propto exp\left\{-\frac{1}{2}\left[\beta'\left(\Sigma_\beta^2 + X'\,\left(\Sigma^{-1} \otimes I_T\right)X\right)\beta - 2\beta'\,\left(\Sigma_\beta^2\,\mu_\beta + X'\left(\Sigma^{-1} \otimes I_T\right)Y\right)\right]\right\}$$

$$\propto N(\overline{\mu_\beta}, \overline{\Sigma_\beta^2})$$

where

$$\overline{\Sigma_\beta^2} = \left[\Sigma_\beta^2 + X'\,(\Sigma^{-1} \otimes I_T)X\right]^{-1}$$

and

$$\overline{\mu_\beta} = \left[\Sigma_\beta^2 + X'\,(\Sigma^{-1} \otimes I_T)X\right]^{-1}\left[\,\Sigma_\beta^2\,\mu_\beta + X'(\Sigma^{-1} \otimes I_T)Y\right]$$

$\overline{\mu_\beta}$ is the posterior mean and $\overline{\Sigma_\beta^2}$ is the variance of the normal multivariate distribution, namely the Generalized Least Square Estimators.

While we assumed the prior distribution for Σ to be an Inverse-Wishart, the posterior distribution for Σ is equal with the multiplication of the likelihood and the prior which is a M-dimensional Wishart:

$$p(\Sigma|\beta, Y, X) \propto \mathrm{L}\,(Y|\beta, X\,)\,p(\Sigma)$$

$$\propto (2\pi)^{-\frac{MT}{2}}|\Sigma|^{-\frac{T}{2}}exp\left\{-\frac{1}{2}\,(Y-\,X\,\beta)'(\Sigma^{-1}\otimes I_T)(Y-\,X\,\beta)\right\}$$

$$\frac{|\Omega_\Sigma|^{\frac{d_\Sigma}{2}}}{|\Sigma|^{\frac{d_\Sigma+M+1}{2}}2^{\frac{d_\Sigma M}{2}}\Gamma_M\left(\frac{d_\Sigma}{2}\right)}\,exp\left\{-\frac{1}{2}\,(\Omega_\Sigma\Sigma^{-1})'\right\}$$

In this posterior distribution, we have many multiplicative constants that can be safely removed without affecting the shape of the function. These constants are $(2\pi)^{-\frac{MT}{2}}$, $|\Omega_\Sigma|^{\frac{d_\Sigma}{2}}$, $2^{\frac{d_\Sigma M}{2}}$ and $\Gamma_M\left(\frac{d_\Sigma}{2}\right)$. Removing them, we can see that the posterior distribution is:

$$p(\Sigma|\beta, Y, X)\ \propto |\Sigma|^{-\frac{T}{2}}exp\left\{-\frac{1}{2}\,(Y-\,X\,\beta)'(\Sigma^{-1}\otimes I_T)(Y-\,X\,\beta)\right\}$$

$$|\Sigma|^{-\frac{d_\Sigma+p+1}{2}}exp\left\{-\frac{1}{2}\,(\Omega_\Sigma\Sigma^{-1})'\right\}$$

By taking into account the properties of the determinant and trace operators, we obtain:

$$\propto |\Sigma|^{-\frac{T}{2}}|\Sigma|^{-\frac{d_\Sigma+M+1}{2}}\,exp\left\{-\frac{1}{2}\,(Y-\,X\,\beta)'(\Sigma^{-1}\otimes I_T)(Y-\,X\,\beta)\right\}\,exp\left\{-\frac{1}{2}\,(\Omega_\Sigma\,\Sigma^{-1})'\right\}$$

$$\propto |\Sigma|^{-\frac{(T+d_\Sigma)+M+1}{2}}\,exp\left\{-\frac{1}{2}\,[(Y-\,X\,\beta)'(\Sigma^{-1}\otimes I_T)(Y-\,X\,\beta)'+\Omega_\Sigma]\Sigma^{-1}\right\}$$

$$\propto IW(\,\overline{d_\Sigma}, \overline{\Omega_\Sigma})$$

Where

$\overline{d_\Sigma} = T + d_\Sigma$

is the posterior mean with T degrees of freedom while

$$\overline{\Omega_{\Sigma}} = [(Y - X\,\beta)'(\Sigma^{-1} \otimes I_T)(Y - X\,\beta)' + \Omega_{\Sigma}]$$

is the variance of the Inverse-Wishart distribution.

We are applying the Gibbs sampling algorithm in order to generate draws of β and Σ from their posterior distributions. Given a starting value for the β (assuming that is β^0), the j-th iteration of Gibbs sampler is completed by simulating the next two steps:

1. Draw β^j from $p(\beta|\Sigma^{j-1}, Y, X)$
2. Draw Σ^j from $p(\Sigma|\beta^{j-1}, Y, X)$

The MCMC theory suggests that after sufficient draws from the conditional probabilities, the Markov chain would converge to the desired posterior distribution whereas "burn in" are discarded from the simulation because they are not from the stationary distribution of the MCMC Markov chain.

2.2.2. Posterior distributions for the MS-SUR model

For the posterior densities of the MS-SUR model, Gibbs sampler can be described by the following posterior conditional distributions known as full conditional distributions which are proportional with the posterior density.

Full conditional distributions for β_1, β_0 is the product between the complete likelihood and the prior distributions:

$$p(\beta_1, \beta_0|\Sigma_1\,, \Sigma_0, \theta_{11}, \theta_{00}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T) \propto$$

$$\propto \mathrm{L}\,(\mathbf{Y}, \mathbf{S}|\beta_1, \beta_0\,, \Sigma_1, \Sigma_0, \mathbf{X}, \theta_{11}, \theta_{00})\; N(m_1, \Upsilon_1^2) N(m_0, \Upsilon_0^2)$$

We continue the description of the full conditional distributions by specifying the prior distributions within states with $j = 0, 1$ instead of writing both distributions:

$$p(\beta_1, \beta_0|\Sigma_1\,, \Sigma_0, \theta_{11}, \theta_{00}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T) \propto$$

$$\propto exp\left\{-\frac{1}{2}(\beta_j - m_j)'\,\Upsilon_j^{-2}(\beta_j - m_j)\right\}$$

$$\sum_{t\in T_j} exp\left\{-\frac{1}{2}\left[Y_t-\left(\beta_1 S_t+\beta_0\,(1-\,S_t)\right)X_t\right]'\Lambda_t^{-1}\,[Y_t-(\beta_1 S_t+\beta_0\,(1-\,S_t))X_t]\right\}$$

$$\propto exp\left\{-\frac{1}{2\Upsilon_j^2}(\beta_j'\beta_j-2\beta'_j\,\mu'_j) -\frac{1}{2}\sum_{t\in T_j}\left[Y_t-\left(\beta_1 S_t+\beta_0\,(1-\,S_t)\right)X_t\right]'\Lambda_t^{-1}\,[Y_t-(\beta_1 S_t + \beta_0\,(1-\,S_t))X_t]\right\}$$

$$\propto exp\left\{-\frac{1}{2}\left(\beta_j'(\frac{T_j}{\Lambda_t^{-1}}+\frac{1}{\Upsilon_j^2})\beta_j\right)-2\beta\left[\frac{1}{\Lambda_t^{-1}}\sum_{t=1}^{T}{Y_t}^2+\frac{m_j}{\Upsilon_j^2}\right]\right\}$$

$$\propto N(\bar{m}_1,\bar{\Upsilon}_1^2)N(\bar{m}_0,\bar{\Upsilon}_0^2)$$

Where the means of the posterior conditional distributions are:

$\bar{m}_j=\bar{\Upsilon}_j^2\left(\frac{1}{\Lambda_t^{-1}}\sum_{t=1}^{T}{Y_t}^2+\frac{m_j}{\Upsilon_j^2}\right)$ for $j=0,1.$

The variance of the full conditional distributions for β_1, β_0 are:

$\bar{\Upsilon}_j^2=\left(\frac{T_j}{\Lambda_t^{-1}}+\frac{1}{\Upsilon_j^2}\right)^{-1}$ with $T_j=\{t|X_t=j\}, T_j=Card\left(T_j\right)$ and $j=0,1.$

Full conditional distributions for Σ_1, Σ_0 will follow an Inverse-Wishart distributions as we multiply the complete likelihood by the fairly informative Inverse-Wishart prior distributions. It is standard to assume that the precision matrix is positive definite. We include the identification constraint $\Sigma_{1,jj}<\Sigma_{0,jj}$ for differentiating the hidden states by using the indicator function $I_{\Sigma_{1,jj}<\Sigma_{0,jj}}$. Following the same approach used for the previous full conditional distributions, the states - specific parameters and their prior distributions were indexed with j. Below are presented these distributions:

$$p(\Sigma_1, \Sigma_0 \mid \beta_1, \beta_0, \theta_{11}, \theta_{00}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T) \propto$$

$$\propto \mathrm{L}\,(\mathbf{Y}, \mathbf{S} | \beta_1, \beta_0, \Sigma_1, \Sigma_0\, , \mathbf{X}, \theta_{11}, \theta_{00})\; IW(a_1, b_1) IW(a_0, b_0)\; I_{\Sigma_{1,jj} < \Sigma_{0,jj}}$$

$$\propto\; |\Lambda_t|^{-\frac{T}{2}} \textstyle\sum_{t \in T_j} exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t - \right.\right.$$

$$\left.\left.\left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]\right\} \frac{|a_j|^{\frac{b_j}{2}}}{|\Lambda_t|^{\frac{b_j + M + 1}{2}} 2^{\frac{b_j M}{2}} \Gamma_M\!\left(\frac{b_j}{2}\right)} exp\left\{-\frac{1}{2}\,\left(a_j \Lambda_t{}^{-1}\right)'\right\}\; I_{\Sigma_{1,jj} < \Sigma_{0,jj}}$$

$$\propto\; |\Lambda_t|^{-\frac{T}{2}} \textstyle\sum_{t \in T_j} exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t - \right.\right.$$

$$\left.\left.\left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]\right\} |\Lambda_t|^{-\frac{b_j + M + 1}{2}} exp\left\{-\frac{1}{2}\,\left(a_j \Lambda_t^{-1}\right)'\right\}\; I_{\Sigma_{1,jj} < \Sigma_{0,jj}}$$

$$\propto\; |\Lambda_t|^{-\frac{T + b_j + M + 1}{2}} \textstyle\sum_{t \in T_j} exp\left\{-\frac{1}{2}\left(\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]\left[Y_t - \right.\right.\right.$$

$$\left.\left.\left.\left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]' + a_j\right)\Lambda_t^{-1}\right\}\; I_{\Sigma_{1,jj} < \Sigma_{0,jj}}$$

$$\propto IW\left(\bar{a}_j, \bar{b}_j\right) I_{\Sigma_{1,jj} < \Sigma_{0,jj}}$$

In our Inverse-Wishart distributions

$$\bar{a}_j = \left[Y_t - \left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1 - \; S_t)\right)X_t\right]' + a_j$$

is the mean of the full conditional distribution for Σ_1, Σ_0 and

$$\bar{b}_j = T + b_j$$

is the variance of the posterior distribution being composed of the prior distribution b_j and T degrees of freedom.

The full conditional distribution for θ_{00} of the latent variable is a beta distribution:

$$p(\theta_{00} | \beta_1, \beta_0, \; \Sigma_1, \Sigma_0, \theta_{11}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T) \propto$$

$$\propto \mathrm{L}\,(\mathbf{Y}, \mathbf{S} | \beta_1, \beta_0\, , \Sigma_1, \Sigma_0, \mathbf{X}, \theta_{11}, \theta_{00})\; \mathcal{Be}(c_{00}, d_{00})$$

$$\propto \ |\Lambda_t|^{-\frac{T}{2}} \sum_{t \in T_j} exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]\right\} \frac{\Gamma(c_{11}+d_{11})}{\Gamma(c_{11})+\Gamma(d_{11})} \pi^{c_{00}-1}(1-\pi)^{d_{00}-1}$$

$$\propto \ \sum_{t \in T_j} exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]\right\} \pi^{c_{00}-1}(1-\pi)^{d_{00}-1}$$

$$\propto \pi^{c_{00}+T_{00}-1}(1-\pi)^{d_{00}+T_{01}-1}$$

$$\propto \mathcal{B}e(\bar{c}_{00}, \bar{d}_{00})$$

with parameters

$$\bar{c}_{00} = c_{00} + T_{00}$$
$$\bar{d}_{00} = d_{00} + T_{01}$$

where $T_{ij} = \{t|S_t = j, S_{t-1} = i\}\ and\ T_{ij} = Card(T_{ij}), i = 0{,}1$.

In order to obtain the full conditional distribution for θ_{11}, we conjugate the beta prior distribution with the likelihood function:

$$p(\theta_{11}|\beta_1, \beta_0,\ \Sigma_1, \Sigma_0, \theta_{00}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T)\ \propto$$

$$\propto \mathrm{L}\ (\mathbf{Y}, \mathbf{S}|\beta_1, \beta_0\ , \Sigma_1, \Sigma_0, \mathbf{X}, \theta_{11}, \theta_{00})\ \mathcal{B}e(c_{11}, d_{11})$$

$$\propto \ |\Lambda_t|^{-\frac{T}{2}} \sum_{t \in T_j} exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]\right\} \frac{\Gamma(c_{11}+d_{11})}{\Gamma(c_{11})+\Gamma(d_{11})} \pi^{c_{11}-1}(1-\pi)^{d_{11}-1}$$

$$\propto \sum_{t\in T_j} exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t\right.\right.$$

$$\left.\left.- \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]\right\}\pi^{c_{11}-1}(1-\pi)^{d_{11}-1}$$

$$\propto \pi^{c_{11}+T_{11}-1}(1-\pi)^{d_{11}+T_{01}-1}$$

$$\propto \mathcal{Be}(\bar{c}_{11}, \bar{d}_{11})$$

with parameters

$\bar{c}_{11} = c_{11} + T_{11}$

$\bar{d}_{11} = d_{11} + T_{01}$

where $T_{ij} = \{t|S_t = j, S_{t-1} = i\}$ and $T_{ij} = Card\left(T_{ij}\right), i = 0{,}1$

The full conditional distribution for the hidden state S_t is the product of the normal likelihood function and the transition probability. We will start computing the full conditional distribution as follows:

$$p(\mathrm{S}_t|\beta_1, \beta_0\,, \Sigma_1, \Sigma_0, \theta_{11}, \theta_{00}, \mathrm{Y}_1, \dots \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_{t-1}, \mathrm{S}_{t+1}, \dots \mathrm{S}_T)\ \propto$$

$$p(\mathrm{S}_t|\beta_1, \beta_0\ \Sigma_1, \Sigma_0, \theta_{11}, \theta_{00}, \mathbf{Y}, \mathbf{X}, \mathrm{S}_{t-1}, \mathrm{S}_{t+1})\ \propto$$

$$\propto exp\left\{-\frac{1}{2}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]' \Lambda_t^{-1}\left[Y_t - \left(\beta_1 S_t + \beta_0\,(1-\ S_t)\right)X_t\right]\right\}$$
$$\theta_{11}{}^{S_t S_{t-1}}\,(1-\ \theta_{11})^{(1-S_t)S_{t-1}}\theta_{00}{}^{(1-S_t)(1-S_{t-1})}(1-\theta_{00})^{S_t(1-S_{t-1})}\theta_{11}{}^{S_{t+1}S_t}(1-$$
$$\theta_{11})^{(1-S_{t+1})S_t}\theta_{00}{}^{(1-S_{t+1})(1-S_t)}(1-\theta_{00})^{S_{t+1}(1-S_t)}$$

$$\propto \mathcal{Bin}(1, \bar{\xi}_t)$$

Which is a binomial distribution with one trial and the next success probability:

$$\bar{\xi}_t = \frac{\theta_{1t} exp\left\{-\frac{1}{2}\,[Y_t - \beta_1 X_t]' \Lambda_t^{-1}\,[Y_t - \beta_1 X_t]\right\}}{\theta_{1t} exp\left\{-\frac{1}{2}\,[Y_t - \beta_1 X_t]' \Lambda_t^{-1}\,[Y_t - \beta_1 X_t]\right\} + \theta_{0t} exp\left\{-\frac{1}{2}\,[Y_t - \beta_0 X_t]' \Lambda_t^{-1}\,[Y_t - \beta_0 X_t]\right\}}$$

Where the success probability of the trial are the transitional probability of our process:

$$\theta_{1t} = {\theta_{11}}^{S_{t-1}} {\theta_{11}}^{S_{t+1}} \, (1 - \theta_{11})^{1-S_{t+1}} (1 - \theta_{00})^{(1-S_{t-1})}$$

$$\theta_{0t} = {\theta_{00}}^{1-S_{t-1}} {\theta_{00}}^{1-S_{t+1}} \, (1 - \theta_{11})^{S_{t-1}} (1 - \theta_{00})^{(1-S_{t+1})}$$

The Gibbs sampler for the MS-SUR is different than the one used for the SUR model because it includes some more parameters to sample which are the hidden states of the Markov chain and the regimes specific parameters. Given a starting value for the parameter (assuming that is β^0), at the j-th iteration, the Gibbs sampler is completed by simulating the next steps:

1. Draw ${\beta_1}^j, {\beta_0}^j$ from $p(\beta_1, \beta_0 | \, {\Sigma_1}^{j-1}, {\Sigma_0}^{j-1}, {\theta_{11}}^{j-1}, {\theta_{00}}^{j-1}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T)$;
2. Draw ${\Sigma_1}^j, {\Sigma_0}^j$ from $p(\Sigma_1, \Sigma_0 \,|\, {\beta_1}^{j-1}, {\beta_0}^{j-1}, {\theta_{11}}^{j-1}, {\theta_{00}}^{j-1}, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T)$;
3. Draw ${\theta_{11}}^j$ from $p(\theta_{11} | {\beta_1}^{j-1}, {\beta_0}^{j-1}, \, {\Sigma_1}^{j-1}, {\Sigma_0}^{j-1}, {\theta_{00}}^j, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T)$;
4. Draw ${\theta_{00}}^j$ from $p(\theta_{00} | {\beta_1}^{j-1}, {\beta_0}^{j-1}, \, {\Sigma_1}^{j-1}, {\Sigma_0}^{j-1}, {\theta_{11}}^j, \mathrm{Y}_1, \dots, \mathrm{Y}_T, \mathrm{X}_1, \dots, \mathrm{X}_T, \mathrm{S}_1, \dots, \mathrm{S}_T)$.

It is important to assess the convergence of the MCMC Markov chain algorithm that is to check if the chain reached the stationary distribution (the desired posterior distribution). Some models can have slow convergence of the Markov chain. This happens more often when there is high correlation between parameters.

3. Empirical results

3.1. Preliminary analysis

In the empirical application, we used monthly data from January 2002 to April 2014 for five developed countries (Eurozone, United Kingdom, United States of America, Canada, and Sweden) from the Bloomberg database. These countries have strong currencies that are globally traded and are long-term stable. We developed a code in MATLAB programming language for performing estimation, modelling and testing. The dataset and the code are available on request.

When downloading the time series from the database, we are asked if we want to select the closing values for all the variables or the average in the case of the daily or weekly time series (in our case the exchange rates). It is better to choose the closing value because at the end of the month, our variable absorbed all the shocks. Choosing the average, we could lose precious information about our time series.

Euro, British pound, American dollar, Canadian dollar and the Swedish krona are classified among the first eleven traded currencies in the world[1]. Forecasting their pairs would be useful for the trading activities because together they have a high percentage share on the total transactions.

It is known that the exchange rates are highly positive correlated between them so there is a linear relationship where their quotes are going in the same direction most of the times.

We turn into discussion the exchange rates known as currency pairs that are the values of the base currency over the quote currency. From the pair EURUSD (or EUR/USD), the euro is the base currency and the American dollar is the quote currency. For example, if the price of EURUSD is 1.5, we need 1.5 dollars to buy 1 euro.

---

[1] BIS Triennial Central Bank Survey. *Foreign exchange turnover in April 2013: preliminary global results.* Monetary and Economic Department, September 2013: available at www.bis.org

I focused my attention on EURUSD and GBPUSD which are the main exchange rates and the most traded in the Foreign Exchange Market (FOREX).

This market is very volatile and the most liquid. It has started after the Bretton Woods agreement ended and it is known to be based on the implementation of the global free-floating currency system. The online currency trading market was available at the late of 1990's and it starts in the Asia-Pacific area. Having an open 24 hours program and five days a week, FOREX continues its activity through Middle Asia, Europe and America.

In the Triennial Central Bank Survey[2] April 2013 edition of the Bank for International Settlements, FOREX market was estimated with a daily turnover that may exceed 5.3 trillion dollars per day in April 2013. The most traded currency is the dollar with almost 87 % of all transactions. The next currency is euro with approximatively 33 % of all trades. The common currency lost 6 % from 2010 because of the sovereign debt crisis in the euro area.

Despite the fact that euro shrunk in the last years, EURUSD remains the most traded currency pairs in the FOREX market with approximatively 24.1 % from the entire daily volume which is about 1.28 trillion dollars of daily transactions.

GBPUSD is the third most traded currency pair after the USDJPY and is called the "Cable" through traders/investors. It is present in the market with 8.8 % of the total daily volume.

It is expected that these two exchange rates are positive correlated due to the strong relationship between the euro and the British pound when American dollar is assumed to be the quote currency.

In order to introduce the Canadian dollar and the Swedish krona, I chose other four exchange rates which have them as base currencies:

- Two have the Canadian dollar as the base: CADEUR and CADGBP;
- Two have the Swedish krona as the base: SEKCAD and SEKUSD.

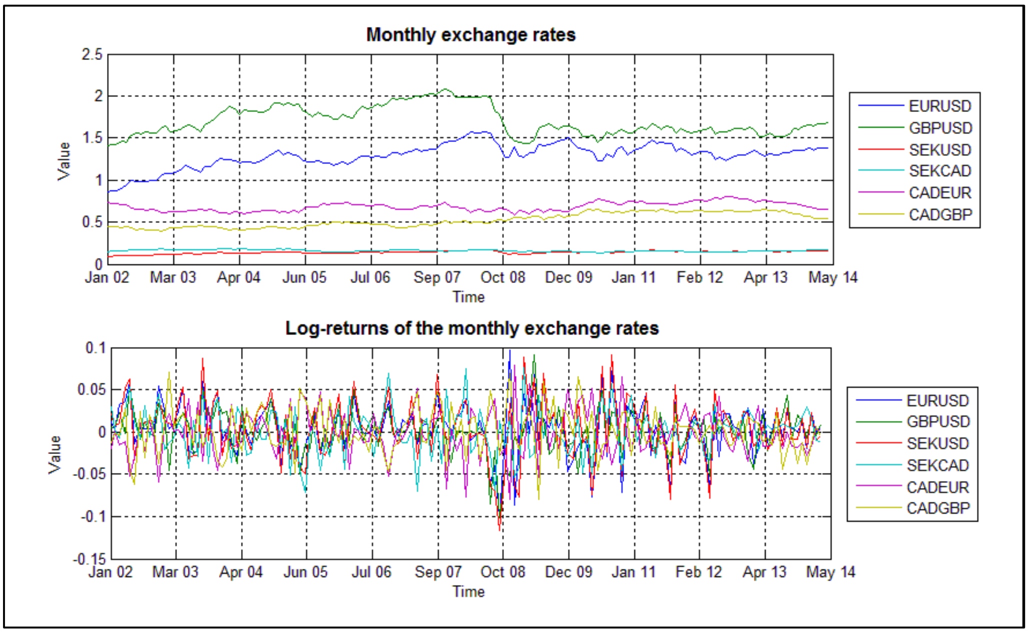
---

[2] BIS Triennial Central Bank Survey. *Foreign exchange turnover in April 2013: preliminary global results.* Monetary and Economic Department, September 2013: available at www.bis.org

There is also the pair that contains both of them with the Swedish krona as the base currency: SEKCAD.

We take log-returns of the exchange rates and the output is in the Figure 1.

Figure 1: Differentiation of the exchange rates



The log-returns of the exchange rates can be represented as follows:

$\Delta EURUSD_t = log(EURUSD)_t - \log(EURUSD)_{t-1}$ with t=2:148 observations.

Our sample has 148 observations but after differentiation, we lose one observation.

In the empirical application are being used ten explanatory variables for every country which is supposed to influence the value of the exchange rates. They are called *fundamentals* and were chosen from the literature and from specialized FOREX websites[3] because of their explanatory power. It is believed that this set of fundamentals and the exchange rates are in a strong relationship over time.

Even in Engle and West (2005), the authors consider a random walk models augmented with the inclusion of fundamentals (interest rate, consumer production index (inflation), money supply and gross domestic product). The authors showed

---

[3] http://www.forexfactory.com/

that fundamentals might help to predict the floating exchange rates. Ghalayini 2014 presents ARIMA model of the regression between the nominal exchange rates and the same four explanatory variables: inflation (CPI), interest rate, business cycle (can be interpreted as GDP) and money aggregate (supply). Another paper, which investigates the relationship between exchange rates and its fundamentals, is Frommel, 2005. The author extends the monetary exchange rate model for the real interest rate differential by introducing Markov regime switches in the model coefficients. The fundamentals in this paper are money supply, GDP, short term interest rate and long term interest rate.

In the present thesis, we take into consideration almost all variables specified in the papers discussed in this section except GDP[4] and we consider a wider set of macroeconomic variables for increasing the forecast accuracy. This set with ten variables covers more sectors of the economy such as growth, inflation, employment, Central Bank, Government and business surveys.

There are many other fundamentals which affect the exchange rates but the next table is presenting the fundamentals used in our models. The Annex 5 provides the name of the indexes and a short description of the explanatory variables available from the Bloomberg database.

Table 1: The list of fundamentals and their clusters

Growth	**Inflation**	**Employment**	**Central Bank**	**Government**	**Business survey**
Trade balance	CPI	Unemployment rate	Interest rate	10 years bond yields	PMI
IPI	PPI		Money supply		
Leading index					

---

[4] This indicator is not available at a monthly frequency so we choose other three factors that are related to the Gross Domestic Product.

We initiate the discussion of each fundamentals and their effects on exchange rates of the relationship with other variables. First of all, the section 2A of the Federal Reserve Act[5] is presenting the monetary policy objectives of the Federal Reserve regarding the stability of the dollar's fundamentals: "*The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the* ***monetary and credit aggregates*** *commensurate with the economy's long run potential to increase* ***production****, so as to promote effectively the goals of maximum* ***employment****, stable* ***prices****, and moderate* ***long-term interest rates****.*".

On the other hand, the Treaty of the functioning European Union[6] establish the "***price*** *stability of maximum 11/2 percentage points than the best three member states*" as the main objective of the ESCB (ESCB=ECB+all central banks) while the criterion of the convergence of the interest rates is "***long-term interest rate*** *maximum 2 % higher than the three best members states*". As the inflation is the main concern of the central banks, we are starting to discuss it:

Consumer Price Index (CPI) A change in the consumer prices is known as the inflation rate. Inflation is important in forecasting the exchange rate or the currency because if the prices are rising, the central banks would raise interest rates to mitigate the inflation so the currency of that country would depreciate;

Producer Price Index (PPI) This is another indicator of inflation as it takes into account that the higher costs of the producer's goods are usually paid by the consumer.

Interest rate (INT) Short term interest rate is the main factor which influence the direction of the value of the exchange rate. Interest rates are manipulated by the central banks as their monetary policy in order to affect inflation and exchange rates. Higher interest rates would appreciate the exchange rates but would lower the inflation;

---

[5] Federal Reserve Act can be accessed at http://www.federalreserve.gov/aboutthefed/section2a.htm

[6] The Treaty of Functioning of the European Union can be accessed in all the languages of the EU members at http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:12012E/TXT

Monthly Supply (MS) It measures the stock of money in circulation within a country and it is determined through the monetary policy of the central bank. It's negatively correlated with interest rates - early in the economic cycle an increasing supply of money leads to additional spending and investment, and later in the cycle expanding money supply leads to inflation and depreciation of the exchange rate;

Trade Balance (TB) This is an important factor because it is determined by the difference between export and import of a specific country. Usually, a positive trade balance indicates that more goods and services were exported than imported meaning that the demand increased and the currency appreciated. However, central bank intervenes in depreciating its currency because it stimulates the export and afterwards the trade balance;

Industrial Production Index (IPI) It's a leading indicator because the industrial production reacts quickly to ups and downs in the business cycle and is correlated with consumer conditions such as employment levels and earnings. It is expected that if the IPI is increasing then the exchange rate appreciates because the economy is growing;

Unemployment rate (UN) It's generally viewed as a lagging indicator, the number of unemployed people is an important signal of overall economic health because consumer spending is highly correlated with labour-market conditions. If the economy is slowing down, the unemployment is increasing and the people are losing their jobs. The demand is decreasing in the same time. We should expect a depreciation of the country's currency and further exchange rates depreciation;

Leading Index (LI) This factor is composed of some economic indicators related to money supply, building approvals, profits, exports, inventories and interest rate spreads. Tends to move before changes in the overall economy and should have a negative impact on the exchange rates;

Purchasing Managers Index (PMI) It's a leading indicator of economic health - businesses react quickly to market conditions, and their purchasing managers hold perhaps the most current and relevant insight into the company's view of the economy. A rising index will indicate economic expansion;

Bonds Yields (10yBond) Yields are set by bond market investors and can be used to read investors' outlook on future interest rates and expected inflation. It is known in the theory as the long term interest rates and it is expected to be highly positive correlated with the exchange rates.

We have two types of interest rates in our model:

- **Short term interest rate** - money market interest rate decided by the Central Bank and captures liquidity effects;
- **Long term interest rate** - government bond yields which captures the expected inflation.

Also we have two types of inflation:

- Consumer Price Index (CPI)
- Producer Price Index (PPI)

The next step of modelling our data is the creation of indexes of ten explanatory macroeconomic variables for each exchange rates like it is suggested in Frommel (2005) and it is presented in the uncovered interest rate theory.

The returns of our explanatory variables which we will call them differentials, are the following:

1. Inflation (CPI) differential
2. Inflation (PPI) differential
3. Interest rate differential
4. Money Supply differential
5. Trade Balance differential
6. Industrial Production (IPI) differential
7. Unemployment differential
8. Leading Index differential
9. Purchasing Managers Index (PMI) differential
10. 10 years bond yields differential

All the monthly differential indexes were calculated as difference between the fundamental of the base currency and the same fundamental of the quote currency.

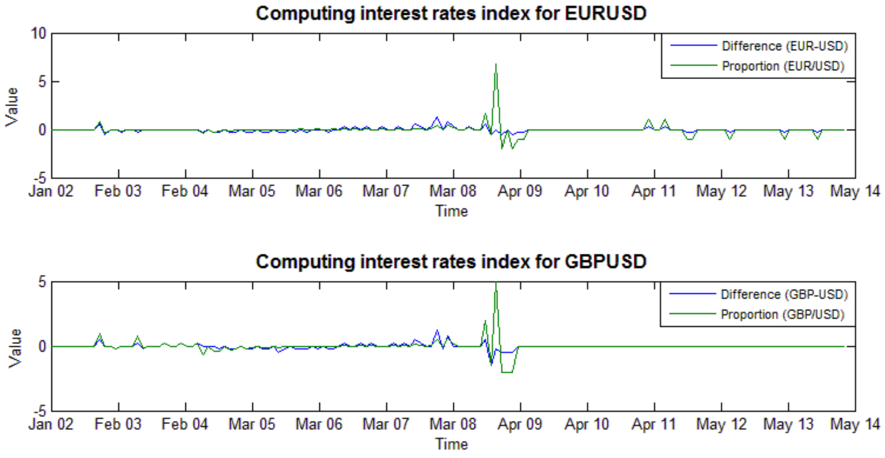
It was explained earlier that for the currency pair EURUSD, EUR is the base currency and USD is the quote currency. The inflation differential was calculated as the Eurozone Harmonized Consumer Price Index subtracted by the American Consumer Price Index then we differentiated to solve the unit root problem.

${\Delta CPI_{EURUSD}}^{t} = (CPI_{EUR} - CPI_{USD})^{t} - (CPI_{EUR} - CPI_{USD})^{t-1}$ with t=2:148 observations.

We cannot take log-returns for the fundamentals because they contain negative values and there is a positive sign constraint in order to compute the natural logarithm ($CPI \ngtr 0$).

In order to motivate the choice of the interest rate differential, we plot in Figure 2 the difference and the ratio between of the Euro and the US interest rate differentials. Also the UK and the US interest rate differential are given in Figure 2. From the graphical inspection, we conclude that the proportion are not stationary due to the spikes appearing in the 2008-2009 period, thus we prefer the use of the difference.

Figure 2: Different versions of computing interest rate differential



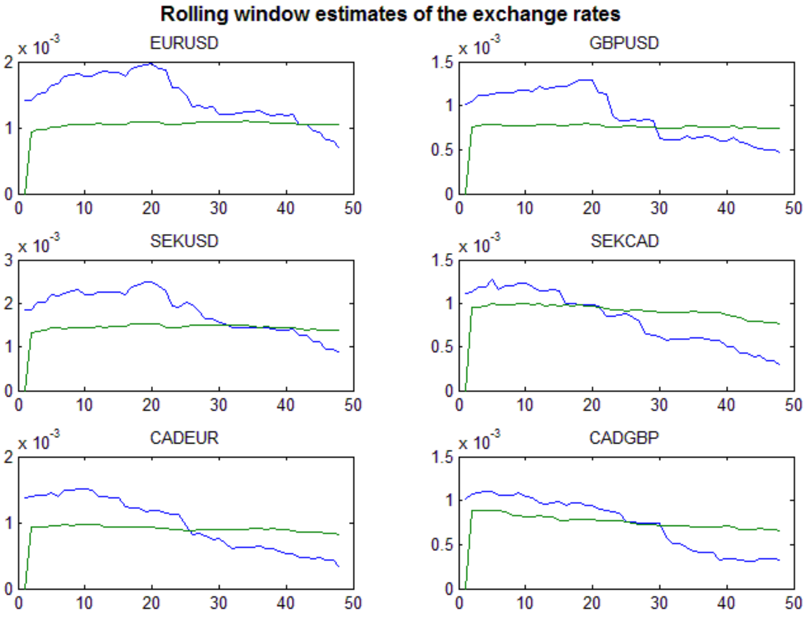
We apply the same approach for all variables except for the trade balance differential. The difference of this variable exhibit spikes whereas the proportions are more stable. The time series of the trade balance is composed from dispersed values meaning excessive standard deviation. Applying difference model, we obtain coefficient estimates with two digits and standard deviation with three digits with a large degree of heterogeneity across exchange rates. For this reason, we choose to model the trade balance differential as follows:

$$\Delta TB_{EURUSD}{}^{t} = \left(\frac{TB_{EUR}}{TB_{USD}}\right)^{t} - \left(\frac{TB_{EUR}}{TB_{USD}}\right)^{t-1}$$

which contributes to reduce the cross currency heterogeneity and makes comparable the coefficients.

As regard to the other explanatory variables, we check with rolling window estimates in order to see if the variances of the exchange rates changes are constant over time. We choose a window size of 40 observations out of 100 and we initiate the process. As we can clearly see from the next figure that the variances (marked with green lines) are stable around 0.001 while the means (blue lines) are decreasing along the time.

Figure 3: Rolling window estimates

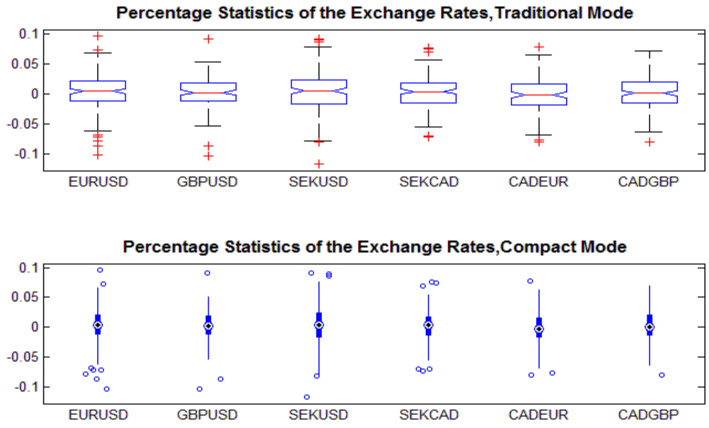


From this figure, we can see that SEKUSD have the highest variance and the highest mean over the chosen window. On the opposite side is GBPUSD with a stable variance of 0.0007 and is constant over time. EURUSD have an increasing variances as gained more power in the last year and is still increasing in volatility since euro in the last years passed through many financial crisis and the pair fluctuates on the most global factors. CADGBP pair has a decreasing variance over time.

Another interesting fact is that all rolling window means are decreasing which indicates that after 2008, most of central banks are trying to depreciate their currencies in order to increase the exports and stimulating the economy. This empirical fact might suggests possible break in the relationship between exchange rates and their fundamentals.

Using “boxplot” function in MATLAB, we can easily check some important statistics of our exchange rates. We watch figure 4 and we see the red line which indicates the median of the data. All the medians are concentrated around 0 but EURUSD and SEKUSD have higher medians. The "central box" representing the central 50% of the data. SEKUSD has the biggest “box” which mean that most of its values are concentrated in the half of the entire data. Its lower and upper boundary lines are at the 25%/75% quantile of the data so SEKUSD has the highest interval which is followed by EURUSD and CADGBP. We can conclude that SEKUSD is quite unstable and dispersed.

Figure 4: Percentage statistics of the exchange rates

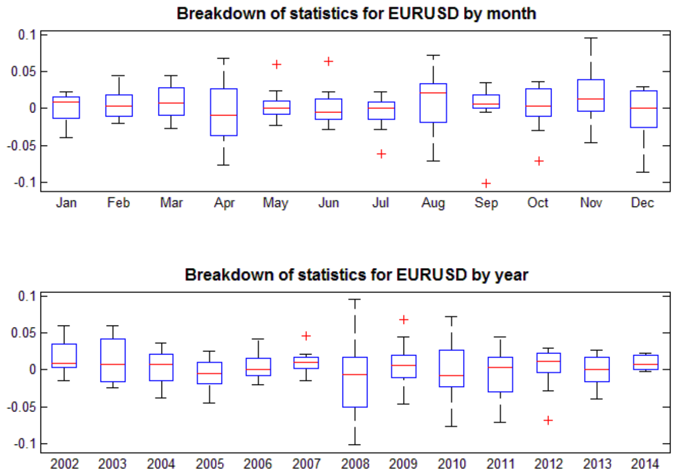

Percentage Statistics of the Exchange Rates,Traditional Mode

Percentage Statistics of the Exchange Rates,Compact Mode

We are using the same “boxplot” function to check the fluctuations of the exchange rates on the monthly and yearly basis. We can easily see in figure 5 the seasonal patterns of our EURUSD exchange rate. April, August, November and December are the months which the most fluctuations because of events like: Easter, summer holidays, Thanksgiving, Christmas or other financial events. December has the lowest log-returns as the investors are selling their assets. August has the highest median and can be considered the most productive month. On the monthly basis, the most probable question would be the interpretation of the red crosses named “whiskers” which are the extreme data points and not outliers. One explanation would be the periods of high values: upper crosses for high appreciation and lower crosses for high depreciation in that months.

By applying our mixture of Markov Switching model, we would see if it catches the structural breaks of the exchange rates in 2003, 2008 and 2010 which seem to fluctuate the most as we clearly see the big “boxes” in the yearly basis. The highest median of 2007 can be interpreted as the year with high returns and small symmetric fluctuations. In recent years, the exchange rate become more stable without high variance on yearly basis.

Figure 5: Breakdown statistics by month and by year for EURUSD



3.2. Descriptive statistics

First of all, we evaluated the most important moments in statistics in order to know better our exchange rates and other variables. For example, the mean is the first central moment of a random variable and it is known as expectation or the average of one sample. The second moment is the variance with $k = 2$ which measures the dispersion or the risk of one financial investment from the following equation:

$$\mu_k = E[\{X - E(X)\}^k]$$

The next two moments help to characterize the shape of a probability distribution. The skewness coefficient measures the degree of asymmetry and is:

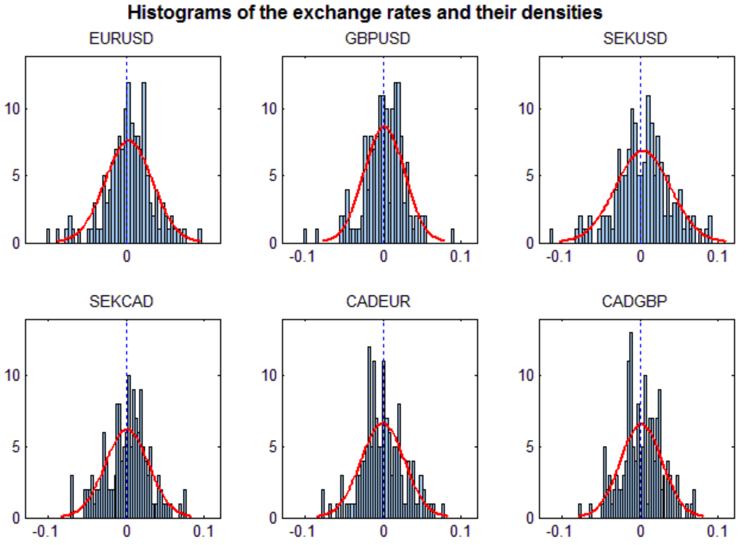
$$Sk\ (X) = \frac{\mu_3}{(\mu_2)^{3/2}}$$

While the kurtosis is a measure of the peak's distribution:

$$Kur\ (X) = \frac{\mu_4}{(\mu_2)^3}$$

After running some tests and plotting the histograms of the log-returns of the pairs presented in the Figure 6, we have checked if the changes of exchange rates have better descriptive statistics like lower variances which is a key parameter in statistics.

Figure 6: Histograms of the exchange rates and their densities



We spot immediately that SEKUSD follow a long tail distribution and this can be demonstrated by the high negative skewness. The first three exchange rates have higher variances while the other three have lower variances.

The main descriptive statistics of the monthly exchange rates are in the Table 2 while for the whole sample they can be found in Annexes 1-4. The mean of the log-returns are not significantly different from 0 and the standard deviation is approximately equal to 0.3. The first three exchange rates are left skewed while the other three are not significantly skewed. We shall see this difference in Figure 6 where is plotted the histogram with 50 bins. On the other hand, we discover the same results for high level of kurtosis in first three time series. The highest skewness (-0.5213) and kurtosis (5.1204) is present in the GBPUSD pair. The meaning of high skewness and kurtosis would be explained later when we discuss the normality tests.

Table 2: Descriptive statistics of the monthly exchange rates

	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
Mean	0.0033	0.0012	0.0034	0.0008	-0.0007	0.0013
Standard deviation	0.0305	0.026	0.0354	0.028	0.0279	0.0267
Skewness	-0.4248	-0.5213	-0.1921	-0.0748	0.0461	0.0167
Kurtosis	4.4656	5.1204	3.6312	3.2392	3.3663	3.1645

Checking the descriptive statistics of the fundamentals differential in the Annexes 2-4, all the means are around 0 except trade balance differential of the EURUSD which has -2.2301. The trade balance differential of EURUSD and SEKCAD have the highest standard deviations of 141.3691 and 56.5036. In general, the trade balance differentials have high standard deviation, skewness and kurtosis. Interest rate and monthly supply differentials are not normal because of high skewness and excess kurtosis. The interest rate differential of CADGBP pair have 3.3622 right skewness and 29.0849 of kurtosis.

Correlation

The correlation is an important measure of linear relationship of two variables that is bounded between -1 and 1. When the coefficient is -1, there is perfect negative correlation and the variables are changing in the same time but in opposite directions. When the correlation is 0, the variables are not related. The last correlation (1) is the case when the variables are moving together and have a linear relationship. Watching the next table about correlation between the exchange rates, we can see that they contain highly positive correlated data between countries. First of all, the exchange rates with the dollar as the quote currency have correlation higher than 0.65. Secondly, the pairs that have Canadian dollar as the base currency are negatively correlated with the exchange rates. One explanation of high correlation is that the markets are correlated between them which is called the “domino effect” or “contagion”. This is another reason for selecting Markov switching mechanism which allows correlation between exchange rates. In case of crisis, all the markets are sinking together especially if there are involved developed countries.

Table 3: Correlation matrix of the exchange rates

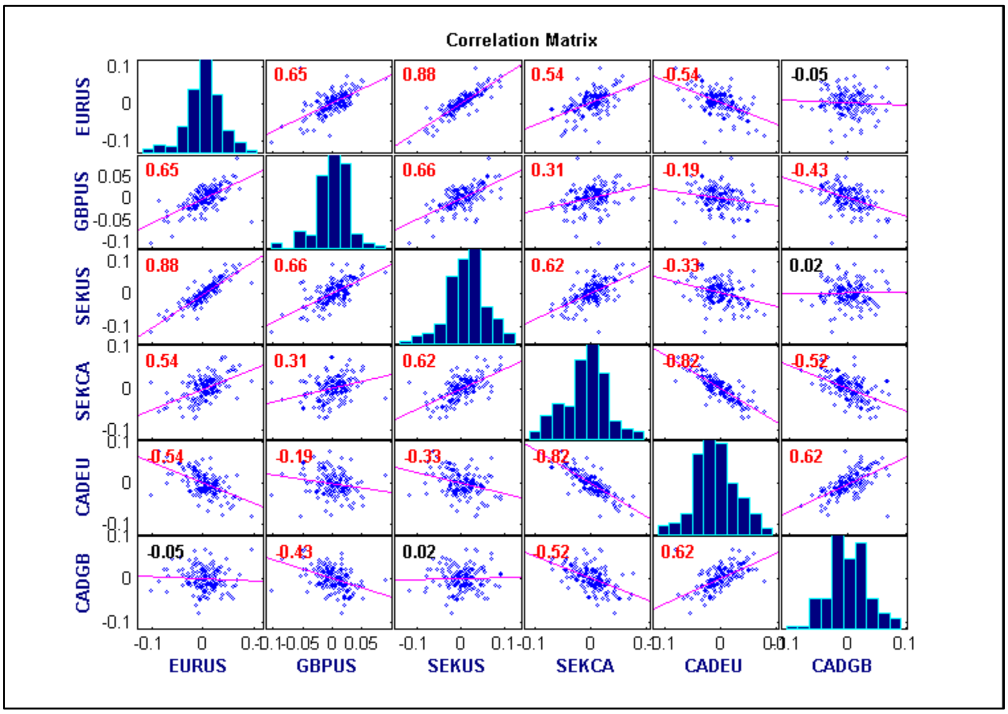
Correlation matrix of the exchange rates						
	EURUSD	GBPUSD	SEKUSD	SEKCAD	CADEUR	CADGBP
EURUSD	1.0000	0.6536	0.8774	0.5439	-0.5362	-0.05
GBPUSD	0.6536	1.0000	0.6582	0.3053	-0.1945	-0.426
SEKUSD	0.8774	0.6582	1.0000	0.6227	-0.3275	0.0247
SEKCAD	0.5439	0.3053	0.6227	1.0000	-0.8170	-0.525
CADEUR	-0.5362	-0.1945	-0.3275	-0.8170	1.0000	0.6176
CADGBP	-0.05	-0.426	0.0247	-0.525	0.6176	1

🟩	high positive correlation	> 0.2
🟥	high negative correlation	< -0.1
🟦	positive correlation	< 0.2
🟧	negative correlation	> -0.1

Another way of computing the correlation matrix in MATLAB is “corrplot” function which is displaying useful information about the exchange rates, their densities and the correlation between them. This graph can be viewed in the Figure 7. Histograms appears along the matrix diagonal and scatter plots of variable pairs appear off diagonal. The lines represent the slopes which are equal with the correlation coefficient while the blue crosses are the residuals.

All the pairs are highly correlated especially EURUSD with SEKUSD with a 0.88 correlation where the residuals are concentrated near the slope. We see that there is also low negative correlation between EURUSD and CADGBP. The lowest positive correlation is between SEKUSD with the same CADGBP which seem to be low correlated and asymmetric with the most exchange rates.

Figure 7: Correlation matrix with histograms



Regarding the explanatory variables, the interest rates and the 10 years bond yields are highly and positive correlated with exchange rates because it shows the importance of short and long term interest rate on the evolution of the exchange rates. We can conclude that if the quotation of interest rate is increasing then the quotation of the 10 years bond yields and the exchange rates are increasing too.

3.3. Unit root and normality tests

We already know that plenty financial time series are non-stationary. Augmented Dickey Fuller (ADF) and Phillip Peron (PP) are the most widely used statistical tests for checking if time series have unit roots. It is noted that our time series are stationary at the 5% level and both tests can confirm this fact.

The Jarque-Bera and Lilliefors test are checking if our time series follow normal distributions. Jarque-Bera test accounts for asymmetry and heavy tails. It is based on computing skewness and kurtosis into one formula which measures the previous issues. For a normal distribution, skewness should be 0 and kurtosis – 3. EURUSD and GBPUSD are negatively skewed and exhibit excess kurtosis so we reject the null hypothesis of a normal distribution for this pairs. The other four exchange rates changes follow normal distributions as their skewness and kurtosis are in the limits. In our opinion, the Lilliefors test is failing to assess the normality of our time series because it finds that our exchange rates are normal distributions. Maybe in the future research, it would not be considered reliable.

Table 4: Results of the stationary and normality tests

Unit root tests (Stationary)						
	EURUSD	GBPUSD	SEKUSD	SEKCAD	CADEUR	CADGBP
ADF Test	1 (0.001)	1 (0.001)	1 (0.001)	1 (0.001)	1 (0.001)	1 (0.001)
PP Test	1 (0.001)	1 (0.001)	1 (0.001)	1 (0.001)	1 (0.001)	1 (0.001)
Tests for normality						
	EURUSD	GBPUSD	SEKUSD	SEKCAD	CADEUR	CADGBP
JB Test	1 (0.004)	1 (0.001)	0 (0.133)	0 (0.5)	0 (0.5)	0 (0.5)
Skewness	-0.4248	-0.5213	-0.1921	-0.0748	0.0461	0.0167
Kurtosis	4.4656	5.1204	3.6312	3.2392	3.3663	3.1645
Lillie Test	0 (0.097)	0 (0.196)	0 (0.5)	0 (0.398)	0 (0.126)	0 (0.5)

ADF h=1 (p)-stationary (no unit root); PP h=1 (p)-stationary (no unit root);
JB h=0 (p)-normal distribution; Lillie h=0 (p)-normal distribution;

In the Annex 4, the majority of the fundamentals of CADEUR and CADGBP pairs are normal distributions with respect to the JB test except consumer production indexes, interest rates and trade balances. The majority of other fundamentals are not statistically significant to be considered normal distributions as they exhibit asymmetry and long tails.

The results of these two tests for the exchange rates are reported in the Table 4 and Annex 1 while the results of the fundamentals are stored in Annex 2-4.

3.4. Heteroscedasticity and autocorrelation tests

The Engle test for heteroskedasticity (ARCH) would check if the variables are heteroskedastic and Ljung Box Q test would test if there is autocorrelation. The test for conditional heteroskedasticity concludes that there is significant volatility in the SEKUSD. We accept the null hypothesis of no autocorrelation in the log-returns of the exchange rates.

Table 5: Results of heteroskedasticity and autocorrelation tests.

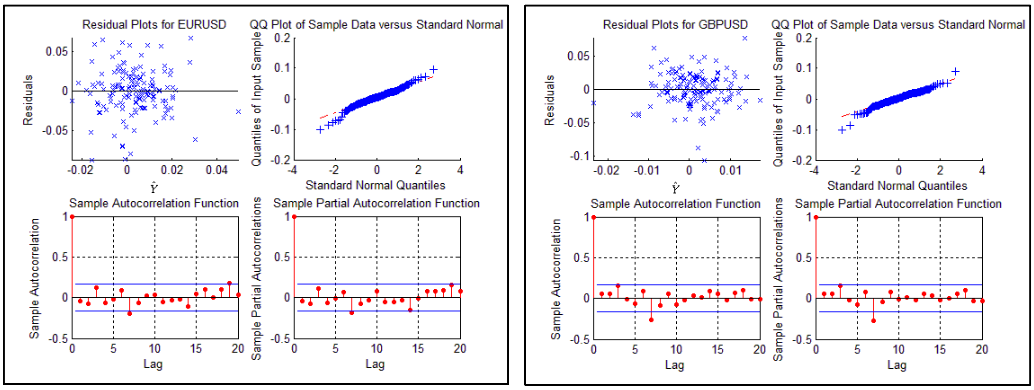
Tests	**EURUSD**	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
Tests for heteroskedasticity						
Arch Test (h)	0	0	1	0	0	0
P value	0.2569	0.6945	0.0175	0.8514	0.2535	0.1214
Statistics	1.2854	0.1543	5.6435	0.0351	1.3041	2.3995
Tests for autocorrelation						
LBQ Test (h)	0	0	0	0	0	0
P value	0.2900	0.0913	0.4913	0.1722	0.1027	0.0768
Statistics	22.9745	28.8210	19.4734	25.8083	28.2913	29.5901

Arch h=1 - heteroskedastic; LBQ h=1 - autocorrelated.

Another way of testing the heteroskedasticity and autocorrelation is the White's heteroscedasticity robust estimates assuming a linear model. In MATLAB it is presented with the function “hac” and plotting sample autocorrelation functions (ACF) and partial autocorrelation function (PACF) tests which are known in MATLAB as “autocorr” and “parcorr” functions. Using „hac” function in the program, we demonstrate the usage of SUR model with heteroscedastic errors because the residuals are dispersed.

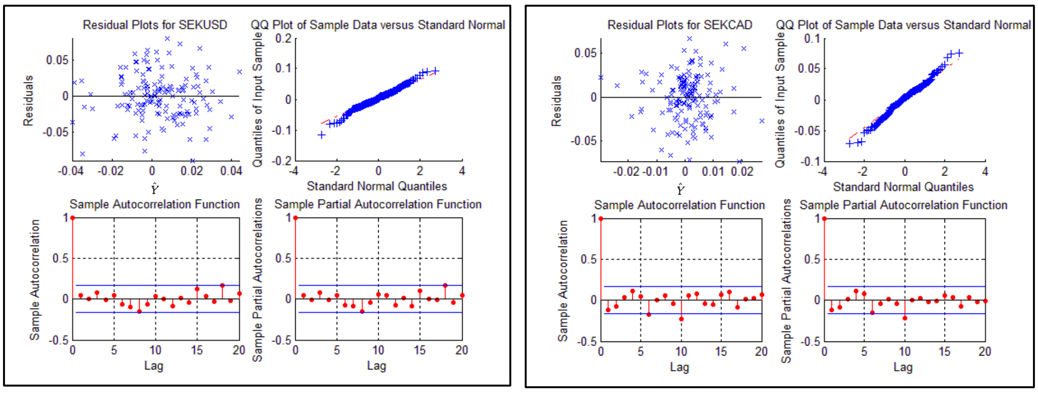
The upper right graph is the Quantile-Quantile plot which assess the normality of the time series in case of linear relationship. On the top left side there are residual plots that provides valuable information about our data. The bottom graphs are the ACF and PACF plots in order to assess the qualitative autocorrelation in the data. Figures 11-12 are showing that there is a significant spike within the boundary in GBPUSD residuals at lag 7 which can be concluded the existence of autocorrelation. LBQ test did not catch any serial correlation in the error terms for the default lag as we did not specify a particular lag to be tested.

Figures 8: ACF and PACF tests for EURUSD and GBPUSD



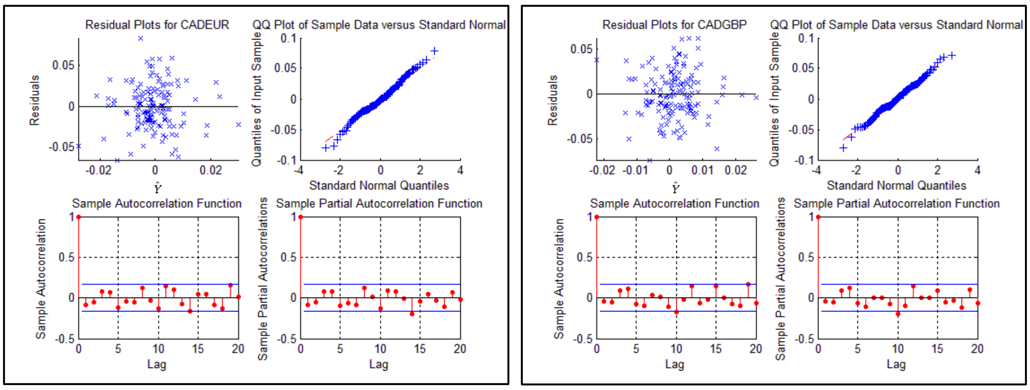
In the SEKCAD pair, there is a spike out of the 5% significance limits at the 10th lag. This proves the existence of autocorrelation but in general the exchange rate pair has not significant serial correlation in the residuals. The residuals of the sample data versus a standard normal are not significantly different so there is an evidence of normality in both exchange rates with Swedish krona as the base currency.

Figure 9: ACF and PACF tests for SEKUSD and SEKCAD



An examination of the last two figures with ACF and PACF tests at 95% confidence indicates that there is no autocorrelation. The residuals are concentrated between -0.2 and 0.2. The Q-Q plot shows the evidence of normality in the exchange rates changes of the Canadian dollar over the other currencies as was found with JB test and with the descriptive statistics.

Figure 10: ACF and PACF tests for CADEUR and CADGBP



In conclusion we can say that there are not major significant spikes in our exchange rates.

Almost all fundamentals of the exchange rates are significant autocorrelated at the default lag in the LBQ test and the results are presented in Annex 2-4. We have few variable which are not autocorrelated so we will enumerate them: long term interest rates differentials (except for CADEUR), purchasing managers differential (except EURUSD and CADEUR), producer price differential and money supply differential of the CADGBP and unemployment rates differential of the CADEUR and CADGBP with the highest p-values.

Also, we detect heteroskedasticity in variables and this calls for the use of time varying volatility model. The null hypothesis of heteroskedasticity is rejected in variables like purchasing managers differentials. Short term interest rates (except SEKCAD), money supply (except EURUSD and GBPUSD), unemployment rates (except EURUSD and SEKCAD) and long term interest rates (except GBPUSD and CADEUR) are not heteroskedastic along with industrial production differential of the EURUSD, leading index differential of the GBPUSD and producer price indexes of CADEUR and of CADGBP.

3.5. MCMC estimation results

The Gibbs sample was run to obtain a sample of 4000 iterations from the posterior distributions of the parameters and then we discard the first 1000 draws. The MCMC sampling algorithm retains 3000 iterations for summarizing the posterior distribution and we compute the posterior mean and standard errors.

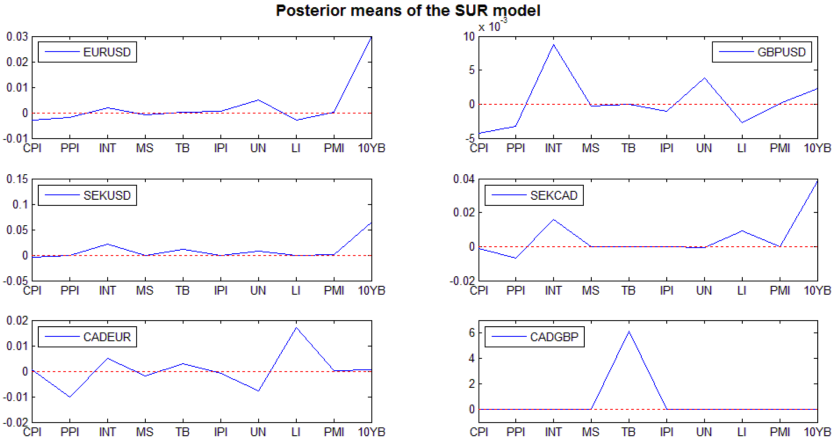
The iterations of the SUR model started by specifying values of variance-covariance matrix of the error term with $d_{\Sigma} = 0$ and $\Omega_{\Sigma} = 0_{6\times 6}$ and prior of regression coefficients $\mu_{\beta} = 0_{K\times 6}$ and $\Sigma_{\beta}^{2} = \Sigma \otimes I_{T}$. For the MS-SUR model's iterations, we start with the regression coefficients $m_{j} = 0_{K\times 6}$ and $\Upsilon_{j}^{2} = \Sigma \otimes I_{T}$ for $j = 1, 0$ for our states. We set $a_{j} = 0$ and $b_{j} = 0_{6\times 6}$ for the prior information about the covariance matrix for both states $j = 1, 0$. Nevertheless, $c_{ii} = 0.9$ and $d_{ii} = 0.9$ are from the prior transitional probabilities ($i = 0,1$). With these fairly informative priors, the simulation process begins by generating draws from β parameters and the simulation continues by generating draws from other parameters like $\boldsymbol{\Sigma}$ in the SUR model or $\boldsymbol{\Sigma}$ and θ for the MS-SUR model as it is explained at the end of the posterior computation of the conditional distributions.

To check if the number of iterations are sufficient, we run a convergence diagnostic for Markov chains proposed by Geweke (1992) based on a test for equality of the means of the first and last part of a Markov chain (by default the first 10% and the last 50%). If the samples are drawn from the stationary distribution of the chain, the two means are equal and Geweke's statistic has an asymptotically standard normal distribution. For the SUR model, only 20 out of 60 posterior means are drawn from the stationary distribution at 5% of significance. For the state 1 of the MS-SUR model just 5 out of 60 are significant while for the state 0 only 3 out of 60 variables. Our model is complex and contain many autocorrelated variables so would be hard to achieve convergence.

The regression coefficients of our Bayesian SUR model measure the change in the predicted value of our fundamentals on the exchange rates performance. We summarize the results of the posterior means in the following figures and the numerical values in the Tables 7-8.

First, we note that our all estimates of the SUR model are not significant at 10% level of significance and we can see them in the following figure.

Figure 11: Posterior means of the SUR model



The posterior means of the MS-SUR model state 1 are similar with our SUR model as we can see in the Figure 12 and most of them are significant at 1%.
The state of low volatility (state 1) has the most correctly significant estimates but we find structural breaks in parameters of consumer production indexes and

unemployment rates. Another differences are identified in trade balance and industrial production index of CADGBP which do not comply with the theory.

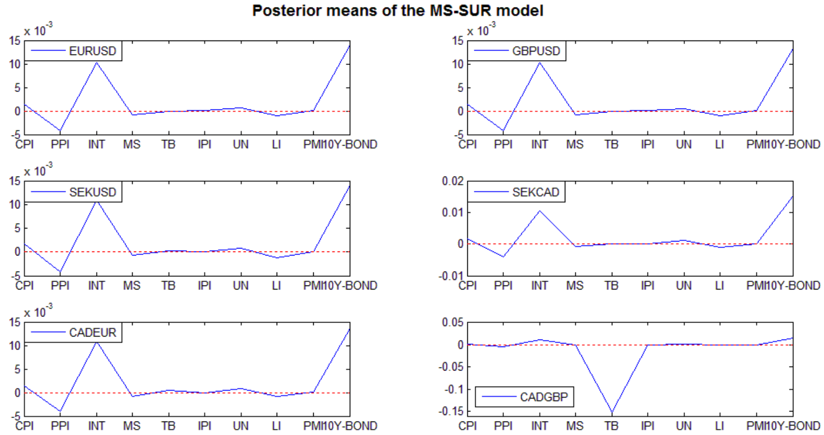
We obtain these significant results regards the changes in the MS-SUR coefficients:

- We found that producer price index influence negatively the performance of the exchange rates so higher inflation would depreciate the exchange rates which is true by the theory. In this state, the consumer price index coefficients are wrongly signed across all equations.
- In this model, short term interest rate and long term interest rate have positive impact on the exchange rates which comply according the theory. Both list of coefficients are significant at 1% level.
- An increase in monthly supply, should cause the fall of the interest rate and the depreciation of the exchange rate. This is described as opposite relationship so we correctly found negative significant coefficients for all equations.
- Trade balance shows the difference between exports and imports. If the trade balance increase, we should expect an increase of the exchange rate. We obtain good estimates in order to show the impact of trade balance on exchange rates. Nevertheless, for the CADGBP pair, we have a very high positive significant coefficient (5.35) at 1% level while for the SEKUSD, we find that the posterior mean is significant at 10% level.
- Industrial production index is a growth indicator because is showing whether the economy is healthy. There should be a positive relationship with the exchange rates. Half of the industrial production index coefficients are insignificant (SEKUSD, CADEUR and CADGBP) and one is wrongly signed (CADGBP). Significant and correctly signed coefficients are in the EURUSD, GBPUSD and SEKCAD equations.
- If there is an increase in the unemployment rate, central bank is decreasing the interest rate so the exchange rates are depreciating. Another structural

break is found in these parameters because there should be a negative relationship between unemployment and exchange rates. Our regression coefficients of the unemployment rate are found correctly only in the CADGBP equation within the 1% level of significance.

- The increase in the leading index, would depreciate the exchange rates because there is an indication of the future state of the economy. The coefficients are negative and significant at 1% for all equations.

Figure 12: Posterior means of the MS-SUR model of state 1



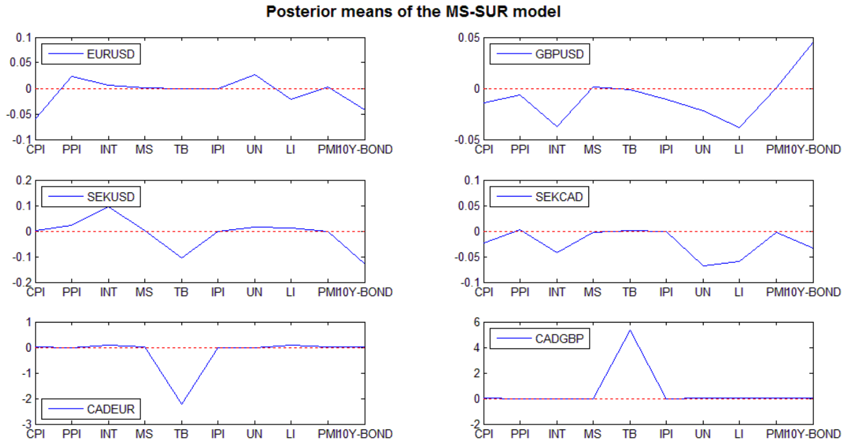
- Purchasing managers index is a survey about the present state of the economy. An increasing indicator means that the economy is good, the interest rates are increasing and the exchange rates are increasing too. We find that the statement is true for our coefficients which turns to influence the exchange rates positively and significant.

We draw Table 6 in order to show the difference between SUR and MS-SUR models coefficients.

In the state 0 of the MS-SUR, the results are heterogeneous and inconsistent with the theory. We have estimates which seem to be more diversified in signs but most of them significant at 1% of significance level.

For example, three coefficients of the consumer price indexes are correctly signed but the estimate with wrong sign of the CADGBP is insignificant, of the SEKUSD is significant at 10% and the CADEUR coefficient is significant at 5%.
We see that most of estimates are equally divided in two categories of signs as previous example with the consumer price index.

Figure 13: Posterior means of the MS-SUR model of state 0



As you probably see the Figure 13, the results are not realistic because we do not find any correct signed estimates across equations.

The trade balance fluctuates from -2.21 in CADEUR equation to 5.3 in CADGBP pair and four estimates are showing that the trade balance have a negative impact on exchange rates which is not true according to the theory.
The interest rates estimates are wrongly signed in the right plots of the Figure 13 (GBPUSD, SEKCAD and CADGBP equations).
The money supply estimates are correctly signed just in SEKCAD and CADGBP.
One fairly good result is in the estimates of the purchasing managers index where four of them have the correct sign and the fifth one (SEKUSD) is insignificant.

We draw a table which is an important output from the thesis. Table 6 is presenting the signs of the estimates in order to assess the difference between SUR and MS-SUR model state 1. These two groups of estimates are almost the same across equations but Markov switching model detected structural instabilities in consumer price indexes and unemployment rates. Apart from this, the estimates of coefficients in the two states are significant at 1% level and generally respect the correct impact predicted by the theory.

There are some improvements especially in the industrial production parameters, trade balance, leading index and purchasing managers parameters where found better estimates which are consistent with the theory.

In the next table, the difference from the theory is highlighted and the asterisks refer to the significance level.

Table 6: The signs of the SUR and MS-SUR state 1 estimates

	True	SUR model						MS-SUR model state 1					
		EUS	GUS	SUS	SCA	CEU	CGP	EUS	GUS	SUS	SCA	CEU	CGP
CPI	-	-	-	-	-	+	-	+*	+*	+*	+*	+*	+*
PPI	-	-	-	-	-	-	-	-*	-*	-*	-*	-*	-*
INT	+	+	+	+	+	+	+	+*	+*	+*	+*	+*	+*
MS	-	-	-	-	-	-	-	-*	-*	-*	-*	-*	-*
TB	+	+	+	+	+	+	+	+*	+*	+***	+*	+*	-*
IPI	+	+	-	-	-	-	-	+*	+*	+	+*	+	-
Un	-	+	+	+	-	-	+	+*	+**	+*	+*	+*	-*
LI	-	-	-	-	+	+	-	-*	-*	-*	-*	-*	-*
PMI	+	+	+	+	+	+	-	+*	+*	+*	+*	+*	+*
10yB	+	+	+	+	+	+	+	+*	+*	+*	+*	+*	+*

Asterisks refer to level of significance *1%, **5%, ***10%.

EUS=EURUSD, GUS=GBPUSD, SUS=SEKUSD.

SCA=SEKCAD, CEU=CADEUR, CGP=CADGBP.

In the Table 7, there are the estimates of the posterior distributions for three exchange rates while the others are found in the Table 8.

Table 9 shows the estimates of the variance-covariance matrix and the Table 10 presents the prior and posterior transitional probabilities.

Table 7: MCMC estimates of posterior distributions of the first three exchange rates

Exchange rates	Variables	Posterior β_{SUR}	Std β_{SUR}	Posterior 1 β_{MS-SUR}	Std 1 β_{MS-SUR}	Posterior 0 β_{MS-SUR}	Std 0 β_{MS-SUR}
EURUSD	CPI	-0.0029	0.0148	0.0015*	0.0001	-0.0590*	0.0016
	PPI	-0.0019	0.0106	-0.0041*	0.0001	0.0239*	0.0006
	INT	0.0018	0.0366	0.0102*	0.0004	0.0054*	0.0020
	MS	-0.0010	0.0013	-0.0008*	0.0000	0.0013*	0.0001
	TB	0.0000	0.0001	0.0000*	0.0000	-0.0001*	0.0000
	IPI	0.0006	0.0039	0.0001*	0.0000	-0.0006*	0.0002
	Un	0.0048	0.0220	0.0006*	0.0002	0.0269*	0.0019
	LI	-0.0028	0.0114	-0.0011*	0.0002	-0.0221*	0.0006
	PMI	0.0003	0.0016	0.0000*	0.0000	0.0023*	0.0001
	10yBond	0.0298	0.0359	0.0139*	0.0006	-0.0423*	0.0016
GBPUSD	CPI	-0.0043	0.0155	0.0013*	0.0001	-0.0144*	0.0007
	PPI	-0.0033	0.0088	-0.0041*	0.0001	-0.0062*	0.0004
	INT	0.0088	0.0331	0.0103*	0.0004	-0.0379*	0.0027
	MS	-0.0003	0.0012	-0.0008*	0.0000	0.0009*	0.0001
	TB	0.0000	0.0004	0.0000*	0.0000	-0.0014*	0.0000
	IPI	-0.0010	0.0039	0.0000*	0.0000	-0.0106*	0.0003
	Un	0.0039	0.0239	0.0004**	0.0002	-0.0224*	0.0016
	LI	-0.0028	0.0109	-0.0011*	0.0002	-0.0383*	0.0009
	PMI	0.0002	0.0005	0.0000*	0.0000	0.0004*	0.0000
	10yBond	0.0023	0.0597	0.0133*	0.0006	0.0449*	0.0021
SEKUSD	CPI	-0.0036	0.0129	0.0016*	0.0001	0.0009***	0.0006
	PPI	-0.0017	0.0061	-0.0041*	0.0001	0.0215*	0.0007
	INT	0.0211	0.0297	0.0108*	0.0004	0.0951*	0.0024
	MS	-0.0007	0.0012	-0.0008*	0.0000	0.0028*	0.0001
	TB	0.0115	0.0875	0.0002***	0.0002	-0.1040*	0.0037
	IPI	-0.0006	0.0021	0.0000	0.0000	0.0000	0.0003
	Un	0.0074	0.0156	0.0008*	0.0002	0.0141*	0.0011
	LI	-0.0008	0.0106	-0.0012*	0.0002	0.0130*	0.0007
	PMI	0.0003	0.0029	0.0000*	0.0000	-0.0001	0.0002
	10yBond	0.0650	0.0579	0.0140*	0.0006	-0.1291*	0.0034

Asterisks refer to level of significance *1%, **5%, ***10%. Std=standard deviation.

Table 8: MCMC estimates of posterior distributions of the last three exchange rates

Exchange rates	Variables	Posterior β_{SUR}	Std β_{SUR}	Posterior 1 β_{MS-SUR}	Std 1 β_{MS-SUR}	Posterior 0 β_{MS-SUR}	Std 0 β_{MS-SUR}
SEKCAD	CPI	-0.0010	0.0107	0.0017*	0.0001	-0.0230*	0.0017
	PPI	-0.0068	0.0061	-0.0041*	0.0001	0.0018*	0.0003
	INT	0.0159	0.0364	0.0106*	0.0004	-0.0416*	0.0024
	MS	-0.0003	0.0033	-0.0008*	0.0000	-0.0026*	0.0004
	TB	0.0001	0.0001	0.0000*	0.0000	0.0002*	0.0000
	IPI	-0.0001	0.0017	0.0000*	0.0000	-0.0002	0.0001
	Un	-0.0007	0.0234	0.0011*	0.0002	-0.0678*	0.0020
	LI	0.0091	0.0216	-0.0010*	0.0002	-0.0600*	0.0016
	PMI	0.0001	0.0013	0.0000*	0.0000	-0.0029*	0.0001
	10yBond	0.0388	0.0683	0.0151*	0.0006	-0.0330*	0.0030
CADEUR	CPI	0.0005	0.0098	0.0015*	0.0001	0.0012**	0.0008
	PPI	-0.0101	0.0089	-0.0040*	0.0001	-0.0311*	0.0008
	INT	0.0051	0.0492	0.0108*	0.0004	0.0947*	0.0047
	MS	-0.0019	0.0094	-0.0008*	0.0000	0.0173*	0.0005
	TB	0.0029	0.1193	0.0005*	0.0002	-2.2153*	0.1351
	IPI	-0.0010	0.0022	0.0000	0.0000	-0.0016*	0.0001
	Un	-0.0077	0.0430	0.0009*	0.0002	-0.0336*	0.0022
	LI	0.0170	0.0409	-0.0009*	0.0002	0.0926*	0.0032
	PMI	0.0003	0.0010	0.0000*	0.0000	0.0020*	0.0001
	10yBond	0.0006	0.0417	0.0137*	0.0006	0.0159*	0.0035
CADGBP	CPI	-0.0040	0.0110	0.0014*	0.0001	0.0010	0.0009
	PPI	-0.0093	0.0090	-0.0040*	0.0001	-0.0171*	0.0006
	INT	0.0045	0.0410	0.0106*	0.0004	-0.0217*	0.0019
	MS	-0.0022	0.0066	-0.0007*	0.0000	-0.0029*	0.0002
	TB	6.1180	9.9398	-0.1519*	0.0178	5.3477*	0.5760
	IPI	-0.0005	0.0024	-0.0001	0.0000	-0.0025*	0.0001
	Un	0.0010	0.0431	0.0006*	0.0002	0.0647*	0.0018
	LI	-0.0006	0.0137	-0.0010*	0.0002	0.0052*	0.0008
	PMI	-0.0001	0.0005	0.0000*	0.0000	0.0005*	0.0000
	10yBond	0.0036	0.0728	0.0140*	0.0006	0.0151*	0.0025

Asterisks refer to level of significance *1%, **5%, ***10%. Std=standard deviation.

Table 9: Posterior variance-covariance matrix for both models

Posterior Variance Covariance matrix of SUR Model						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
EURUSD	0.0080	0.0005	0.0008	0.0004	-0.0004	-0.0001
GBPUSD	0.0005	0.0078	0.0005	0.0002	-0.0002	-0.0003
SEKUSD	0.0008	0.0005	0.0082	0.0005	-0.0003	0.0000
SEKCAD	0.0004	0.0002	0.0005	0.0078	-0.0005	-0.0003
CADEUR	-0.0004	-0.0002	-0.0003	-0.0005	0.0078	0.0003
CADGBP	-0.0001	-0.0003	0.0000	-0.0003	0.0003	0.0078
Posterior Variance Covariance matrix state 1 of MS-SUR Model						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
EURUSD	0.0008	0.0004	0.0008	0.0004	-0.0004	-0.0001
GBPUSD	0.0004	0.0006	0.0005	0.0002	-0.0001	-0.0003
SEKUSD	0.0008	0.0005	0.0010	0.0005	-0.0003	0.0000
SEKCAD	0.0004	0.0002	0.0005	0.0006	-0.0005	-0.0003
CADEUR	-0.0004	-0.0001	-0.0003	-0.0005	0.0007	0.0003
CADGBP	-0.0001	-0.0003	0.0000	-0.0003	0.0003	0.0006
Posterior Variance Covariance matrix state 0 of MS-SUR Model						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
EURUSD	0.0009	0.0005	0.0011	0.0006	-0.0005	0.0000
GBPUSD	0.0005	0.0006	0.0003	-0.0001	-0.0006	-0.0004
SEKUSD	0.0011	0.0003	0.0016	0.0011	-0.0003	0.0005
SEKCAD	0.0006	-0.0001	0.0011	0.0010	0.0002	0.0007
CADEUR	-0.0005	-0.0006	-0.0003	0.0002	0.0007	0.0005
CADGBP	0.0000	-0.0004	0.0005	0.0007	0.0005	0.0007

The variance-covariance matrix of the SUR model indicates that the variances are concentrated around 0.008 across equations. The CADEUR has negative covariance coefficients with the majority of exchange rates.

The MS-SUR model has two variance-covariance matrixes, each for one state. As we expected, the variances are lower than the standard SUR model but the variances of the state 0 are higher than the first one due to the high volatility regime. From all variances, we clearly see the high values for the SEKUSD pair in both models.

In table 10, we have registered the probabilities of switching between the states. The posterior probability for the state 1 is 0.964 while for state 0 is 0.808. We should mention that our regimes are highly persistent as the probabilities for remaining in the same state is 81% and 96 %.

Table 10: Prior and posterior transition probabilities of the MS-SUR model

Prior transition probability matrix		Posterior transition probability matrix	
0.9	0.1	0.808	0.036
0.1	0.9	0.192	0.964

The expected duration of the regimes is calculated from the posterior transition probability matrix. Let denote D_j the number of periods when the system is in state j then the expected duration of the regime in that state is

$$E[D_j] = \frac{1}{1-\theta_{jj}} \text{ for } j = 0{,}1.$$

Expected duration of the regimes are the following:

- Regime 0 of high volatility $E[D_0] = \frac{1}{1-\theta_{00}} = \frac{1}{1-0.808} = 5{,}21$ (months).
- Regime 1 of low volatility $E[D_1] = \frac{1}{1-\theta_{11}} = \frac{1}{1-0.964} = 27.75$ (months).

The expected duration of high volatility regime is approximately 5 months and for low volatility 28 months.

Another relevant result in this thesis is the extraction of the regimes following the MS-SUR model which was used to check if there are structural breaks in the parameters.

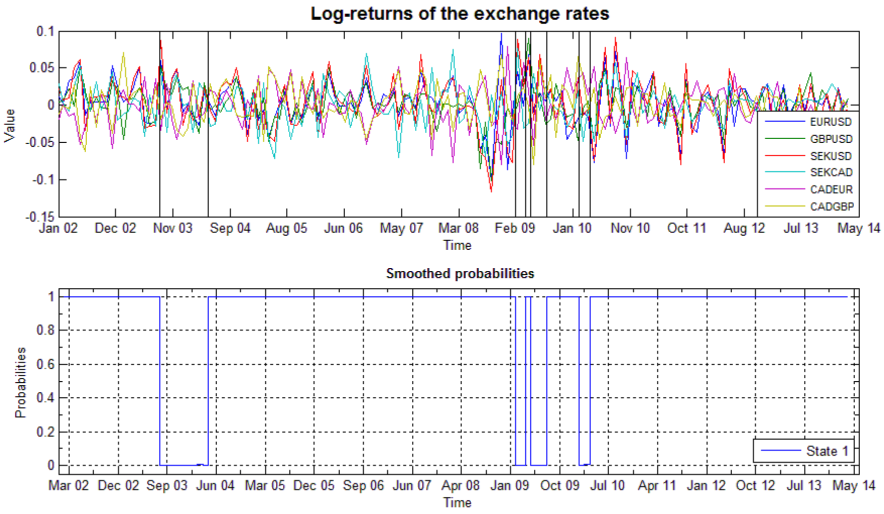
Figure 14 is showing the log-returns of the exchange rates in the upper plot. The bottom plot represents the smoothed probabilities of the states.

Surprisingly, the regime switching model did not detect volatility changes for the 2007-2008 financial and subprime crisis, Lehman Brothers collapse, etc.

Our model found some structural breaks in the parameters which can be seen in the smoothed probabilities of the states in four periods:

August 2003 – April 2004: The first period is the longest and lasted 9 months. One important event was the reduction of the interest rate in United States where the Federal Reserve Chairman, Alan Greenspan cut the key interest rate to 1 % two months before the starting of our period. This was the lowest interest rate in 45 years and was kept until May 2004 which was exactly one month after this interval. In the same time, the changes in exchange rates had huge volatility. The inflation rates have increased and the bonds yields decreased dramatically.

Figure 14: The switching regimes of MS-SUR



February 2009 – March 2009: It should be noted that in these two months, the global economy was in recession. In February, the President of the United States signs American Recovery and Reinvestment Act for stimulating the economy.

The European Central Bank cuts the interest rate by 1.25 point in four month and the Bank of England cuts interest rates to a record low of 1% from 1.5% – the fifth interest rate cut since October 2008. Even the Swedish and Canadian central bank reduced their key interest rate.

Therefore, the both rates of inflation increased sharply while all the exchange rates depreciated. The government bonds and the trade balances decreased. The industrial production indexes were at low levels and the leading indexes had negative coefficient so the overall economy was weak. The exit from this period was ensured by the G20 meeting when they announced one stimulus package of $5 trillion for global recovery.

May 2009 – July 2009: Haven't passed one month that we have another simultaneous switch with a duration of three months. More probably has been switched because ECB decreased the interest rates again at 1%. The Federal Reserve and the Canadian Central Bank established 0.25% interest rate followed by the Swedish Central Bank in July 2009. The unemployment rose in all countries and industrial production index of EU was around -17 % in that period.

February 2010 – March 2010: One month before, the European Commission presented "Report on Greek Government Deficit and Debt Statistics"[7] where they reviewed the deficit and the debt of this country. In this short period, the European Commission have created first round recommendations for the Greek government to help with the recovery from the crisis. Sovereignty debt crisis of the EU was so strong that euro felt against the American dollar and weakened against the British pound especially because of the Greece difficulties to cut the deficit and debt. Portugal, Ireland and Spain were still recovering after the crisis which created some fluctuations in the exchange rate. The economy was weak and still recovering after the sovereign debt crisis. In May 2010, ECB started to improve the liquidity in the system by buying the Eurozone government bonds which stabilised the bond yields.

The main exchange rates have depreciated. The money supply and the government bond yields have decreased constantly across all the equations.

---

[7] *„Report on Greek Government Deficit and Debt Statistics*" available at http://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:52010DC0001

4. Conclusions

We have proposed, analysed and discussed a new model which extracted regimes based on high and low volatility. The new model, that is MS-SUR model, allows for structural breaks in the parameters and offer the possibility to study the relationship between the exchange rates and their fundamentals.

We also discuss and illustrate the computation of the conditional distributions of the Bayesian inference approach which are used in the Markov chain Monte Carlo framework.

In the thesis, the results of the regression coefficients across equations are satisfactory and reflects what is predicted by the economic theory and previous empirical studies.

Mainly, the changes in short term interest rates (monetary policy) and in the long term interest rates (government bond yields-include expected inflation) have more explanatory power than the other fundamentals. These fundamentals affect positively the exchange rates according to our results and by the theory.

Consumer price index and producer price index are affecting negatively in the long run and are known as the inflation rates. Another important fundamentals are money supply which characterizes a negative impact on the exchange rate performance.

We obtain good estimates for the trade balance and purchasing managers index which provides positive relationships with the exchange rates.

As regards the regimes, one of the estimated MS-SUR regimes is highly persistent and has coefficients which are similar to the one of the standard SUR model.
In the analysed period, we found evidence of parameter switches during August 2003 – April 2004, February 2009 – March 2009, May 2009 – July 2009 and February 2010 – March 2010. We find that in these periods, the economies were

weak or in recession. The central banks cut interest rates to historical low levels and the exchange rates have depreciated.

MS-SUR model can be useful for studying relationships with structural breaks in many other subjects. Further research will be done in order to add the high-low mean constraint and to study the forecasting accuracy of the out-of-sample observations. We found that our explanatory variables are heteroskedastic so we would try to build a time varying volatility model.

Acknowledgements

The author would like to thank Biblioteca di Economia of University Ca' Foscari Venice for the access to the Bloomberg database.

References

BIS Triennial Central Bank Survey, 2013. *Foreign exchange turnover in April 2013: preliminary global results.* Monetary and Economic Department, available at www.bis.org

Cassel, Gustav, 1918. *Abnormal Deviations in International Exchanges*. The Economic Journal, 112, 413 415.

Casarin, R., Tronzano, M., Sartore, D., 2013. *Bayesian Markov Switching Stochastic Correlation Models.* Department of Economics Ca' Foscari University of Venice, Working Papers, No. 11.

Chib, S., Greenberg, E., 1995. *Hierarchical analysis of SUR models with extensions to correlated serial errors and time-varying parameter models*. Journal of Econometrics 68, 339-360.

Chib, S., 1996. *Calculating posterior distributions and modal estimates in Markov mixture models*. Journal of Econometrics 75, 79-97.

Engel, Charles, Hamilton, James D., 1990. *Long swings in the dollar. Are they in the data and do markets know it?* American Economic Review 80, 689– 713.

Engel, Charles, 1994. *Can the Markov switching model forecast exchange rates?* Journal of International Economics 36, 151– 165.

Engel, C. and West, K. D., 2005. *Exchange Rates and Fundamentals*. Journal of Political Economy, 113, 485-517.

European Commission, Economic and financial department, 2014. *EU response to the crisis.* Available at http://ec.europa.eu/economy_finance/crisis/index_en.htm

European Central Bank, *Timeline of the financial crisis.* Available at https://www.ecb.europa.eu/ecb/html/crisis.en.html

Federal Reserve Bank of St. Louis, *The Financial Crisis – A timeline of events and policy actions.* Available at http://timeline.stlouisfed.org/index.cfm?p=timeline#

Franke, Jurgen, 2012. *Markov Switching Time Series Models.* Time Series Analysis: Methods and Applications, Vol. 30.

Frommel, M, MacDonald, R, Menkhoff, L, 2005. *Markov switching regimes in a monetary exchange rate model.* Economic Modelling 22, 485– 502.

Frühwirth-Schnatter, Sylvia, 2006. *Finite Mixture and Markov Switching Models.* Springer Series in Statistics, New York.

Ghalayini, Latife, 2014. *Modelling and Forecasting the US Dollar/Euro Exchange Rate.* International Journal of Economics and Finance; Vol. 6, No. 1.

Goldfeld, S.M. and R.E. Quandt, 1973, *A Markov model for switching regressions*. Journal of Econometrics I, 3-16.

Greene, W.H., 2002. *Econometric Analysis* (5th ed.). New Jersey: Prentice-Hall.

Griffiths, William E., 2001. *Bayesian Inference in the Seemingly Unrelated Regressions.* Chapter prepared for Model.Computer-Aided Econometrics.

Hamilton, J.D., 1989. *A new approach to the economic analysis of nonstationary time series and the business cycle*. Econometrica, 57, 357-384.

Hamilton, James D., 1994. *Time Series Analysis*. Princeton University Press, Princeton, United States of America.

Koop, Gary, 2004. *Bayesian Econometrics,* John Wiley & Sons Ltd, Chichester, England.

Lee, Hsiu-Yun, Chen, Show-Lin, 2006. *Why use Markov-switching models in exchange rate prediction?* Economic Modelling, 23, 662–668.

Lin, Winston T., 2005. *Currency Forecasting Based on an Error Components-Seemingly Unrelated Nonlinear Regression Model*. Journal of Forecasting, 24, 593–605.

Nydick, Steven W., 2012. *The Wishart and Inverse Wishart Distributions*. International Journal of Electronics and Communication, 22, 119-139.

Percy, D., 1992. *Predictions for Seemingly Unrelated Regressions*. Journal of the Royal Statistical Society Series B, 54, 243-252.

Perlin, M., 2009. *MS Regress: A Package for Markov Regime Switching Models in Matlab.* Matlab Central File Exchange, available at: http://www.mathworks.com/matlabcentral/fileexchange/authors/21596.

Ruppert, David, 2011. *Statistics and Data Analysis for Financial Engineering.* Springer Texts in Statistics.

Srivastava, V. K. and Giles, D. E. A., 1987. *Seemingly Unrelated Regression Equations Models*. Dekker, New York.

Zellner, A., 1962. *An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias*. Journal of the American Statistical Association 57 (298): 348-368.

Zellner, A., Ando, T., 2010. *A Direct Monte Carlo Approach for Bayesian Analysis of the Seemingly Unrelated Regression Model*. Journal of Econometrics, 159, 33–45.

Annex 1: Statistical test for analysing the exchange rates

General statistics for exchange rates						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
Mean	0.003255	0.001198	0.003364	0.000832	-0.00073	0.001308
Standard deviation	0.030497	0.026044	0.035439	0.027996	0.027896	0.026698
Kurtosis	4.46559	5.120368	3.631235	3.239183	3.366297	3.164464
Skewness	-0.42483	-0.52132	-0.19215	-0.07477	0.046058	0.016677
Augmented Dickey Fuller Test for stationary/unit root (ADF)						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
decision	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001
statistics	-12.1591	-10.7184	-11.7115	-13.2753	-13.4902	-12.6411
Phillips Perron Test for stationary/unit root (PP)						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
decision	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001
statistics	-12.1591	-10.7184	-11.7115	-13.2753	-13.4902	-12.6411
Jarque Bera Test for normality (JB)						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
decision	1	1	0	0	0	0
p value	0.004125	0.001	0.132822	0.5	0.5	0.5
statistics	17.57804	34.19634	3.345126	0.487374	0.873785	0.172485
Lilliefors Test for normality (Lillie)						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
decision	0	0	0	0	0	0
p value	0.097158	0.195567	0.5	0.397622	0.12653	0.5
statistics	0.067973	0.061182	0.049348	0.052579	0.065507	0.048801
Engle test for residual heteroskedasticity test (Arch)						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
decision	0	0	1	0	0	0
p value	0.256892	0.694492	0.01752	0.851381	0.253458	0.121373
statistics	1.285428	0.154266	5.643541	0.035102	1.30414	2.399512
Ljung Box Q Test for residual autocorrelation (LBQ)						
	EURUSD	**GBPUSD**	**SEKUSD**	**SEKCAD**	**CADEUR**	**CADGBP**
decision	0	0	0	0	0	0
p value	0.290049	0.091343	0.491274	0.172237	0.102681	0.076774
statistics	22.97449	28.82104	19.47339	25.80829	28.29131	29.59009

Explanations of the results:

ADF h=1 (p value)-stationary (no unit root); PP h=1 (p value)-stationary (no unit root);
JB h=0 (p value)-normal distribution; Lillie h=1 (p value)-not a normal distribution;
Arch h=1 (p value)-heteroskedastic; LBQ h=1 (p value)-autocorrelated

Annex 2: Statistical tests for the fundamentals indexes of EURUSD and GBPUSD

General statistics	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
Mean	0.0001	-0.0095	-0.0102	-0.0510	-2.2301	0.0351	0.0259	0.0061	0.1721	-0.0075	0.0041	-0.0061	-0.0136	-0.0068	-0.1913	0.0106	0.0122	-0.0048	0.2966	0.0010
Std	0.6149	0.7535	0.2025	6.3022	141.3691	1.9926	0.3666	0.6827	4.7677	0.2059	0.6066	0.8767	0.2318	6.3207	20.8253	1.9276	0.3673	0.6884	15.4811	0.1280
Skew ness	-0.1014	0.0347	1.8599	0.2645	0.4796	-0.4615	-0.5044	-0.0641	-0.0795	0.1620	0.0731	0.4082	-0.5125	0.3892	-0.1912	0.2300	-0.7919	0.0664	-0.2148	-0.2479
Kurtosis	3.2293	3.0929	14.0203	8.5530	3.9796	4.2881	3.0465	2.5881	2.5322	4.5651	3.0942	5.0976	19.8296	8.5402	3.4290	6.0371	3.4191	3.8948	3.1242	4.2894
ADF Test for stationarity	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
statistics	-16.2077	-20.9006	-10.7325	-18.3081	-16.9912	-13.6207	-10.1823	-15.4013	-17.1858	-14.5103	-20.6081	-19.5647	-11.5729	-18.6114	-20.6241	-23.1814	-10.7590	-20.0136	-15.8868	-10.8161
PP Test for stationarity	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
statistics	-16.2077	-20.9006	-10.7325	-18.3081	-16.9912	-13.6207	-10.1823	-15.4013	-17.1858	-14.5103	-20.6081	-19.5647	-11.5729	-18.6114	-20.6241	-23.1814	-10.7590	-20.0136	-15.8868	-10.8161
JB Test for normality	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	0	0	1	1	1	1	1	0	0	1	0	1	1	1	0	1	1	0	0	1
p value	0.5000	0.5000	0.0010	0.0010	0.0114	0.0058	0.0404	0.5000	0.4066	0.0055	0.5000	0.0010	0.0010	0.0010	0.2976	0.0010	0.0049	0.0616	0.4907	0.0110
statistics	0.5740	0.0824	828.6126	190.5806	11.5127	15.3793	6.2474	1.1400	1.4952	15.6464	0.1852	31.0311	1741.2480	191.7106	2.0235	57.7932	16.4386	5.0120	1.2247	11.6898
Lilliefors Test for normality	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	0	0	1	1	1	0	1	0	0	0	0	1	1	1	1	0	1	1	0	1
p value	0.4366	0.2546	0.0010	0.0010	0.0049	0.5000	0.0010	0.2645	0.5000	0.4254	0.4551	0.0470	0.0010	0.0010	0.0091	0.2057	0.0010	0.0048	0.5000	0.0067
statistics	0.0514	0.0584	0.3711	0.1263	0.0906	0.0395	0.1244	0.0579	0.0436	0.0517	0.0508	0.0742	0.3746	0.1202	0.0866	0.0607	0.1590	0.0909	0.0421	0.0887
Arch test for heteroskedasticity	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	0	1	1	0	1	1	0	0	1	1	0	1	1	1	0	0	0	1
p value	0.0000	0.0068	0.7560	0.0049	0.0002	0.2959	0.0260	0.0351	0.4299	0.2747	0.0000	0.0001	0.3687	0.0040	0.0000	0.0001	0.1422	0.0600	0.8732	0.0101
statistics	20.3836	7.3326	0.0966	7.9177	13.7994	1.0927	4.9558	4.4412	0.6230	1.1930	19.4442	14.9393	0.8081	8.2690	18.2354	15.2545	2.1535	3.5362	0.0255	6.6193
LBQ Test for autocorrelation	**EURUSD**										**GBPUSD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0
p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0002	0.0000	0.0762	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2073	0.7720
statistics	209.3622	78.7161	77.6213	62.2208	215.2367	51.7306	295.3327	50.3447	62.0701	29.6230	190.0080	68.8523	66.9752	67.0457	85.1113	71.4154	274.2245	63.8505	24.8473	15.0772

Explanations of the results:

ADF h=1 (p value)-stationary (no unit root); PP h=1 (p value)-stationary (no unit root);

JB h=0 (p value)-normal distribution; Lillie h=0 (p value)-normal distribution;

Arch h=1 (p value)-heteroskedastic; LBQ h=1 (p value)-autocorrelated.

Annex 3: Statistical tests for the fundamentals indexes of SEKUSD and SEKCAD

General statistics	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
Mean	0.0034	-0.0120	-0.0102	0.0276	0.0022	0.0361	0.0190	0.0084	0.0245	-0.0060	0.0032	0.0018	-0.0136	0.0009	-0.0639	0.0378	0.0238	0.0244	-0.0027	-0.0021
Std	0.6154	1.2330	0.2463	6.7180	0.0929	3.9790	0.4941	0.7222	2.6500	0.1334	0.7034	1.2096	0.2166	2.1912	56.5036	4.3363	0.3176	0.3438	5.6087	0.1106
Skew ness	0.2984	-0.2504	0.2110	0.5781	0.1632	-0.0019	-0.3155	0.0794	-0.4155	-0.2920	-0.3381	-0.1391	-1.1004	0.4428	-0.2714	0.1515	0.2010	0.6966	-0.0731	-0.4086
Kurtosis	3.5946	4.7830	10.8077	8.2286	5.5936	4.4463	2.7296	2.7970	3.6599	4.0734	3.6160	3.4762	7.9521	4.4390	17.1810	3.8219	2.8722	3.2235	2.8356	4.2552
ADF Test for stationarity	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.043153	0.001	0.001
statistics	-17.2692	-19.3788	-9.6454	-16.7718	-17.9432	-24.6337	-13.4316	-13.4005	-15.7662	-9.6009	-21.0858	-18.7377	-14.2035	-13.7076	-20.6292	-22.2835	-17.3271	-2.0078	-14.5506	-9.5320
PP Test for stationarity	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.043153	0.001	0.001
statistics	-17.2692	-19.3788	-9.6454	-16.7718	-17.9432	-24.6337	-13.4316	-13.4005	-15.7662	-9.6009	-21.0858	-18.7377	-14.2035	-13.7076	-20.6292	-22.2835	-17.3271	-2.0078	-14.5506	-9.5320
JB Test for normality	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	0	1	1	1	1	1	0	0	1	1	0	0	1	1	1	0	0	1	0	1
p value	0.0814	0.0026	0.0010	0.0010	0.0010	0.0089	0.1728	0.5000	0.0334	0.0190	0.0588	0.3278	0.0010	0.0042	0.0010	0.0702	0.5000	0.0100	0.5000	0.0076
statistics	4.3462	21.0083	374.4670	175.6373	41.8538	12.8125	2.8864	0.4067	6.8970	9.1456	5.1250	1.8630	179.8750	17.4881	1233.5512	4.7004	1.0903	12.1945	0.2966	13.7398
Lilliefors Test for normality	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	0	0	1	1	0	0	1	0	0	0	0	0	1	0	1	0	1	1	0	0
p value	0.1567	0.0898	0.0010	0.0010	0.0865	0.5000	0.0288	0.5000	0.2687	0.0585	0.5000	0.5000	0.0010	0.1893	0.0010	0.5000	0.0477	0.0040	0.1586	0.5000
statistics	0.0634	0.0687	0.3134	0.1158	0.0690	0.0477	0.0783	0.0412	0.0578	0.0724	0.0390	0.0496	0.3142	0.0615	0.3364	0.0345	0.0741	0.0920	0.0633	0.0467
Arch test for heteroskedasticity	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	0	0	1	1	0	1	0	0	1	1	1	0	1	1	1	1	0	0
p value	0.0023	0.0000	0.9548	0.0920	0.0000	0.0000	0.6390	0.0308	0.1198	0.5962	0.0014	0.0185	0.0005	0.8205	0.0000	0.0000	0.0050	0.0000	0.5022	0.4203
statistics	9.3168	19.9101	0.0032	2.8394	21.1194	28.1046	0.2201	4.6643	2.4200	0.2808	10.2062	5.5521	12.2859	0.0515	30.5595	20.0044	7.8663	116.1260	0.4502	0.6495
LBQ Test for autocorrelation	**SEKUSD**										**SEKCAD**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0
p value	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.1024	0.2021	0.0000	0.0000	0.0016	0.0092	0.0000	0.0000	0.0000	0.0000	0.1184	0.3345
statistics	120.7060	78.8248	103.8597	51.7651	60.5201	73.8515	88.2053	66.9499	28.3027	24.9822	131.3831	63.3694	43.7703	37.8577	56.4456	70.3648	57.1547	577.3422	27.6332	22.1104

Explanations of the results:

ADF h=1 (p value)-stationary (no unit root); PP h=1 (p value)-stationary (no unit root);

JB h=0 (p value)-normal distribution; Lillie h=0 (p value)-normal distribution;

Arch h=1 (p value)-heteroskedastic; LBQ h=1 (p value)-autocorrelated.

Annex 4: Statistical tests for the fundamentals indexes of CADEUR and CADGBP

General statistics	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
Mean	0.0001	-0.0042	0.0136	0.0778	-0.0003	-0.0368	-0.0306	-0.0222	-0.1449	0.0036	-0.0040	-0.0076	0.0170	0.0335	0.0000	-0.0123	-0.0170	-0.0113	-0.2694	-0.0049
Std	0.7367	0.8411	0.1623	0.7938	0.0627	3.3714	0.1674	0.1811	7.6562	0.1811	0.6912	0.8638	0.1866	1.1235	0.0007	3.1722	0.1718	0.5527	17.1723	0.1058
Skew ness	0.4796	0.2840	0.2491	0.1749	-0.0217	0.0056	-0.0541	-0.3404	-0.1742	-0.1934	0.4503	0.0640	3.3622	0.0757	0.4911	-0.2352	0.1593	0.0871	0.0837	0.0831
Kurtosis	3.3080	3.0916	5.0708	3.0458	33.7628	3.8073	3.3096	3.0690	3.8621	3.8104	3.4879	3.4622	29.0849	2.5802	7.9960	3.7211	3.1913	3.5125	2.9093	3.4566
ADF Test for stationarity	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.0411	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
statistics	-22.4118	-16.5287	-10.5763	-10.9163	-20.8770	-18.4833	-13.1012	-2.0287	-16.8873	-14.9679	-26.6225	-15.1225	-10.2623	-14.0322	-17.1659	-21.7510	-12.4396	-8.9834	-16.0219	-9.7057
PP Test for stationarity	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p value	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.0411	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
statistics	-22.4118	-16.5287	-10.5763	-10.9163	-20.8770	-18.4833	-13.1012	-2.0287	-16.8873	-14.9679	-26.6225	-15.1225	-10.2623	-14.0322	-17.1659	-21.7510	-12.4396	-8.9834	-16.0219	-9.7057
JB Test for normality	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	0	1	0	1	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0
p value	0.0408	0.2968	0.0012	0.5	0.001	0.0951	0.5	0.1748	0.0552	0.0635	0.0383	0.4290	0.001	0.4929	0.0010	0.0750	0.5000	0.3413	0.5	0.4190
statistics	6.2166	2.0278	27.7858	0.7627	5796.4218	3.9926	0.6589	2.8678	5.2953	4.9388	6.4258	1.4087	4444.5367	1.2199	158.7864	4.5403	0.8462	1.7943	0.2218	1.4463
Lilliefors Test for normality	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	0	0	1	0	1	0	1	0	0	0	1	0	1	0	1	0	1	0	0	0
p value	0.0986	0.0687	0.001	0.5	0.001	0.3418	0.0010	0.1416	0.5	0.4165	0.0243	0.5	0.001	0.5	0.0327	0.1526	0.0010	0.4332	0.0732	0.5000
statistics	0.0678	0.0710	0.3633	0.0493	0.4120	0.0546	0.1284	0.0644	0.0412	0.0520	0.0796	0.0471	0.3799	0.0484	0.0772	0.0637	0.1549	0.0515	0.0705	0.0491
Arch test for heteroskedasticity	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	0	0	0	1	1	0	1	0	1	1	0	0	0	1	1	0	1	0	0
p value	0.0030	0.3449	0.1293	0.6336	0.0000	0.0000	0.9948	0.0000	0.3466	0.0275	0.0004	0.4587	0.6307	0.5103	0.0000	0.0000	0.6579	0.0174	0.3086	0.2618
statistics	8.7996	0.8922	2.3005	0.2273	34.2955	25.8708	0.0000	122.5196	0.8858	4.8610	12.7764	0.5491	0.2311	0.4335	23.1757	27.4456	0.1961	5.6522	1.0366	1.2592
LBQ Test for autocorrelation	**CADEUR**										**CADGBP**									
	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND	CPI	PPI	INT	MS	TB	IPI	UN	LI	PMI	10Y-BOND
decision	1	1	1	1	1	1	0	1	1	1	1	0	1	0	1	1	0	1	0	0
p value	0.0000	0.0002	0.0036	0.0000	0.0000	0.0005	0.2815	0.0000	0.0060	0.0117	0.0000	0.0809	0.0198	0.0684	0.0232	0.0000	0.4828	0.0000	0.0640	0.0594
statistics	282.9756	50.6710	41.1285	62.5516	88.2457	47.4671	23.1506	550.4952	39.3403	37.0076	481.5608	29.3630	35.0509	30.0908	34.4584	72.7411	19.6055	130.6701	30.3767	30.6937

Explanations of the results:

ADF h=1 (p value)-stationary (no unit root); PP h=1 (p value)-stationary (no unit root);

JB h=0 (p value)-normal distribution; Lillie h=0 (p value)-normal distribution;

Arch h=1 (p value)-heteroskedastic; LBQ h=1 (p value)-autocorrelated.

Annex 5: Explanatory variables and their Bloomberg indices.

Country	Consumer Price Index	Producer Price Index	Interest rate	Monthly Supply	Trade Balance	Industrial Production Index	Unemployment rate	Leading Index	Purchasing Managers Index	10-Year Government Bond Yield
United Kingdom	UKRPCJMR The monthly growth rates represent the inflation rate.	UKPPIOC 2005 = 100 (rebased in Oct. 2008 from 2000=100)	UKBRBASE Overnight lending rate.	UKMSM41M The MS 4 is the total amount of money in circulation.	UKTBTTBA MIL. GBP Exports minus imports.	UKIPIMOM Percentage change of the volume of the production.	UKUEILOR Percentage which follows the ILO unemployment definition.	UKCBLIMM Month on month percentage.	LTSBBCNX % LLOYD BANK COMERCIAL BUSINESS BAROMETER	OEGBR006 Bond Market 10-Year Government Bond Yield
United States of America	CPI CHNG The monthly growth rates represent the inflation rate.	PPI CHNG Average changes in prices received by domestic producers of commodities.	FDTR A target interest rate set by the central bank to influence short-term interest rates.	M2% CHNG Monthly supply M2 as percentage change.	USTBTOT BIL. USD Exports minus imports.	IP CHNG 2007=100 Percentage change of the volume of the production.	USURTOTN Number of unemployed persons as the percentage of the labor force (the total number of employed plus unemployed).	LEI CHNG Month on month %. Includes economic variables that move before the overall economy.	NAPMPMI %. Around 300 supply management individuals which respond to a survey of the economic conditions.	OEUSR007 Bond Market 10-Year Government Bond Yield
Canada	CACPICHG 2002=100	CAIPMOM 2010=100 not seasonally adjusted.	CABROVER Target interest rate.	MSCAM3YY Monthly supply M3.	CATBTOTB BIL. CAD Exports minus imports.	CAMFCHNG Industrial/ MNFC Sales	CANLXEMR Number of unemployed/ % of total labor force.	OLE3CANA %. OECD Composite Leading Indicator.	IVEYPRIC %. ~300 purchasing managers-Survey.	OECAR009 10-Year Government Bond Yield
Sweden	SWCPMOM 1980=100	SWPPIMOM 2005=100 Month on Month - %.	SWRRATE Decision Rate-REPO Rate.	SWMSM3YY Monthly supply M3.	SWTBAL BIL. SEK Exports – imports.	SWIPIMOM 2010=100 Volume	SWUESART Number of unemployed/% of labor force.	OLE3SWED %Composite Leading Indicator.	SZPUI % BALANCE/ DIFFUSION RATE	OESER003 10-Year Government Bond Yield
Eurozone and some indexes are for EU 28.	ECCPEMUM The harmonised index of consumer prices, used primarily within the EU.	EUPPEMUM 2005=100 Total industry: Monthly % Changes, Seasonally Adjusted.	EURR002W The main refinancing rate is the rate for the Eurosystem's regular open market operations.	ECMSM3YY Monthly supply M3.	XTTBEZ MIL. EUR Exports minus imports.	EUIPEMUY Percentage change of the volume of the production.	UMRTEMU Number of unemployed/% of labor force.	OLE3EURA % change. Includes economic variables that tend to move before the overall economy.	EUICUK %. The managers of 1500 companies respond to a survey.	GECU10YR The rates are comprised of Generic Euro government bills and bonds.

The description of all the variables starts with the **index name** from **Bloomberg** database.
The method of calculation is the second information which follows by a short description.